Equivalent Network of a Perfectly Matched Layer

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Abstract – *An absorbing boundary condition for computational electromagnetics called the perfectly matched layer (PML) was previously introduced [1]. In this paper, equivalent transmission line models for PML in both right-handed (RH) and left-handed (LH) materials are derived and proposed. Simulations employing the proposed model agree with those using the characteristic impedance of free space (η_0). These models have application in numerical methods for reducing the calculation complexity and simulation time.*

Keywords: Perfectly matched layer (PML), Transmission-line.

1 Introduction

The Perfectly Matched Layer (absorbing boundary condition) is a technique which simplifies free-space unbounded electromagnetic problems. Using a PML, problems involving non-perpendicular propagation may be easily solved in a single problem rather than two problems with the normal and tangential components.

Berenger devised the PML technique in [1], however, no model has yet been published for using this technique with transmission lines. In this paper we will investigate the transmission-line model of a perfectly matched layer (PML) for Left-Handed (LH) and Right-Handed (RH) materials to obtain faster computational results. We will use this method in one-dimensional transmission-lines and will compare the PML results with the characteristic impedance of the transmission-line. To verify that PML can provide the desired characteristic impedance with a limited number of layers, we will measure the input impedance of the PML region and it will be shown that by adjusting transmission-line parameters, it is possible that the PML behaves as the characteristic impedance showing no reflections at the material/PML boundary.

The paper is organized as follows. In Section 2, the PML transmission-line formulas and models are derived. In Section 3, the numerical experiments and simulations related to the PML transmission-lines equations are demonstrated. Section 4 presents a discussion on the obtained numerical results and enables us to prove calculation-time saving in our optimized PML modeling.

2 Theory and Formulation

The two-dimensional equations describing the PML for TE (transverse electric) modes are given in this section. In Cartesian coordinates, we consider a problem that is without variation along the *z*-axis, with the electric field lying in the (x, y) plane. The electromagnetic field has three components: E_x, E_y, and H_z in a medium with an electric conductivity σ and a magnetic conductivity σ^*, Maxwell's equations can be written as [1]

$$\varepsilon_0 \frac{\partial E_x}{\partial t} + \sigma E_x = \frac{\partial H_z}{\partial y} \tag{1.a}$$

$$\varepsilon_0 \frac{\partial E_y}{\partial t} + \sigma E_y = -\frac{\partial H_z}{\partial x} \tag{1.b}$$

$$\mu_0 \frac{\partial H_z}{\partial t} + \sigma^* H_z = \frac{\partial E_x}{\partial y} - \frac{\partial E_y}{\partial x}\,. \tag{1.c}$$

With the condition:

$$\frac{\sigma}{\varepsilon_0} = \frac{\sigma^*}{\mu_0}\,, \tag{2}$$

the impedance of the medium equals that of vacuum and there will be no reflection at the interface.

2.1 PML for Right-Handed Materials

In this part, we change the TE mode PML formulas to TM and generate a transmission-line model for PML. Comparing the formulas with transmission-line equations [3], the one-dimensional transmission-line model for PML in Figure 1 is obtained.

$$\mu_0 \frac{\partial H_x}{\partial t} + \sigma^* H_x = -\frac{\partial E_z}{\partial y} \tag{3.a}$$

$$\mu_0 \frac{\partial H_y}{\partial t} + \sigma^* H_y = \frac{\partial E_z}{\partial x} \qquad (3.b)$$

$$-\varepsilon_0 \frac{\partial E_z}{\partial t} - \sigma E_z = \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} . \qquad (3.c)$$

