Robust Feedback Linearization Control for Uncertain Nonlinear Systems, Implemented on Helicopter Laboratory Model

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Abstract— *In this paper in order to suppress the effect of model uncertainties and disturbances, a robust feedback linearization control scheme for a large class of multi-input/multi-output nonlinear systems with unknown modeling terms based on Lyapunov function is proposed.*
To verify the validity and effectiveness of the designed method, the suggested technique has been applied to a Twin Rotor system. The results of computer simulations with MATLAB on the complete system and implementation on real model for various types of inputs and disturbances have been presented. The comparative study of these results with those obtained in nominal feedback linearization control architecture, state feedback method and PID controller establishes the performance of this new control scheme.

Keywords: Uncertain Nonlinear Systems, Robust Feedback Linearization, Twin Rotor.

1 Introduction

Feedback linearization is an approach to nonlinear control design which has attracted a great deal of research interest, but in the practice it suffers from two major limitations [1].

One shortcoming of this theory is due to that it relies on a precise model of the system for exact cancellation of nonlinear terms. Second problem comes from the fact that it requires certain structural and regularity conditions such as involutivity or existence of relative degree. Usually however, feedback linearization control does not guarantee exact linearization and robustness in the presence of uncertainties. In the control sense, two kinds of uncertainties could be considered: modeling uncertainties including unknown-model and environmental condition called disturbances and parametric uncertainties [2].

An often strategy to deal with model uncertainties is to introduce some kinds of integral action [3]. But in most cases it is not an easy task to prove that such control strategy yields robust regulation and signal tracking in the presence of uncertainties. Besides, there does not exist a clear procedure to tune the resulting controller [4].

In the case of parametric uncertainties, adaptive control has been used as a natural tool and interesting solution. In 1999 adaptive control design method for highly nonlinear systems has been used with feedback linearization to regulate the temperature of a bed reactor [4]. The strategy is an input-output linearizing feedback control scheme which involves an uncertainties dynamic estimator.

Considering the disabilities of adaptive methods like to matching condition, considerable progresses have been made in robust feedback linearization [5-8]. But most of them are applicable for single input nonlinear systems or just in the presence of parametric uncertainties.

In the past decades, fuzzy logic control, as one of the most useful approaches for collecting human knowledge and expertise appeared. Considering the disabilities explained before for adaptive and robust based methods, it has been used for plants that are mathematically poorly modeled or the model uncertainty in the dynamics is either unknown or impossible [9]. In some previous researches, adaptive techniques were applied and adaptive fuzzy feedback linearization methods were suggested to guarantee robustness [10-12].

Sliding control has been used as a secondary procedure with feedback linearization method for control of uncertain nonlinear systems. In [13] has been shown that when feedback linearization control failed to stabilize the uncertain system, by using a sliding mode control with an appropriate choice of the sliding surface, the uncertain system can be stabilized.

In this paper we propose a robust nonlinear controller for a class of MIMO nonlinear system based on feedback linearization approach. A robust term has been added based on lyapuanov function. The designed controller has been applied on Twin Rotor model two degrees of freedom (elevator and azimuth).

The work is organized as follows. Section 2 presents a brief review of the input-output feedback linearizationmethod . In section 3 the mathematical model of Twin Rotor has been proposed. In section 4 the controller design based on feedback linearization has been done.

Section 5 presents the modifications onthe basic feedback linearization method to suppress the effect of model uncertainties and unmeasured disturbances.

In section 6 the results of computer simulation using MATLAB and the implementation of the proposed controller on a Twin Rotor model (laboratory helicopter) are presented. Finally the work is closed with conclusions in section 7.

2 Feedback Linearization

The input-output feedback linearization technique is based upon linearized relationship between input and output. Without loss of generality, the multi-input (m) multi-output (m) nonlinear system of the form below is considered:

$$\dot{x} = f(x) + g_1(x)u_1 + \dots + g_m(x)u_m \tag{1}$$

$$\begin{cases} y_1 = h_1(x) \\ \dots \\ y_m = h_m(x) \end{cases}$$

where $x \in \Re^n$ is the state vector, f, g_i's are smooth vector fields and h_i's are smooth scalar functions in $\Re^m$. The control and output vectors are represented by $u=[u_1,\dots,u_m]^T$, $y=[y_1,\dots,y_m]^T$ (both $\in \Re^m$).

