



Based on the equivalent circuit of STATCOM shown in figure1 we can achieve an equation state space for STATCOM as below:

$$\frac{d}{dt} \begin{bmatrix} i_d \\ i_q \\ V_{dc} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & -\frac{D_d}{3L} \\ -\omega & -\frac{R}{L} & -\frac{D_q}{3L} \\ \frac{3}{2C}D_d & \frac{3}{2C}D_q & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ V_{dc} \end{bmatrix} + \begin{bmatrix} \frac{1}{3L} \\ 0 \\ 0 \end{bmatrix} V_m \quad (1)$$

From the model we can see that the states of the STATCOM dynamics loop are  $i_d, i_q, V_{dc}$ .  $V_m$  can be a system constant parameter. The control variables are  $D_d$  and  $D_q$ . Note that this is a nonlinear system. In the model, we selected the circuit parameters for the circuit shown in (1), and circuit parameters are selected as below:

$$R = 0.02\Omega; L = 2.8e-03H; C = 0.1F; V_m = 5882V; \omega = 377(\frac{rad}{s})$$

Desired steady states:

$$V_{dc} = 10000V$$

$$i_d = 0A$$

$$i_q = 178A$$

$$D_d = 0$$

$$D_q = 0.6445$$

The problem is how we can deal with the nonlinear characteristics in the controlled STATCOM. There are different methods, which can be used to control a nonlinear system. Since we know the operating points of the STATCOM we can use the linearization method to cope with this nonlinear problem. Matrixes linearized around the operating points of  $x^0$  and  $u^0$ , are shown below:

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= cx \end{aligned} \quad (2)$$

$$A = \left. \frac{\partial f}{\partial x} \right|_{(x^0, u^0)} \quad (3)$$

$$B = \left. \frac{\partial f}{\partial u} \right|_{(x^0, u^0)} \quad (4)$$

With operating points as below:

$$x^0 = \begin{bmatrix} i_{d0} \\ i_{q0} \\ V_{dc0} \end{bmatrix}, u^0 = \begin{bmatrix} D_{d0} \\ D_{q0} \end{bmatrix} \quad (5)$$

Here we compute the matrixes Jacobian as below:

$$A = \left. \frac{\partial f}{\partial x} \right|_{(x^0, u^0)} = \begin{bmatrix} -\frac{R}{L} & \omega & -\frac{D_{d0}}{3L} \\ -\omega & -\frac{R}{L} & -\frac{D_{q0}}{3L} \\ \frac{3D_{d0}}{2C} & \frac{3D_{q0}}{2C} & 0 \end{bmatrix} \quad (6)$$

$$B = \left. \frac{\partial f}{\partial u} \right|_{(x^0, u^0)} = \begin{bmatrix} -\frac{V_{dc0}}{3L} & 0 \\ 0 & -\frac{V_{dc0}}{3L} \\ \frac{3i_{d0}}{2C} & \frac{3i_{q0}}{2C} \end{bmatrix} \quad (7)$$

Equations state space that we want to control are as below:

$$\begin{bmatrix} i_d \\ i_q \\ V_{dc} \end{bmatrix} = \begin{bmatrix} -\frac{R}{L} & \omega & -\frac{D_{d0}}{3L} \\ -\omega & -\frac{R}{L} & -\frac{D_{q0}}{3L} \\ \frac{3D_{d0}}{2C} & \frac{3D_{q0}}{2C} & 0 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ V_{dc} \end{bmatrix} + \begin{bmatrix} -\frac{V_{dc0}}{3L} & 0 \\ 0 & -\frac{V_{dc0}}{3L} \\ \frac{3i_{d0}}{2C} & \frac{3i_{q0}}{2C} \end{bmatrix} u \quad (8)$$

$$Y = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_d \\ i_q \\ V_{dc} \end{bmatrix} \quad (9)$$

Operating points for STATCOM are:

$$\begin{aligned} V_{dc} &= 10000V; i_d = 0; i_q = 178A; \\ D_{d0} &= 0; D_{q0} = 0.6445; \end{aligned}$$

Thus we get

$$A = \begin{bmatrix} -7.1429 & 377.0000 & 0 \\ -377.0000 & -7.1429 & 9.6675 \\ 0 & 9.6675 & 0 \end{bmatrix}$$

$$B = 1.0e+006 \times \begin{bmatrix} -1.1905 & 0 \\ 0 & -1.1905 \\ 0 & 0.0027 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

