Full Wave Analysis Of Narrow Slot In The Ground Plane Of The Microstrip Line

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Abstract – A general approach to derive a solution method for the problem of microstrip line with an inclined slot in the ground plane is presented. The formulation is based on the Galerkin's method to find the unknown electric field which is generated in the slot area. Using the spectral domain transformation, reciprocity theorem and some other techniques such as integral transformation, a full wave solution for this configuration is obtained. Effects of various parameters such as slot dimensions and inclination are studied. Finally computed results are compared with measurements and previously computed data.

1 Introduction

The configuration of the problem is depicted in the Fig. 1, where the slot in the ground plane has an arbitrary angle with the feed line (θ_s).

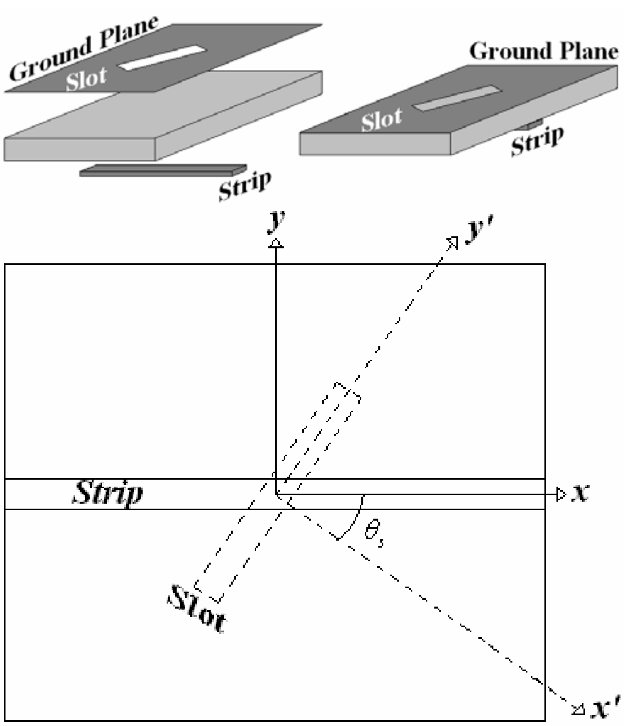

Fig. 1: Geometry of an inclined slot in the ground plane of the microstrip line

This configuration has some useful applications in the microwave devices such as filters, patch antennas, phase shifters, wide band couplers, hybrids, slot-coupled double layer microstrip lines, etc.

In the past researches, there has been some analysis dealing with the problems related to this configuration. Analysis of a slot with 90° inclination using the moment method is reported by Pozar [1], in which only transverse modal fields of the microstrip line have been considered. However in the inclined configuration, longitudinal component of the magnetic modal field is playing an essential role in the generation of the electric field in the slot region. Therefore applying this method for an inclined slot would generate non-accurate results. In the reference [2] the longitudinal component of the magnetic field has been considered, but the effect of the TE and TM poles in the microstrip Green's function has not been considered in the computation of the final integrals and residues. The TM poles have a zero cut off frequency while TE poles appear in the higher frequencies. Therefore at least one TM pole exists in the Green's function. This pole has a negligible effect on the results in the lower frequencies, so we have good results in these frequencies without considering this pole as could be seen in [2]. However for the higher frequencies, not only the effect of the first TM pole increases but also some other TE and TM poles appear in the Green's function so for obtaining more accurate results in the higher frequencies we need to consider the effect of both TE and TM poles in the computation of the integrals and residues.

In this paper, exact Green's function of a grounded dielectric slab is used in a moment method solution to find the unknown electric field in the slot region. Similar to [1] and [2], reciprocity theorem results in a relationship between the electric field of the slot and the transmission properties of the microstrip line. Finally, results are compared with the measured and previously computed data. Effects of the slot inclination, width and length on the transmission properties are discussed.

2 Formulation

Assume a microstrip line with the strip width of W_m, substrate dielectric thickness of H and relative dielectric constant of ε_r. The ground plane and strip both assumed to be perfect conductors. The structure is infinite in the both x and y directions and the strip is extended from $-\infty$ to $+\infty$, in the x direction. Slot is etched on the ground plane. Slot's width (W_s), Length (L_s) and inclination to the feed line (θ_s) are shown in the figures 1 and 2. Coordinate systems (x,y,z) and (x',y',z) are placed in the center of the slot, as shown in the Fig. 1.