The input- output linearization of the system by equation (1) is achieved by differentiating the outputs y with respect to time until the inputs appear explicitly. Thus, by differentiating y, we have:

$$\dot{y}_j = L_f h_j + \sum_{i=1}^{m} (L_{g_i} h_j)u_i, \quad j = 1,\dots, m \tag{2}$$

If $L_{g_i} L_f^{(r_j-1)} h_j(x) = 0$ for all i, the inputs do not appear in equation (2) and further differentiation shall be repeated. Assume that r_j is the lowest integer such that at least one of the inputs will appear in $y_j^{(r_j)}$, which means:

$$y_j^{(r_j)} = L_f^{r_j} h_j + \sum_{i=1}^{m} L_{g_i}(L_f^{r_j-1} h_j)u_i \tag{3}$$

where $L_f^k h(x)$ is called the *Lie derivative* of $L_f^{k-1} h(x)$ along the vector field f, and it is assumed that for at least one i, $1 \le i \le m$, $L_{g_i} L_f^{r_j-1} h_j(x) \neq 0$ holds. This procedure is repeated for each output y_j. In the above derivation if above conditions hold at $x = x_0$, r_1 *to* r_m (correspond to y_1 *to* y_m) are relative degrees for the MIMO system defined by equation (1).

Define the matrix E as follow:

$$E(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1 & \dots & L_{g_m} L_f^{r_1-1} h_1 \\ \dots & & \\ L_{g_1} L_f^{r_m-1} h_m & \dots & L_{g_m} L_f^{r_m-1} h_m \end{bmatrix} \tag{4}$$

then the equations of (3) can be rewritten as follow:

$$\begin{bmatrix} y_1^{r_1} \\ \dots \\ y_m^{r_m} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(x) \\ \dots \\ L_f^{r_m} h_m(x) \end{bmatrix} + E(x) \begin{bmatrix} u_1 \\ \dots \\ u_m \end{bmatrix} \tag{5}$$

If E is nonsingular for a given point x_0, then the (decoupling) control input [1] can be written as:

$$u = -E^{-1} \begin{bmatrix} L_f^{r_1} h_1(x) \\ \dots \\ L_f^{r_m} h_m(x) \end{bmatrix} + E^{-1}(x) \begin{bmatrix} v_1 \\ \dots \\ v_m \end{bmatrix} \tag{6}$$

where $[v_1,\dots,v_m]^T$ are the new set of inputs which can be defined by designer. The resultant dynamics after applying equation (6) to MIMO system is given by,

$$\begin{bmatrix} y_1^{r_1} \\ \dots \\ y_m^{r_m} \end{bmatrix} = \begin{bmatrix} v_1 \\ \dots \\ v_m \end{bmatrix} \tag{7}$$

After achieving the decoupled and linear model, now each control goal could be accessible applying powerful linear control methods to equations (7) which are linear and decoupled.

3 Model Description of Twin Rotor

Among autonomous flying systems, helicopters have particularly interesting dynamic features. The main difficulties in designing controllers for them follow from nonlinearities and couplings [15]. Another problem is that the inputs are not directly applied torques or forces.

Twin Rotor laboratory system has been shown in figure .

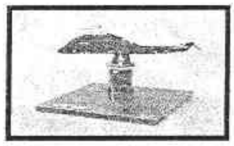

Figure 1: Sketch of the helicopter model

The model consists of a body carrying two DC motors which drive the propellers. The controls of the system are the supply voltages of the motors. Both body position angles, i.e. azimuth angle in horizontal plane and elevation angle in vertical plane are influenced by the rotating propellers simultaneously. The measured signals are the two position angles that determine the position of the beam in space. A dedicated I/O board allows for control, measurements and communication with a PC. The RT toolbox in the MATLAB environment is used to perform real-time experiments.