All of variables can be measured from the power system, which means the system is observable. However we only have two independent control inputs and three system states. We need to see if the system is controllable. The controllability matrix is defined as below:

$$C_O = \begin{bmatrix} B & AB & A^2 B \end{bmatrix}$$

Thus controllability matrix  $C_0$  is

$$C_O = 1.0e+010 \times \begin{bmatrix} 9.9252 & 0.0001 & 0 \\ 0.0001 & 9.9165 & -0.0424 \\ 0 & -0.0424 & 1.0420 \end{bmatrix}$$

$$\det(C_O) = 1.0254 + 032$$

The open loop linearized system's eigenvalues are:

$$\Lambda = \begin{bmatrix} -7.12 + 3.7798i \\ -7.12 - 3.7798i \\ -0.04 \end{bmatrix} \times 1.0e-02$$

Since all the eigenvalues of the system are on the left hand of the plane, the system is stable. In the open loop system response shown in figure 2 there are large oscillations in  $i_d$  and  $i_q$  during 0.0-0.6sec and  $V_{dc}$  is not constant.

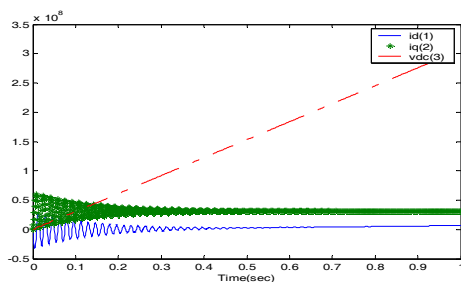


Figure 2: open loop response

Since the STATCOM is used for instantaneous reactive power compensation for power systems, the  $i_q$ 's transient response should be within one period of power system [4]. The frequency of the power system is 60 Hz, which means that one period is about 0.017sec. Thus the  $i_q$  transient response time should be less than 0.017sec. In this time the voltage of the capacitor  $V_{dc}$  should be kept constant [3]. The

control effort of  $D_d$  and  $D_q$  should be within  $[-1, +1]$ , see [5].

When we look at the open loop system response figure 2, we can see several dynamic characteristics need to be improved:

- 1) The eigen values of  $i_q$  and  $i_d$  are poorly damped.
- 2) The settling times of  $i_d$  and  $i_q$  are larger than the time required.
- 3) The overshoot of  $i_d$  and  $i_q$  is large.
- 4) The voltage of the capacitor  $V_{dc}$  doesn't remain around a steady constant value 10000v.

There are two main design methods. Pole assignment design and LQR design. we will discuss these two methods used in STATCOM controller design.

### 3 pole assignment controller design

In this method, closed loop pole selection is an important part of the design which should be correct and provide the desired steady state, and so the control effort should be within acceptable range.

#### 3.1 control algorithm

For a given system as below:

$$\dot{x} = Ax + Bu \quad (10)$$

$$Y = Cx \quad (11)$$

If we set the controller as

$$u(t) = -kx(t) \quad (12)$$

Then the closed loop state equation can be obtained as:

$$\dot{x} = [A - Bk]x(t) \quad (13)$$

The closed loop eigenvalues and eigenvectors are related by the equation:

$$[A - Bk]V_i = \lambda_i V_i \quad (14)$$

This equation can be put in the form

$$\begin{bmatrix} A - \lambda_i I & B \end{bmatrix} \begin{bmatrix} V_i \\ q_i \end{bmatrix} = 0 \text{ for } i = 1 \dots n$$

(15)

Where  $V_i$  is eigenvector and  $q_i = kV_i$  in order to satisfy (14) the vector  $[V_i^T \ q_i^T]^T$  must lie in the kernel or null space of the matrix

$$S(\lambda_i) = \begin{bmatrix} A - \lambda_i I & B \end{bmatrix} \quad (16)$$

The notation  $\ker S(\lambda_i)$  is used to define the null space which contains all the vectors  $[V_i^T \ q_i^T]^T$  for which (14) is satisfied  $q_i = kV_i$  can be used to form the matrix equality:

$$\begin{bmatrix} q_1 & q_2 & \dots & q_n \end{bmatrix} = \begin{bmatrix} kV_1 & kV_2 & \dots & kV_n \end{bmatrix} \quad (17)$$

Since  $k$  can be factorized from the right-hand matrix of Eq(17):

$$k = \begin{bmatrix} q_1 & q_2 & \dots & q_n \end{bmatrix} \begin{bmatrix} V_1 & V_2 & \dots & V_n \end{bmatrix}^{-1} \quad (18)$$

#### 4 LQR controller design

This method determines the feedback gain matrix that minimizes  $J$ , in order to achieve some compromise between the use of control effort, the magnitude, and the speed of response that will guarantee a stable system.