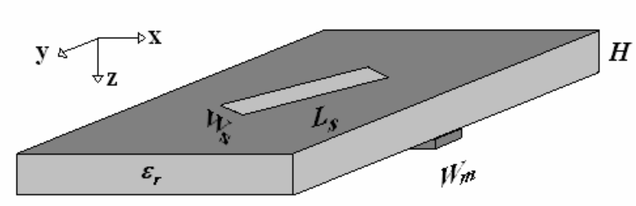

Fig. 2: Inclined slot etched in the ground plane of the microstrip line

Now, imagine an auxiliary source placed at $x = -\infty$ that creates the positive propagating waves. These waves are propagated along the x direction in the perfect microstrip configuration until reach to the slot. Presence of the slot makes some changes to the waves, because of the non-matched condition for the fields in the slot region; Therefore part of the electromagnetic field is reflected and goes back to the source (Γ), part of it is radiated in the free space below the slot and the remaining of the field is propagated in the positive x direction and runs away from the slot (T). This produces some evanescent fields near the slot and a transverse electric field in the slot area which is essential for satisfying the boundary conditions in the slot.

If the modal components of the microstrip are represented by $\vec{e}^{\,+}, \vec{h}^{+}$ for the positive propagating waves and $\vec{e}^{\,-}, \vec{h}^{-}$ for the negative propagating waves, one could say that in the microstrip line, the fields before the slot ($x \ll 0$) are a combination of the positive and negative propagating waves but after the slot ($x \gg 0$), there is only the positive propagating waves. Therefore the field of the feed line in the microstrip-slot structure can be represented by the following expressions:

$$\vec{E}(x,y,z) = \begin{cases} \vec{e}^{\,+} e^{-j\beta x} + \Gamma \vec{e}^{\,-} e^{+j\beta x} & : x<0 \\ \mathrm{T}\vec{e}^{\,+} e^{-j\beta x} & : x>0 \end{cases} \tag{1}$$

$$\vec{H}(x,y,z) = \begin{cases} \vec{h}^{+} e^{-j\beta x} + \Gamma \vec{h}^{-} e^{+j\beta x} & : x<0 \\ \mathrm{T}\vec{h}^{+} e^{-j\beta x} & : x>0 \end{cases} \tag{2}$$

Where Γ and T are the unknown reflection and transmission coefficients of the slot and β is the propagation constant of the modal fields in the microstrip line. The time dependence $e^{j\omega t}$ is assumed for all components. The modal fields can be achieved using the spectral domain approach and the equivalent transmission line model for the microstrip line [3]. Expressions for β and the modal components are presented in the Appendix. The modal fields normalized so that:

$$\int_{z=0}^{z=+\infty} \int_{y=-\infty}^{y=+\infty} \left(\vec{e}^{\,+} \times \vec{h}^{+} \right) \bullet \left(\hat{x} dy dz \right) = 1 \tag{3}$$

Using the reciprocity theorem similar to which has been used in the reference [1] and [2], results:

$$\Gamma = \tfrac{1}{2} \iint_{Slot} \left(\vec{E}_{slot} \times \vec{h}^{+} e^{-j\beta x} \right) \bullet \left(\vec{ds}_z \right) \tag{4}$$

$$T = \tfrac{1}{2} \iint_{Slot} \left(\vec{E}_{slot} \times \vec{h}^{-} e^{+j\beta x} \right) \bullet \left(\vec{ds}_z \right) + 1 \tag{5}$$

$$\vec{ds}_z = -\hat{z} dx dy \tag{6}$$

E_{slot} is the unknown electric field of the slot. As can be seen in the above equations, the unknowns Γ and T are related to the E_{slot}. Therefore the problem is finding the unknown electric field of the slot. By applying the method moments, E_{slot} could be computed.

If the slot is sufficiently narrow, the electric field of the slot only has a transverse component. In the other words, in the slot region, only an electric field in the x' direction exists and the y' component of the electric field of the slot is negligible. Therefore this field could be represented by:

$$\vec{E}_{slot} = E_{slot}\ \hat{x}' \tag{7}$$

By this assumption, the problem is finding a scalar function E_{slot} in the Eq. (7). For finding E_{slot} an auxiliary equation is needed. This equation is obtained by satisfying the boundary condition for the magnetic field of the slot. In the slot area two types of the fields exist:

1. One type of the field is the field which is in the form of the microstrip modal field. This type only exists in the z>0 region and its magnetic field in the slot ($z=0^+$) is represented by H^f. This type of the field never creates any electric field in the slot area.
2. The other type of the field is the field which is not propagative in the microstrip line and only exists near the slot. In the both upper and lower regions (z>0 and z<0) this type of evanescent field exists and presence of this type is the source of the electric field which is generated in the slot (E_{slot}). In the slot area, the magnetic field of this type is represented by H_m, H_m^+ for $z=0^+$ and H_m^- for $z=0^-$.