3.1 Helicopter Body Dynamics in the Horizontal Plane (Azimuth Subsystem)

Figure 2 shows a sketch of the helicopter body seen from the above.

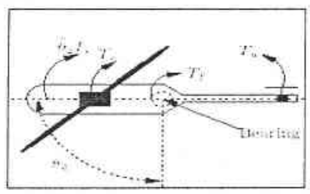

Figure 2: Twin Rotor seen from the above

By balancing the torques acting on the body in the horizontal plane, as shown in figure2, it can be written:

$$I_\psi \ddot{\varphi} = \tau_u - \tau_f - \tau_r \tag{8}$$

where,

ϕ : Azimuth angle (Rad), I_ψ : Moment of Inertia (kg m)

τ_f : Friction torque(Nm), τ_r : Main rotor reaction torque (Nm)

τ_u : Torque generated by the side rotor (Nm)

3.2 Helicopter Body Dynamics in the Vertical Plane (Elevator Subsystem)

Figure 3 shows a sketch of the helicopter body seen from the side.

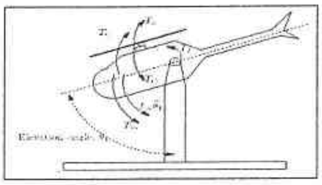

Figure 3: Sketch of the helicopter model seen from the side

By balancing the torques acting on the body in the vertical plane, as shown in figure3, it can be written:

$$I\ddot{\Psi} = \tau_u + \tau_c + \tau_G - \tau_f - \tau_m \tag{9}$$

where,

ψ : Elevation angle (Rad), τ_G: Gyroscopic torque (Nm),
τ_f: Friction torque (Nm), I_m : Moment of Inertia (kg m),
τ_u : Lift torque generated by the main rotor (Nm),
τ_c : Centrifugal torque (Nm),
τ_m : Torque generated by the mass of the body (Nm)

Combining the system denoted by equations (8), (9), motors, propeller and sensors models yields the complete helicopter body dynamics [14] in form $\overset{\circ}{x} = f(x,u)$, $y = g(x)$:

$$f(x,u) = \begin{bmatrix} x_2 \\ \frac{1}{I}(-\tau_g \sin(x_1) + k_{Gyro}(u_1 x_6 \cos(x_1)) - B_\psi x_2 + a_1 x_3^2 + b_1 x_3 \\ x_4 \\ \frac{1}{T_1^2}(u_1 - x_3 - 2T_1 x_4) \\ x_6 \\ \frac{1}{I_\varphi}\left(-B_\varphi x_6 - \left[K_r \frac{T_{or}}{T_{pr}} u_1 + x_9\right] + a_2 x_7^2 + b_2 x_7\right) \\ x_8 \\ \frac{1}{T_2^2}(u_2 - x_7 - 2T_2 x_8) \\ \frac{1}{T_{pr}}\left[K_r(1 - \frac{T_{or}}{T_{pr}})u_1 - x_9\right] \end{bmatrix}$$

$$g(x) = \begin{pmatrix} x_1 k_\psi + y_{\psi_\circ} \\ x_2 k_\varphi + y_{\varphi_\circ} \end{pmatrix} \tag{10}$$

where,

k_φ : Azimuth angle offset, y_φ : Azimuth angle read by sensor
$y_{\varphi_\circ}$: Azimuth angle offset, y_ψ : Elevator angle read by sensor
k_ψ : Elevator constant, $y_{\psi_\circ}$: Azimuth angle offset
u_i : Control voltages applied to rotors

