System Identification of LTI Singular Systems via a Quasi-Static Method

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Abstract: *system identification is one the most important parts in science and technology. This branch of science specially has an important role in control engineering; because only with a proper identification method can one model a phenomenon to control its performance. On the other hand, singular systems have been the subject of interest over the last two decades due to their many practical applications. But it has to be said that system identification of such system is still a challenging area because of the difficulty of identification of such systems for their complex structures. In addition, it seems that by developing a useful method for singular system identification, one can use the useful property of such systems in describing the natural complex phenomena. This paper introduces a new quasi-static identification method for LTI singular systems as a powerful tool to identify this kind of systems. Results clearly demonstrate the advantages of this new method in system identification of LTI singular systems.*

Keywords: LTI singular systems, improper systems, system identification, strictly proper systems, quasi-static method.

1 Introduction

Identification aims at finding a mathematical model from the measurement record of inputs and outputs of a system. A state space model is a most obvious choice for a mathematical representation because of its widespread use in system theory and control. On the other hand, continuous and discrete-time linear control of the respective forms $E\,\dot{x}(t) = A\,x(t) + B\,u(t)$ and $Ex(k+1) = Ax(k) + Bu(k)$ where $x \;\; R^n, u \;\; R^m$ all coefficient matrices are constant and E is singular, have attracted the attention of many authors [1, 2]. These systems are also known as descriptor, semi-state, generalized state-space, algebraic-differential, implicit and degenerate systems arise naturally as a linear approximation of system models or linear system models in many applications such as electrical networks, aircraft dynamics, neutral delay systems, chemical, thermal and diffusion processes, large-scale systems, interconnected systems, systems of partial differential equations [3, 4], economics [5], optimization problems, feedback systems, robotics, biology, etc. The literature on singular systems is extensive and we refer the readers to [6, 7, 8, 9 and 10]. Previously not very much work has been done on the identification of singular systems, especially not in the linear case. Some works on the nonlinear case are [1, 12, and 13].

Here we propose a new algorithm for the identification of linear singular systems by working with some derivative of input/output data directly. The utility of this method is that the identification algorithm is not applied on the singular system directly but on its equivalents sub-systems by decomposing the singular systems to two sub-systems: a strictly proper system and a polynomial one which models the non-causality characteristics of the singular systems.

The remaining of this paper is structured as follows. Section 2 briefly introduced singular or generalized state space systems and presents its properties in compare of ordinary systems. Section 3 is devoted to describe the learning methodology which is used to approximate the parameters of the linear time invariant (LTI) singular systems in frequency domain. Section 4 is devoted to show the performance of the proposed identification algorithm in system identification of the LTI singular systems in compare of other methods via some case studies. The last section contains the concluding remarks.

2 Singular systems

A singular implicit differential equation is an implicit ordinary differential equation which takes the form of

$$F\left(\dot{x}(t), x(t), u(t), t\right) = 0, \qquad \dot{x}(t_0) = x_0 \tag{1}$$

where x is an n-dimensional state vector, u is an m dimensional control vector, t is time and the Jacobian matrix $\frac{\partial F}{\partial \dot{x}}$ is singular [14]. *A* system which is described by singular implicit differential equation is called as a singular system [7]. Singular systems are often referred to as differential algebraic equations because they frequently are a mixture of differential and algebraic equations, that is, they take the form of

$$\begin{aligned} \dot{x}(t) &= f\left(x, u, t\right) \\ 0 &= g\left(x, u, t\right) \end{aligned} \tag{2}$$

Looking to this equation, one can define a matrix E such that

$$\begin{aligned} &E\dot{x}(t) = F\left(x, u, t\right), \\ &E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \\ &F\left(x, u, t\right) = \begin{bmatrix} f\left(x, u, t\right) \\ g\left(x, u, t\right) \end{bmatrix} \end{aligned} \tag{3}$$

In this paper, the time-invariant system of r first order coupled linear differential equations is considered:

$$E\dot{x}(\mathrm{t}) = Ax(t) + Bu(\mathrm{t}) \tag{4a}$$

$$y(\mathrm{t}) = Cx(\mathrm{t}), \qquad \mathrm{t} \geq 0 \tag{4b}$$

Where x is an r-vector of internal variables, u is an m-vector of control inputs or forcing function and y is a p-vector of outputs.