Therefore in the upper region (z>0), the magnetic field is the summation of the feed line and the slot's fields, while in the lower region (z<0), there is only the field of the slot:

$$\vec{H}(x,y,z=0^+) = \vec{H}_m^+(x,y) + \vec{H}^f(x,y) \tag{8}$$

$$\vec{H}(x,y,z=0^-) = \vec{H}_m^-(x,y) \tag{9}$$

In the slot, the continuity of the transverse component of the magnetic field must be satisfied so that:

$$H_{y'}^f(x,y) + H_{my'}^+(x,y) - H_{my'}^-(x,y) = 0 \tag{10}$$

This relation is valid only in the slot area. H^f is the magnetic field of the feed line which is a function of the modal fields, Γ and T. Modal fields are known from the spectral domain solution of the microstrip line (See Appendix) while Γ and T are related to the unknown E_{slot}

(See Eq. (4) and Eq. (5)). Therefore H^f is could be represented as a function of E_{slot}.

If E_{slot} is known, the use of the Green's function of the grounded dielectric slab determines the magnetic field generated by E_{slot} (which is defined as H_m). Therefore in the Eq. (10), the magnetic field H_m can be related to the unknown E_{slot} by using the Green's function.

Since both H^f and H_m can be related to the E_{slot}, Eq. (10) can be expressed by an equation consisting of E_{slot}. This is the key point to find the unknown electric field of the slot. The magnetic components H_m and H^f can be expressed by:

$$\Delta H^{m}_{y'}(x,y) = H^{+}_{my'}(x,y) - H^{-}_{my'}(x,y) = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \tilde{G}_{HM}(k_x,k_y)\tilde{E}_{slot}(k_x,k_y)\, e^{-jk_x x} e^{-jk_y y}\, dk_x\, dk_y \qquad (11)$$

$$H^{f}_{y'}(x,y) = H^{f}_{x}(x,y,0)\sin(\theta_s) + H^{f}_{y}(x,y,0)\cos(\theta_s) \qquad (12)$$

$\tilde{G}_{HM}$ and $\tilde{E}_{slot}$ are the spectral domain transformation of G_{HM} and E_{slot} where G_{HM} is obtained from the Green's function of the grounded dielectric slab (See Appendix). H^f_x and H^f_y are the magnetic field components of the feed line:

$$H^{f}_{x}(x,y) = \begin{cases} h_x(y,0)e^{-j\beta x} + \Gamma h_x(y,0)e^{+j\beta x} & : x<0 \\ Th_x(y,0)e^{-j\beta x} & : x>0 \end{cases} \qquad (13)$$

$$H^{f}_{y}(x,y) = \begin{cases} h_y(y,0)e^{-j\beta x} - \Gamma h_y(y,0)e^{+j\beta x} & : x<0 \\ Th_y(y,0)e^{-j\beta x} & : x>0 \end{cases} \qquad (14)$$

h_x and h_y are the modal components of the magnetic field of the microstrip line.

In order to solve the problem by the method of moments, the electric field of the slot is expressed by a summation of the piecewise sinusoidal basis functions. By using these functions, accurate results can be achieved even by expanding the unknown field only by one term. The form of the basis functions is shown in the figure 3:

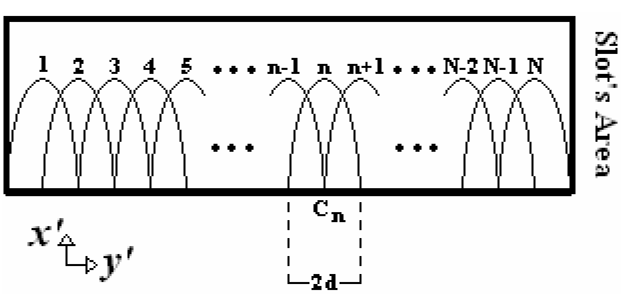

Fig. 3: Definition of the basis functions for the slot

Each basis function has a definition domain of 2d in the y' direction and its domain in the x' direction is W_s (as shown in the Fig. 3 and Eq. (16)). Each basis function has a center of definition which is shown by C_n for n^{th} basis function. The number of the basis functions is limited to N and E_{slot} is expressed by:

$$E_{slot}(x',y') = \sum_{n=1}^{N} a_n f_n(x',y') \qquad (15)$$

Where a_1 to a_N are unknown coefficients and f_n is:

$$f_n(x',y') = \begin{cases} \dfrac{\sin[k_e(d-|y'-C_n|)]}{W_s \sin[k_e d]} & : \begin{cases} |x'| < \frac{W_S}{2} \\ |y'-C_n| < d \end{cases} \\ 0 & : \text{else where} \end{cases} \qquad (16)$$