##### 4.1 control algorithm

For a given system

$$\dot{x} = Ax + Bu \quad (19)$$

Determine the matrix  $k$  of the LQR vector

$$u(t) = -kx(t) \quad (20)$$

So in order to minimize the cost function  $J$

$$J = \int_0^{\infty} (x^T Q x + u^T R u) dt \quad (21)$$

Where  $Q$  and  $R$  are the positive definite ltermition or real symmetric matrix. Thus we get a control law as:

$$u(t) = -kx(t) = -R^{-1} B^T P x(t) \quad (22)$$

In which  $P$  must be determined to solve the Riccati equation as:

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \quad (23)$$

#### 5 simulations

This section shows simulation results for STATCOM. The parameters used here are based on those of a practical STATCOM.

##### 5.1 simulation by using pole assignment method

Considerable open loop response, we need to reduce the over shoot of  $i_d$ , settling time of all three states and steady state error of  $V_{dc}$  further such that the requirement are satisfied. After trial and error we set the poles on  $[-2500, -2500, -300]$ . The system response is shown in figure 3,4,5. The overshoot of  $i_d$  is reduced to 140(A) and settling time for all three states is reduced to less than 0.017(sec) and  $V_{dc}$  is 10000(v). The control effort of  $D_d$  and  $D_q$  are within  $[-1, +1]$ .