4 Feedback Linearization Controller

In this section we try to design a controller using feedback linearization approach for Twin rotor. For the first output y_1 (the elevator angle), relative degree is 4 and for the second output y_2 (the azimuth angle), it is 2 which cause the matrix E(x) mentioned by equation (4) be singular and the control law described by equation (6) could not be calculated. It shows that the MIMO model of Twin Rotor is non-linearizable and now in order to cope with this difficulty, dynamics extension can be considered. It means that integral actions shall be added for some inputs. Applying this method causes the relative degrees to change so that results nonsingular E(x). For example we can add two integrators for u_1 which results nonsingular E(x), but r_1 changes to 6. It means that the resultant controller will be in higher degrees and complex. As a consequence, the implementation will be impossible practically (because of sampling time constraints). Meanwhile in this methodology zero dynamics appear which requires more consideration for stability and make more difficulties. In this paper we have considered another solution for above problem. We consider two separate subsystems (elevator and azimuth) which have interface on each other and then we will design the controller using feedback linearization method separately.

Theorem1. Assume that the system $\overset{\circ}{x} = f(x,u)$, $y = g(x)$ has relative degree r and its zero dynamics is locally asymptotically stable. Let $d(p) = P^r + \alpha_{r-1}p^{r-1} + \ldots + \alpha_1 p + \alpha_0$ be a Hurwitz polynomial. Then the state feedback law $u = \frac{1}{L_g L_f^{r-1} h(x)}(-L_f^r h(x) - \alpha_{r-1} L_f^{r-1} h(x) - \ldots - \alpha_1 L_f h(x) + \alpha_0 h(x))$ leads to a locally asymptotically stable closed –loop system. Proof in [1].

4.1 Controller Design for Elevator Subsystem

The relative degree of elevator subsystem derived from equation (10) is 4 and this system satisfies the feedback linearization conditions. Using the theorem1 and applying the resultant control law results the output and control effort shown in figures 4.1.a, 4.1.b (simulation with MATLAB), 4.1.c and 4.1.d (implementation).

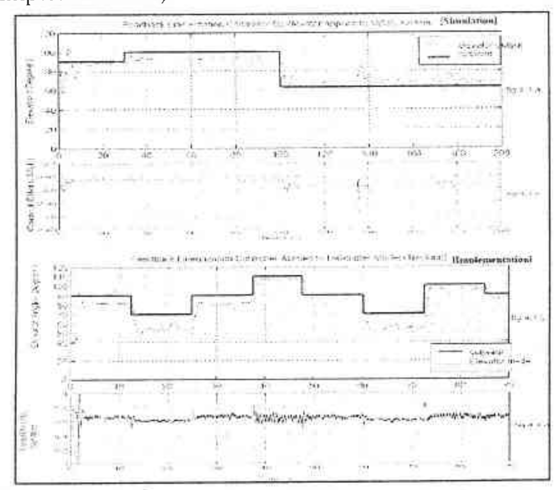

Figure 4.1: FL[1] controller applied to Helicopter (Elevation subsystem) [simulation and Implementation]

4.2 Controller Design for Azimuth Subsystem

The relative degree of azimuth subsystem derived from equation (10) is 4 and this system satisfies the feedback linearization conditions. Using the theorem1 for controller design and applying the resultant control law results the output and control effort shown in figures 4.2.a, 4.2.b (simulation with MATLAB), 4.2.c and 4.2.d (implementation).

[1] Feedback Linearization

Like to elevator controller, for both simulation and implementation on model, minimum order linear observer has been used. The set point has been changed in simulation and implementation and tracking problem has been considered for both subsystems. Meanwhile in simulation the disturbance has been applied in time 135(s) to elevator subsystem and in time 168(s) to azimuth subsystem.

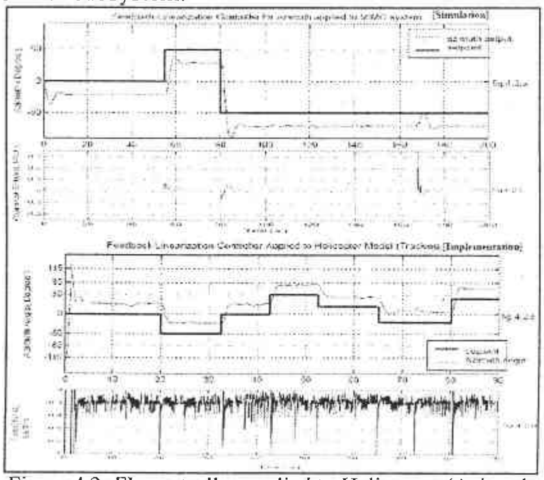

Figure 4.2: FL controller applied to Helicopter (Azimuth subsystem) [simulation and Implementation]