In the above equation C_n, d and k_e are defined as:

$$d = \frac{L_s}{N+1}\ ,\ C_n = -\frac{L_s}{2} + nd\ ,\ k_e = k_0\sqrt{\frac{\varepsilon_r+1}{2}} \qquad (17)$$

k_0 is the free space wave number. To find the unknown coefficients Eq. (10) is used in the following integral form (Galerkin's method):

$$\iint_{slot} [H^{f}_{y'}(x,y) + \Delta H^{m}_{y'}(x,y)] f_m(x,y)\,dxdy = 0 \qquad (18)$$

The above equation is repeated for m=1 to N and this procedure results N independent equations on the electric field of the slot (E_{slot}). Replacing E_{slot} by the expansion of Eq. (15), results in a system of equations that can be used to find unknown coefficients (a_1 to a_N). Elements of this system are some integrals which are presented in the following equations:

$$\begin{bmatrix} D_{1,1} & D_{1,2} & D_{1,3} & \dots & D_{1,N} \\ D_{2,1} & D_{2,2} & D_{2,3} & \dots & D_{2,N} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ D_{N,1} & D_{N,2} & D_{N,3} & \dots & D_{N,N} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_N \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_N \end{bmatrix} \qquad (19)$$

Where:

$$D_{m,n} = G_n \times HG_m + T_n \times HT_m + \Delta H_{m,n} \qquad (20)$$

$$E_m = -HC_m - HT_m \qquad (21)$$

So that:

$$\Delta H_{m,n} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (2\pi)^2 \tilde{G}_{HM}(k_x,k_y)\tilde{f}_n(k_x,k_y)\tilde{f}_m(-k_x,-k_y)\,dk_y\,dk_x \qquad (22)$$

$$G_n = \int_{-X_m}^{+X_m}\int_{-Y_m}^{+Y_m} -\tfrac{1}{2}\left[h_x \sin(\theta_s) + h_y \cos(\theta_s)\right] f_n\, e^{-j\beta x}\,dydx \qquad (23)$$

$$T_n = \int_{-X_m}^{+X_m}\int_{-Y_m}^{+Y_m} -\tfrac{1}{2}\left[h_x \sin(\theta_s) - h_y \cos(\theta_s)\right] f_n\, e^{+j\beta x}\,dydx \qquad (24)$$

$$HG_m = \int_{-X_m}^{0}\int_{-Y_m}^{+Y_m} \left[h_x \sin(\theta_s) - h_y \cos(\theta_s)\right] f_m\, e^{+j\beta x}\,dydx \qquad (25)$$

$$HT_m = \int_{0}^{+X_m}\int_{-Y_m}^{+Y_m} \left[h_x \sin(\theta_s) + h_y \cos(\theta_s)\right] f_m\, e^{-j\beta x}\,dydx \qquad (26)$$

$$HC_m = \int_{-X_m}^{0}\int_{-Y_m}^{+Y_m} \left[h_x \sin(\theta_s) + h_y \cos(\theta_s)\right] f_m\, e^{-j\beta x}\,dydx \qquad (27)$$

X_m and Y_m are defined by the following equations:

$$X_m = \tfrac{1}{2}\sqrt{L_s^2 + W_s^2}\,\sin(\theta_s + \tan^{-1}(\tfrac{W_s}{L_s}))$$
$$Y_m = \tfrac{1}{2}\sqrt{L_s^2 + W_s^2}\,\cos(\theta_s - \tan^{-1}(\tfrac{W_s}{L_s})) \tag{28}$$

Solving these equations, lead to the finding of the unknown coefficients and the electric field of the slot, as well. By determination of the electric field of the slot, Γ and T are obtained by using equations (4) and (5).

$$\Gamma = \sum_{n=1}^{N} a_n \times G_n \quad , \quad \mathrm{T} = 1 + \sum_{n=1}^{N} a_n \times \mathrm{T}_n \tag{29}$$

$\tilde{G}_{HM}$ has some TE and TM poles. $\Delta H_{m,n}$ is a two dimensional spectral domain integral of this function with infinite integral limits. Therefore integrand of this integral has some TM and TE poles in the spectral domain space. As it is said, these poles must be considered for achieving more accurate results. For computation of these integrals the method of [4] is used with some changes and the effect of the mentioned poles is considered in the computation of the integrals. All the other integrals (Eqs. 23 to 27) are defined in the space domain so they could be computed by conventional methods.