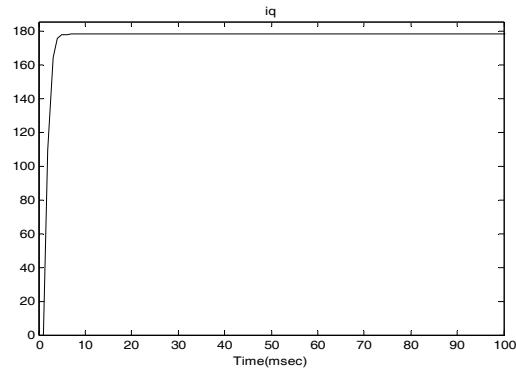


Figure3:  $i_q$  response when poles in  $[-2500, -2500, -300]$

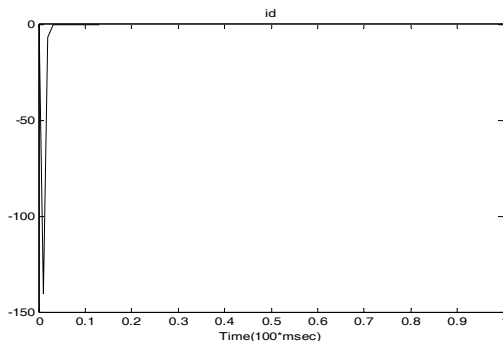


Figure4:  $i_d$  response when poles in  $[-2500, -2500, -300]$

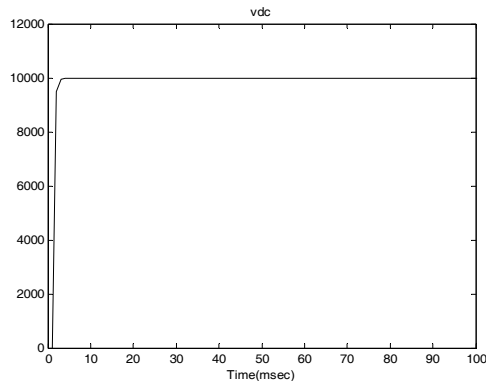


Figure 5:  $v_{dc}$  response when poles in  $[-2500, -2500, -300]$

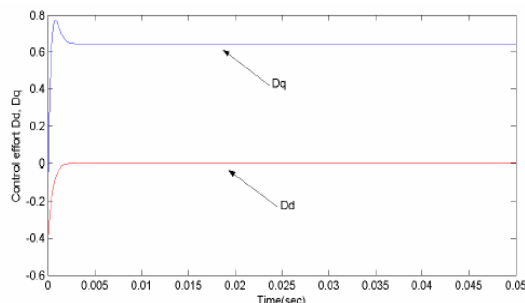


Figure 6: control effort when pole in  $[-2500, -2500, -300]$

## 5.2 simulation by using LQR method

In this method, we first select the diagonal elements of  $Q$  and  $R$  as  $\text{diag}[Q] = [0.005, 0.005, 1500]$  and  $\text{diag}[R] = [45, 100]$ . The system response is shown in figures 7, 8, 9. We can find that the responses  $i_d$  and  $i_q$  are perfect and  $V_{dc}$  converges to 10000(V) and so all three states are reduced to less than 0.017(sec). The control effort of  $D_d$  and  $D_q$  are within  $[-1, +1]$ .

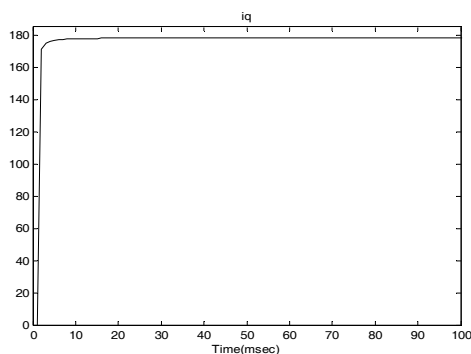


Figure 7:  $i_q$  response when  $\text{diag}[Q] = [0.005, 0.005, 1500]$  and  $\text{diag}[R] = [45, 100]$

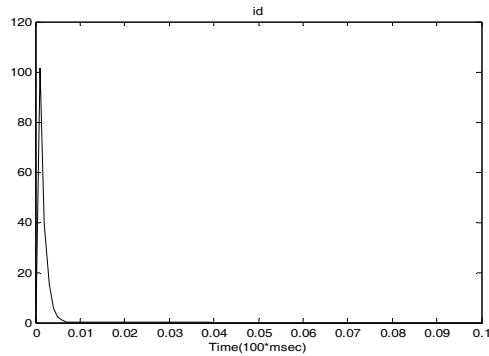


Figure 8:  $i_d$  response when  $\text{diag}[Q] = [0.005, 0.005, 1500]$  and  $\text{diag}[R] = [45, 100]$

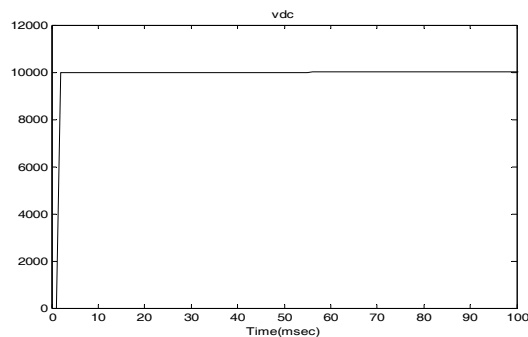


Figure 9:  $v_{dc}$  response when  $\text{diag}[Q] = [0.005, 0.005, 1500]$  and  $\text{diag}[R] = [45, 100]$

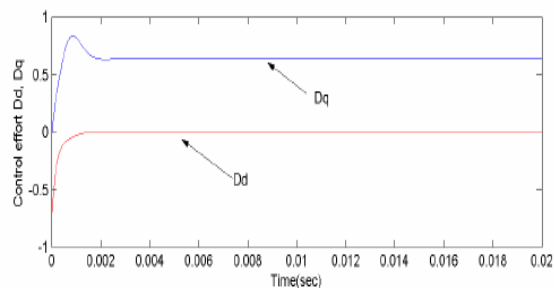


Figure 10: control effort when  $\text{diag}[Q] = [0.005, 0.005, 1500]$  and  $\text{diag}[R] = [45, 100]$

## conclusion

We investigated the dynamic model of a typical STATCOM. Its characteristics show that STATCOM is a nonlinear M.I.M.O system with a control effort saturation requirement. A linearization model method around the operating point can be applied to the nonlinear model to convert it into a linear model. Two kinds of feedback controllers (pole assignment and LQR) are designed. It was found that LQR method is not preferred to pole assignment. However, as a design method the determination of state feedback gains



are easier to obtain using LQR method by simulation observations, we can find that STATCOM is a device for reactive power compensation and a good stabilizer.

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