As it has been shown in figures 4.1.a and 4.2.a, steady states error has been occurred in outputs because of ignoring the interfaces of subsystems (uncertainties) in controller design. Meanwhile both systems show low robustness encountering to unmeasured disturbances. The bad effect of interaction between two outputs of systems is considerable when one of them changes. Implementation of FL controller has been done on MIMO model of Twin Rotor using RT toolbox of MATLAB and by taking 0.01(s) as the fastest sampling time considering the bode diagram of subsystems and hardware constraint.

As it is shown in figures 4.1.c and 4.2.c, steady state error has been occurred both in elevator and azimuth subsystems. Meanwhile large interaction between these two subsystems is considerable when one of outputs changes. It should be noted that using this controller leads to low robustness encountering applied disturbance caused by changing the position of the gravity center of the body via related step motor. In order to show the above statement, the experiments have been done via proper control input applied to relevant step motor. The result is presented in figure 4.3.

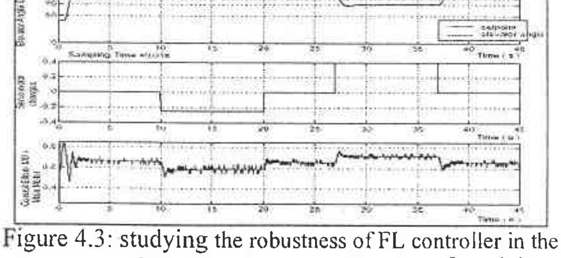

Figure 4.3: studying the robustness of FL controller in the presence of changing the center of gravity of model

As it has been shown in figure 4.3 when the center of gravity changes, a steady state error appears in outputs which implies that nominal FL method is not robust encountering internal disturbances. More experiments have been done on model using FL controller, show that this controller is not robust encountering unmeasured disturbances applied to system via hand flaps to the body.

5 Robust Feedback Linearization

In this section we propose a control law including feedback linearization controller and robust term achieving from lyapunov function to overcome the effect of unknown parts of model and unmeasured disturbances.

Assume the SISO system in form:

$$\overset{\circ}{x} = f(x) + \Delta f(x) + [g(x) + \Delta g(x)]u \qquad (11)$$
$$y = h(x)$$

where $\Delta f(x,t), \Delta g(x)$ show the modeling errors and unmeasured disturbances [16]. Assume the relative degree of system to be r. Applying diffeomorphism transformation $\phi(x) = (\xi, \eta)^T$ to equation (11) results:

$$\begin{cases} \overset{\circ}{\xi}_1 = \xi_2 \\ \dots \\ \overset{\circ}{\xi}_{r-1} = \xi_r \\ \overset{\circ}{\xi}_r = F(x) + \Delta F(x,t) + [G(x) + \Delta G(x)]u \end{cases}$$
$$\overset{\circ}{\eta} = q(\xi, \eta) \qquad (12)$$
$$y = \xi_1$$

Now considering feedback linearization control law as $u = G^{-1}(v - F)$ and applying it to equation (12) gives:

$$\begin{cases} \xi_1^{r} = v + \Delta F + \Delta G.G^{-1}(v - F) \\ \overset{\circ}{\eta} = q(\xi, \eta) \end{cases} \qquad (13)$$

The equations (13) can be written as:

$$\begin{cases} \overset{\circ}{\xi} = A\xi + bv + b\lambda(x, v) \\ \overset{\circ}{\eta} = q(\xi, \eta) \end{cases} \qquad (14)$$

where $\lambda(x,v) = \Delta F + \Delta G.G^{-1}(v - F)$ and A, b are in Brunovskey canonical form. We consider final control law includes robust and linearizing terms as:

$$v = v_L + v_R \qquad (15)$$

v_L can be designed using control law by theorem1. By defining the error as:

$$e = \xi - \bar{y}_d \;, \quad \bar{y}_d = [y_d, \overset{\circ}{y}_d, \dots, y_d^{(r-1)}]^T \qquad (16)$$

we can rewrite the equation (14) for error which gives:

$$\overset{\circ}{e} = A_c e + bv_R + b\lambda \qquad (17)$$

where $A_c = A - b[\alpha_0, \dots, \alpha_{r-1}]$.