When E is singular in (4), resulting in what we shall term a *generalized state-space system* or *singular system*, this behavior is considerably modified. In contrast to *regular state-space system* (E is non-singular) we find the following.

i. The number of degrees of freedom of the system, i.e., the number of independent values that $Ex(0.)$ can take, is now evidently reduced to

$$\mathrm{f} \triangleq \operatorname{rank} E < r \tag{5}$$

We propose the term *generalized order* for *f*.

ii. The transfer function $G(s)$ may no longer be strictly proper, in which case it may be written as the sum of a strictly proper part $\bar{G}(s)$ and a polynomial part $D(s)$.

iii. For this case of singular E,

$$\text{degree of } |sE - A| \triangleq n \leq f < r \tag{6}$$

The free response of the system in this case exhibits exponential motions, as regular systems, at the n finite frequencies $s = \lambda$ (possibly non-distinct) where $(sE - A)$ is singular. In addition, however, it contains f-n *impulsive* (i.e., *distributional*) motions, or "infinite-frequency" modes (corresponding essentially to $(sE - A)$ losing rank at $s = \infty$) [6].

3 System identification LTI singular system via a quasi-static algorithm

Considering a linear singular system such as

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{7}$$

The output and input under zero initial conditions (i.e., $Ex(0.)=0$) are related by the transfer function $G(s)$, as follows:

$$G(s) = C(sE - A)^{-1}B \tag{8}$$

It is found that the transfer function $G(s)$ may no longer be strictly proper, in which case it may be written as the sum of a strictly proper part $\bar{G}(s)$ and a polynomial part $D(s)$. Therefore, we have:

$$G(s)=\bar{G}(s)+D(s) \tag{9}$$

where

$$\bar{G}(s)=\bar{C}(sI-\bar{A})^{-1}\bar{B}, \quad \text{strictly proper} \tag{10a}$$

and

$$D(s)=\tilde{C}(I-s\tilde{E})^{-1}\tilde{B} = \tilde{C}(I+s\tilde{E}+\cdots+s^{v}\tilde{E}^{v})\tilde{B}, \quad \text{polynomial} \tag{10b}$$

$\bar{A}$, $\bar{B}$, $\bar{C}$, $\tilde{B}$, $\tilde{C}$ and $\tilde{E}$ are came from a restricted standard equivalence of the system (8). Here v is less than the size of $\tilde{E}$, since $\tilde{E}$ is *nilpotent* (i.e. has all eigenvalues = 0) [6]. Fig. 1 shows a linear singular system which is decoupled to a strictly proper subsystem and a polynomial part (in discrete domain).

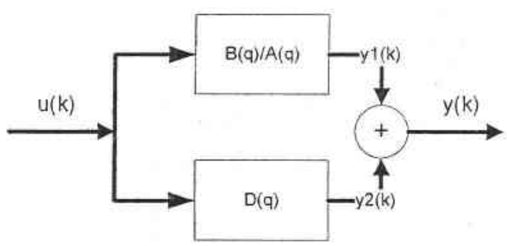


Fig 1: A decoupled singular system.

It is obvious that the polynomial subsystem in discrete domain will be a moving average subsystem. Fortunately, each sub system could be identified by classical identification methods. Therefore, one can adjust the parameters of a singular system by decoupling it in to two subsystems, and then adjust these parameters simultaneously. The quasi-static algorithm to identify parameters of linear singular system is as follows:

1. To identify the parameters of the strictly proper subsystem, consider the output of the polynomial part (which is not identified yet) as a measurement noise to the strictly proper part.
2. Estimate an ARX model $A(q)y(k) = B(q)u(k) + y_2(k)$ from the data $\{\underline{u}(k), \underline{y}(k)\}$ by

$$\hat{\theta}_{ARX}=(\underline{X}^{T}\underline{X})^{-1}\underline{X}^{T}\underline{y} \tag{11}$$

3. Calculate the prediction error of this ARX model

$$e_{ARX}(k)=\hat{A}(q)y(k)-\hat{B}(q)u(k) \tag{12}$$

whose $\hat{A}(q)$ and $\hat{B}(q)$ are determined by $\hat{\theta}_{ARX}$.

4. To identify the parameters of the polynomial part, consider the output of the strictly proper part (which is identified in this iteration) as a measurement noise to the polynomial part.
5. Estimate the d_i parameters of the following FIR model by least squares

$$e_{ARX}(k)=D(q)u(k) \tag{13}$$

This algorithm can be iterated until the convergence is reached. Fig. 2 shows this quasi-static algorithm.

This method yields the linear descriptor systems in frequency domain. In addition, it has to be said that to get to the state space form of the singular system one can use the Silverman-Ho algorithm which gets the state space model of the singular system via its transfer function [10, 15].