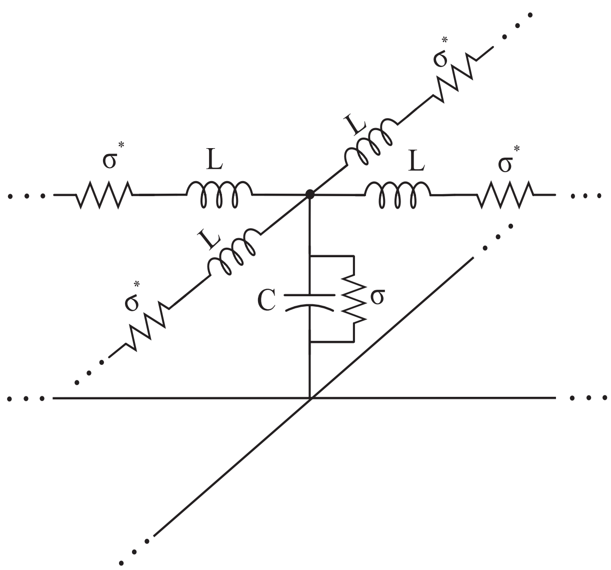

Figure 1. PML transmission line for RH materials

2.2 PML for Left-Handed Materials

Based on the transmission-line model for negative refractive index (NRI) materials [2], the transmission line model for left-handed PML is proposed in Figure 2.

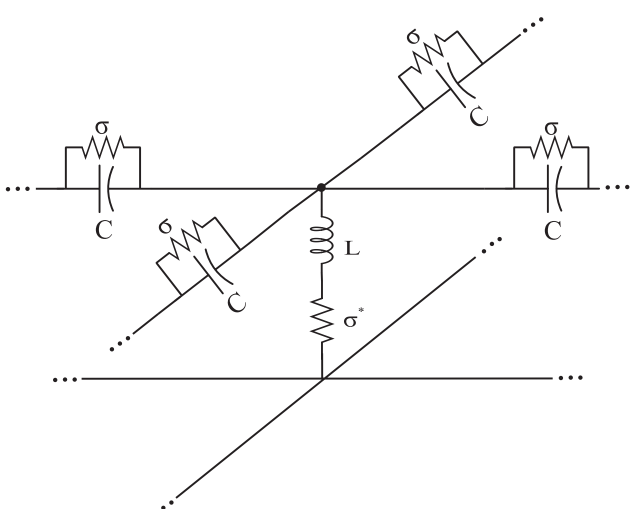

Figure 2. PML transmission line for LH materials

To prevent severe variations in the LH material, we have placed a layer of air between PML and the material. In order to have perfect absorption, the PML layer should have the characteristic impedance of free space. According to our LH PML modeling, characteristic impedance of PML transmission-line can be written as

$$z_c = \sqrt{\frac{j\omega\mu_0 + \sigma^*}{j\omega\varepsilon_0 + \sigma}} \qquad (4.a)$$

to match the impedances between air and PML, the ratio (4.b) should be satisfied. This ratio is similar to RH materials.

$$\frac{\sigma}{\varepsilon_0} = \frac{\sigma^*}{\mu_0} \qquad (4.b)$$

3 Numerical Experiments

In order to evaluate the relations and the models, presented in the previous section, the one-dimensional transmission-line models are simulated according to the diagram shown in Figure 3. An incident wave enters the material, then after passing through air, reaches the PML and is completely absorbed by the PML without any reflections.

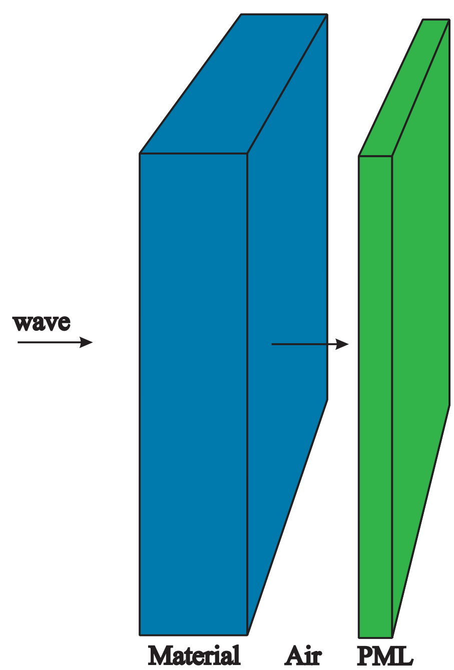

Figure 4. Diagram of the problem to be simulated

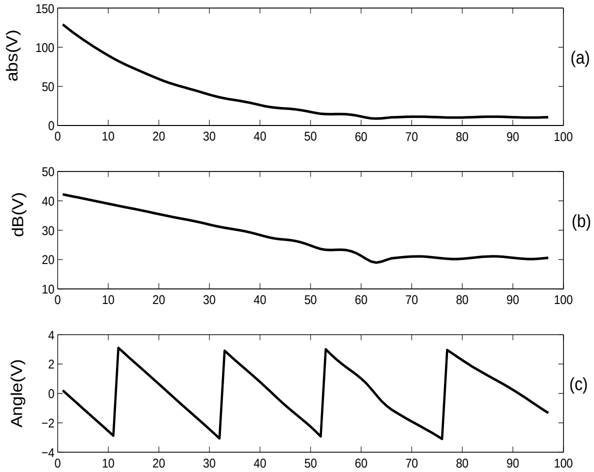

Figure 4. Magnitude and phase of voltage nodes of transmission-line ending in characteristic impedance