3 Numerical Results

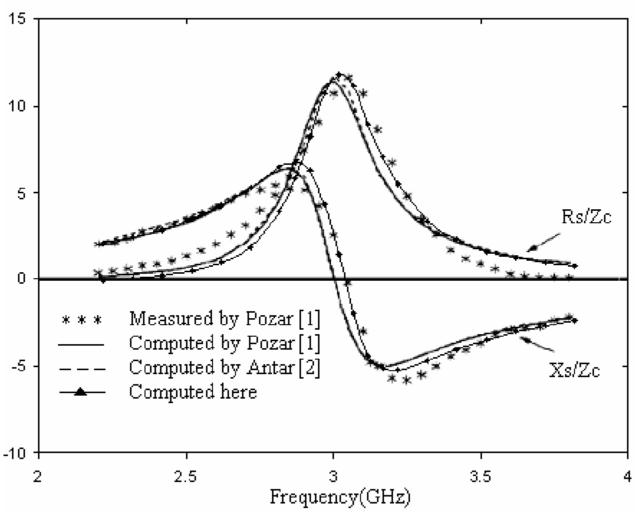

Fig. 4: Measured and computed results for micro strip-slot configuration with ε_r = 2.20, H = 1.6 mm, L_s = 40.2 mm, W_s = 0. 7 mm, W_m = 5 mm, θ_s =0

In order to compare the results, problem of the [1] and [2] is solved by presented method. Pozar reported the measured and computed results by an equivalent circuit for this configuration. Equivalent circuit consists of a series impedance which its normalized value to the microstrip characteristic impedance is represented by $\overline{R} + j\overline{X}$. The equivalent circuit is shown in the figure 5. For this equivalent circuit $\overline{R}$ and $\overline{X}$ are related to the Γ as follows:

$$\overline{Z} = \overline{R} + j\overline{X} = \frac{2\Gamma}{1 - \Gamma} \tag{30}$$

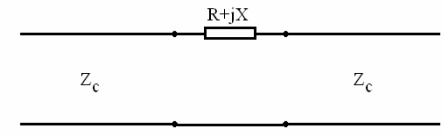

Fig. 5: Equivalent circuit of the slot in the ground plane

Measured and calculated data is shown in the Fig. 4. Computed data is the result of three term expansion of the electric field of the slot (N=3), good accuracy compared to the previously computed results, is achieved only by considering three basis function for the slot.

Finally the effect of the slot inclination on the transmission properties of the slot is discussed. In the Figure 6 reflection coefficient of the slot is depicted for various inclinations. By increasing θ_s, Γ decreases and the resonant frequency increases. An effective orthogonal length for the slot could be defined such that it is proportional to the image of the slot on a vector which is perpendicular to the strip of the feed line. The resonant frequency and the coupling coefficient of the slot with the free space (magnitude of Γ), is directly determined by this effective length. Since, the effective length is decreases, for example by increasing the θ_s, the resonant frequency increases and the coupling factor decreases. Variation of the slot's length results the variation of the effective length in a similar manner. Therefore by decreasing in the length of the slot, the resonant frequency increases and the coupling factor decreases, as well.

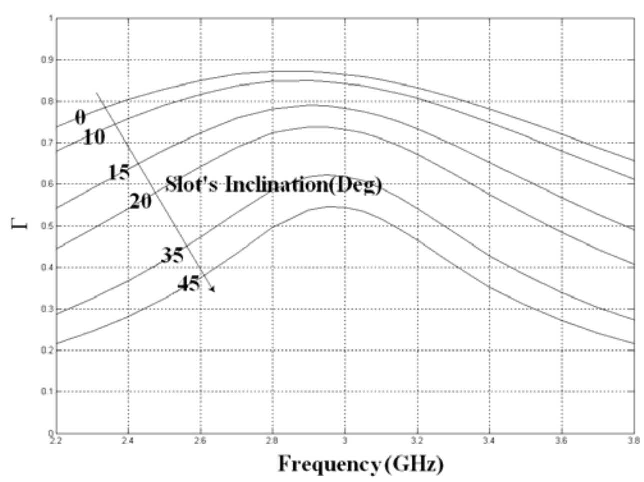

Fig. 6: Variation of the reflection coefficient of the microstrip-slot structure for various inclinations