Now we suppose the lyapunov function as $V(e) = e^T Pe$, where P is the positive definite answer of lyapunov **equation:** $pA_c + A_c^T p + Q = 0$ where -Q is **symmetric** negative definite matrix (assume -I).

Derivation of the considered lyapunov function is:

$$\overset{\circ}{V} = -e^T Qe + 2e^T Pb(v_R + \lambda) \qquad (18)$$

Assume that $|\lambda| \le \gamma$. Now considering the worst case in equation (18) results:

$$\overset{\circ}{V} = \begin{cases} -e^T Qe + 2e^T Pb\ (v_R + \gamma)\ , & e^T Pb > 0 \\ -e^T Qe - |2e^T Pb|(v_R - \gamma)\ , & e^T Pb \le 0 \end{cases} \qquad (19)$$

In order to have $\overset{\circ}{V} < 0$, v_R is obtained:

$$v_R = \begin{cases} -\gamma(x,t) - \alpha\ , & e^T Pb > 0 \\ \gamma(x,t) + \alpha\ , & e^T Pb \le 0 \end{cases} \qquad (20)$$

where α is positive constant. The above form for robust term guarantees the negative $\overset{\circ}{V}$ ($\overset{\circ}{V} \le -e^T Qe - \alpha < 0$) and global stability achieved. The above form of robust term means $|v_R| \ge \gamma$.

Now inspired by the equation of λ and writing recursive equations we can write:

$$|\lambda| \le |\Delta F + \Delta G.G^{-1}(v_L - F)| + |\Delta G.G^{-1}|\gamma \qquad (21)$$

Finally the γ can be expressed as:

$$\gamma(x,t) = \frac{|\Delta F + \Delta G.G^{-1}(v_L - F)|}{|(1 - |\Delta G.G^{-1}|)|} \qquad (22)$$

In order to prevent the chattering the robust term can be written as:

$$v_R = \begin{cases} -\dfrac{e^T Pb}{|e^T Pb|}(\gamma(x,t) + \alpha)\ , & |e^T Pb| > \varepsilon \\ -\dfrac{e^T Pb}{\varepsilon}(\gamma(x,t) + \alpha)\ , & |e^T Pb| \le \varepsilon \end{cases} \qquad (23)$$

As it has been shown, inspired by the lyapunov function and designing the robust term according to this consideration, the error globally converges to zero. It means that the output will track the desired input completely and the steady state error will not occur. Meanwhile all of the state variables remain bounded.

All of the above statements can be written for MIMO systems.

6 Simulations and Implementation

Using the controller designed in section 5, for both subsystems and applying it helicopter model, both in computer simulations and hardware implementation, results the advantages we expected as goal. As it has been shown in figure 5 (The disturbances have been applied in 135(s) to elevator subsystem and in 168(s) to azimuth subsystem) using the robust controller for simulation with MATLAB, causes both subsystems to overcome the effect of model uncertainties and unmeasured disturbances. Steady state error has not been occurred and overshoot is the lowest and other response characteristics are better in comparison with nominal FL or linear controllers.