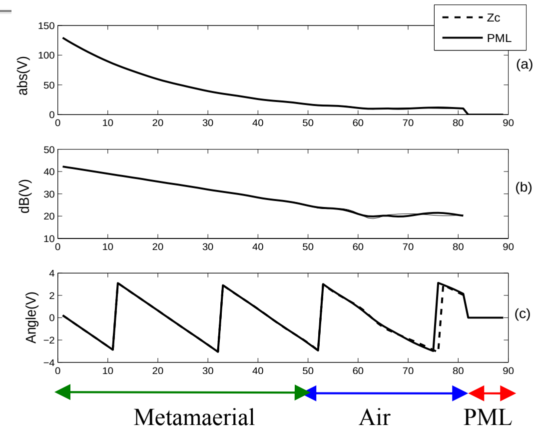

Figure 5. Magnitude and phase of voltage nodes of transmission-line ending in PML

Figure 4 shows a one-dimensional transmission-line terminated by an impedance equal to the line's characteristic impedance (η_0). This serves as the reference for validating our model. We have applied a sinusoidal voltage source to this transmission-line from the left side. In section 3.1 we will compare these results with those obtained using our PML model.

3.1 Numerical Results for Right-Handed Materials

The structure shown in Figure 3 was simulated with our model for the PML. The results are shown in Figure 5. Since the material is taken to be lossy, the propagating wave exhibits exponential attenuation as shown in Figure 5(a) and 5(b). After exiting the material and entering air, the wave no longer attenuates. Finally, after reaching the PML, it will die out immediately. Figure 5(c) shows that PML doesn't change the phase of the wave. Phase variations of the wave remain linear and without any distortion exactly like the wave propagation in free space. The layers' length in material, air and PML are 8, 2, and 1 cm respectively. The simulation frequency is 8 GHz. The number of layers in material, air, and PML are 64, 16, and 8 respectively. If we increase the thickness of the material layer, the PML will still be successful in absorbing the wave.

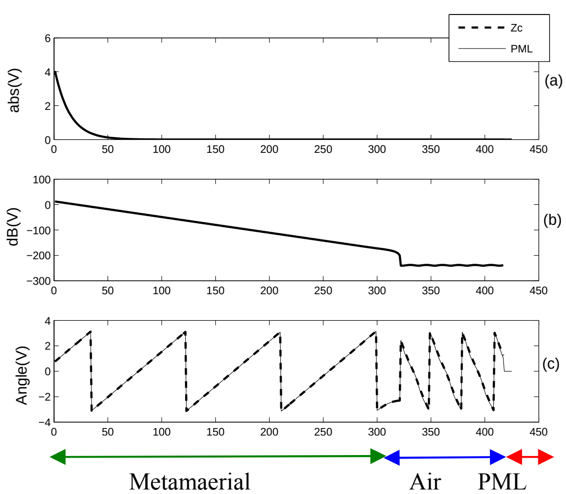

Figure 6. Magnitude and phase of voltage nodes of transmission-line ending in PML

3.2 Numerical Results for Left-Handed Materials

Figure 6(a) shows the same simulation results as Figure 5(a) but for left-handed material. Figure 6(c) clearly shows the change in the phase velocity in the transition from the left-handed material to the right-handed one. The phase variations of the wave remain linear and without distortion similar to wave propagation in free space. It also verifies the model proposed in the Figure 2 is correct for the left-handed materials.