4 CONCLUSION

In this paper, a full wave method has been presented for analyzing the slot discontinuities in the ground plane of the microstrip line. The method is based on the using of non-TEM modes for the microstrip line which gives the ability of the accurate simulation of the problem, for an arbitrary inclined slot in the ground plane. By using this method, very good agreement with the previously measured and computed data is achieved. Considering the effect of TE and TM poles of the Green's function in the calculation of the integrals, introduces an accurate method which could be even applied to the higher frequency problems.

5 APPENDIX

Using the spectral domain techniques lead to finding a transmission line model for solving the microstrip structure and computing the modal fields [3]. In the dielectric slab, the magnetic modal field components, for the microstrip line are:

$$\tilde{h}_x(k_y,z)=\tilde{f}_1\times\left\{\tilde{f}_2-\tilde{f}_3\right\} \tag{A1}$$

$$\tilde{h}_y(k_y,z)=\tilde{f}_1\times\left\{\tilde{f}_2\times\left(\frac{k_y}{\beta}\right)+\tilde{f}_3\times\left(\frac{\beta}{k_y}\right)\right\} \tag{A2}$$

$$\tilde{h}_z(k_y,z)=\tilde{f}_1\times\tilde{f}_2\times\frac{j\left(\beta^2+k_y^2\right)}{\beta k_{z2}}\tan(K_{z2}z) \tag{A3}$$

$$\tilde{f}_1=\frac{\beta I_0}{\pi W_m}\times\frac{1}{\beta^2+k_y^2}\times\sin(k_y\frac{W_m}{2})\times\frac{\cos(k_{Z2}z)}{\cos(k_{Z2}H)} \tag{A4}$$

$$\tilde{f}_2=\frac{k_{Z2}}{k_{Z2}+jk_{Z1}\tan(k_{Z2}H)} \tag{A5}$$

$$\tilde{f}_3=\frac{k_{Z1}\varepsilon_r}{k_{Z1}\varepsilon_r+jk_{Z2}\tan(k_{Z2}H)} \tag{A6}$$

Where k_{Z2} and k_{Z1} are:

$$k_{Z1}^2=k_0^2-\beta^2-k_y^2 \tag{A7}$$

$$k_{Z2}^2=k_0^2\varepsilon_r-\beta^2-k_y^2 \tag{A8}$$

By these equations the spectral domain functions of the magnetic modal fields are known. By using the Fourier transformation integrals, one could obtain the space domain functions for the magnetic modal fields. Modal electric fields are obtained from the following equation:

$$\vec{E}=\frac{\nabla\times\vec{H}}{j\omega\varepsilon} \tag{A9}$$

I_0 in the definition of the modal fields is the quasi static current that creates the modal fields. For normalization of the modal fields (as expressed in Eq. (3)) we must have:

$$I_0=\left(\sqrt{Z_C}\right)^{-1} \tag{A10}$$

Z_C is the characteristic impedance of the microstrip line. β and Z_C could be computed by any conventional method which is used to calculate the characteristic impedance and propagation constant of the microstrip line[5]. Using Green's function of grounded dielectric slab, results:

$$\tilde{G}_{HM}(k_x,k_y)=\frac{\left[j\omega k_x^2\left(\frac{1}{Y_{in}}+\frac{1}{Y_{01}}\right)-\frac{1}{j\omega}k_y^2\left(\frac{1}{Z_{in}}+\frac{1}{Z_{01}}\right)\right]}{k_x^2+k_y^2} \tag{A11}$$

$$Y_{in}=Y_{02}\frac{Y_{01}+jY_{02}\tan(k_{Z2}H)}{Y_{02}+jY_{01}\tan(k_{Z2}H)} \tag{A12}$$

$$Z_{in}=Z_{02}\frac{Z_{01}+jZ_{02}\tan(k_{Z2}H)}{Z_{02}+jZ_{01}\tan(k_{Z2}H)} \tag{A13}$$

$$Y_{02}=\frac{jk_{Z2}}{\varepsilon_0\varepsilon_r},\ Y_{01}=\frac{jk_{Z1}}{\varepsilon_0},\ Z_{01}=\frac{j\mu_0}{k_{Z1}},\ Z_{02}=\frac{j\mu_0}{k_{Z2}} \tag{A14}$$

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