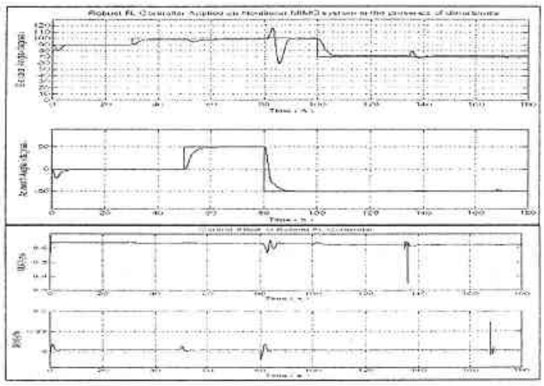

Figure 5: Robust feedback linearization method applied to helicopter model (simulation results)

The powerful results of implementation of proposed controller on helicopter have been shown in figure 6.

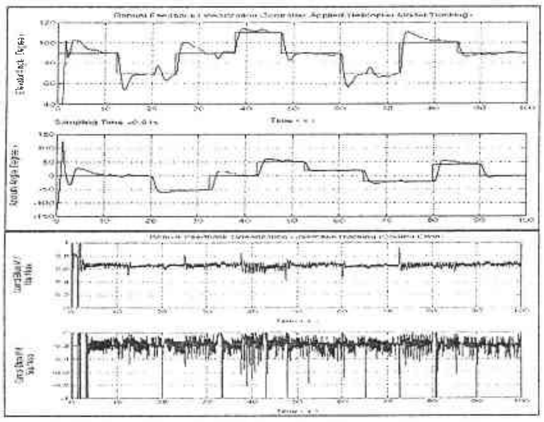

Figure 6: Robust Feedback Linearization applied to helicopter (implementation)

As it has been shown in figure 6, complete tracking is achieved by utilizing modified controller (Robust FL). Meanwhile overshoot and other system response characteristics are better than FL controller.

Using this controller leads to robustness encountering the disturbance caused by changing the position of the gravity center of the body. In order to show the above statement, the experiments have been done by changing the center of gravity via proper control input applied to relevant step motor. The result is presented in figure7.

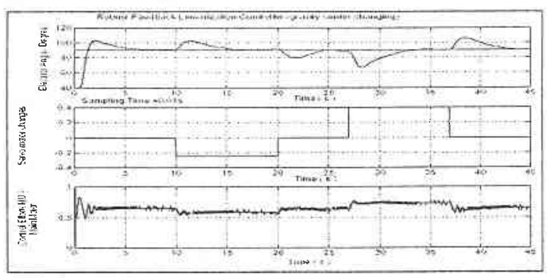

Figure 7: RFL controller applied to Twin Rotoencountering internal disturbance

As it has been shown in figure7 despite of what has been obtained for nominal FL, system with the designed controller rejects the effect of disturbance successfully. The characteristics of time response are well.

For more clarification the result of applying a well known industrial controller (PID) is presented by figure 8. The coefficients of this controller have been adjusted by manufacturer as the best adjustment. It can be seen that the results which obtained via our controller are no worst that the PID ones. For our controller the speed of response is more that PID response, but it seems that the PID response is better than Robust FL in overshoot.

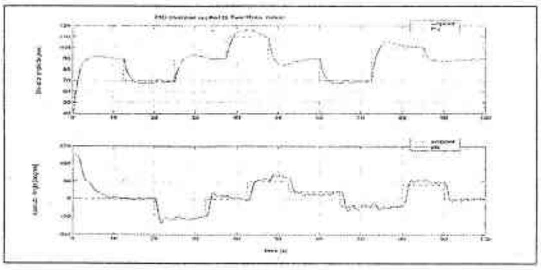

Figure 8: PID controller applied to Twin Rotor

7 Conclusion

In this paper we proposed a robust controller via feedback linearization approach and lyapunov function in order to overcome the effect of modeling errors and unmeasured disturbances. The effectiveness of the designed controller has been tested by applying to Twin Rotor (laboratory helicopter) model. Analysis of the results indicated the good performance and advantages (obviation the steady state error, rejecting the effect of disturbances and desired time response) of the proposed robust controller in comparison with nominal feedback linearization or state feedback. Meanwhile the results are no worst than what have been obtained by well known classic controllers like to PID. Of course the obtained results indicated that the designed robust controller was not successful in overcoming the interaction effect of two outputs completely.

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