3.3 Numerical Results for Input Impedance of the PML layer

For a suitable operating point for any given number of layers of PML, we must alter the parameters while satisfying equation (2) and (5) until the real part of the input impedance of the PML equals the characteristic impedance of free space (η_0) and the imaginary part is zero.

$$\sqrt{\frac{\mu}{\varepsilon}} = 377\Omega \tag{5}$$

In Figure 7, the real and imaginary parts of the input impedance are shown for a PML biased for 8 layers.

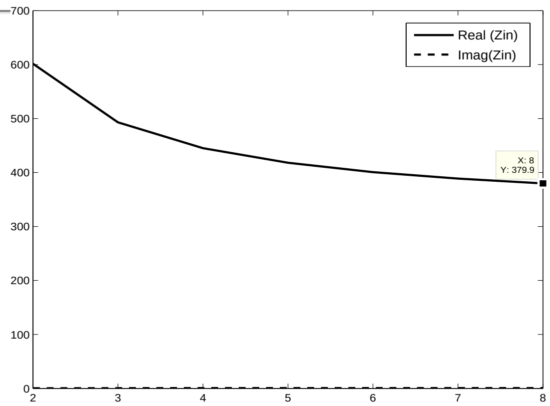

Figure 7. Real and Imaginary part of the input impedance in PML

3.4 Effect of Frequency on PML

The variations of real and imaginary part of the input impedance versus frequency and the number of layers are shown in the Figures 4 and 5. The results show stable behavior for different frequencies. The figures also show that the most suitable results are obtained when there are 8 layers. This is the same operating point for which we tuned the model parameters. By increasing the number of layers the simulation is still accurate but other than increasing the complexity and simulation time, no gain is obtained. Therefore our model can be used for high speed simulations involving LH and RH PML.

4 Conclusions

An equivalent network model for PML was devised for wave propagation in LH and RH one-dimensional materials. In this method, transmission-lines are responsible for modeling wave propagation in these materials. This method is absolutely convergent. With the help of this technique we can simulate free space with fewer calculations. We have shown the validity of our model by comparing it to a transmission line terminated by a matched load.

5 Acknowledgements

HM would like to acknowledge the members of the University of Tehran Antenna lab for their valuable advice and assistance.

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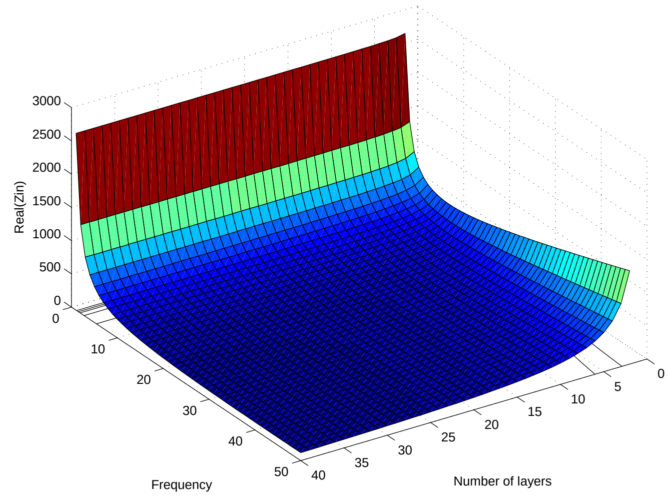

Figure 8. Real part of the input impedance in PML versus the number of the layers and the frequency

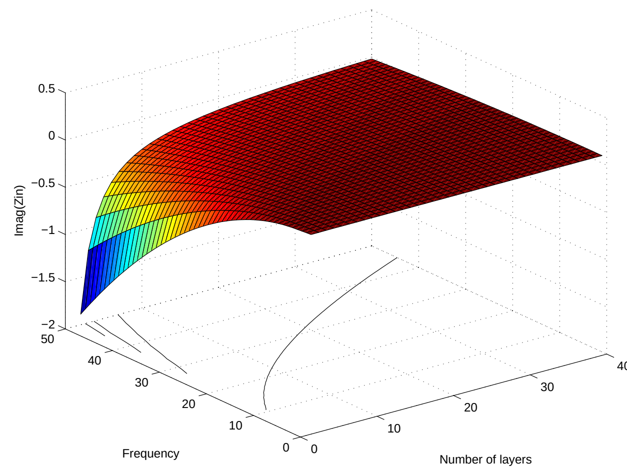

Figure 9. Imaginary part of the input impedance in PML versus the number of the layers and the frequency