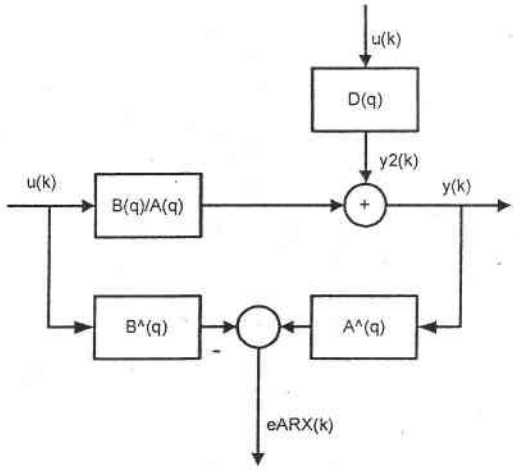
4 Simulation results

In this section, in order to show the performance of the quasi-static method in identifying linear singular systems two case studies are considered. In the first one, the quasi-static algorithm is implemented to model a proper linear descriptor system and its results are compared with the performance of ordinary Least Square Method. In the second case study, it is tried to model an improper singular systems via quasi-static method.

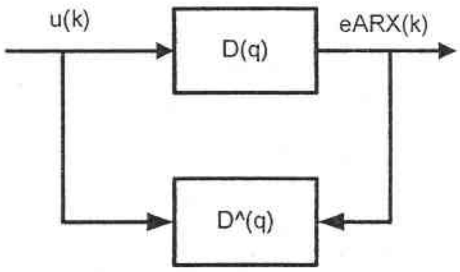
4.1. Case study 1

Consider a linear singular system as:

$$\begin{bmatrix}1 & 1\\ 0 & 0\end{bmatrix}\dot{x}=\begin{bmatrix}-1 & 1\\ 0 & -1\end{bmatrix}x+\begin{bmatrix}-1\\ 2\end{bmatrix}u \qquad y=\begin{bmatrix}1 & 2\end{bmatrix}x \tag{14}$$



(a)



(b)

Fig 2: Singular system identification by decoupling method: (a) approximating the parameters of the strictly proper subsystem, (b) approximating the parameters of the polynomial part after identifying the strictly proper subsystem.

The regular pencil for this system after a linear transformation is:

$$\left(s\tilde{E}-\tilde{A}\right)=\begin{bmatrix} s+1 & 0 \\ 0 & 1 \end{bmatrix} \tag{15}$$

It can be seen that the nilpotent part of the system has an index equal to zero; therefore, the degree of polynomial part of the transfer function is a zero order polynomial. The transfer function for this system is:

$$\begin{aligned} G(s) &= \frac{2s+5}{s+1} = 2+\frac{3}{s+1} \\ G_T(z) &= 2+\frac{3T}{z+(T-1)} \end{aligned} \tag{16}$$

It is obvious that discrete system is unstable for $T>1$. We use T=0.9 and consider Gaussian white noise for input signal and white noise as a measurement noise. 300 input/output data is used for tuning the coefficients of the model via quasi-static algorithm. Table 1 shows the performance of the quasi-static method in comparison with ordinary least square method in approximating the parameters of this linear singular system:

Table 1. Performance of the quasi-static method in estimating the parameters of proper singular system in comparison with the ordinary least square method

	a_0	P_0	b_0	RMSE
Real Parameters	0.1	2	2.7	---
LSE Method	0.588	0.026	-0.03	0.178
Quasi-static Method	0.095	2.027	2.583	0.007

Fig. 3 shows the performance of the model which is obtained by ordinary least square algorithm in comparison with the singular system's output. It is obvious that the performance of the LSE method is poor in approximating the parameters of this singular system.

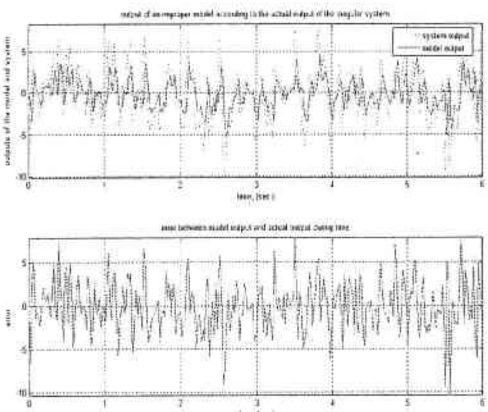


Fig 3: upper: performance of the LSE method in modeling a proper linear singular system; lower: modeling error

Fig. 4 shows the performance of the model which is obtained by quasi-static method in comparison with the singular system's output. It is obvious that the performance of this method is much better in approximating the parameters of this singular system in comparison with the performance of the ordinary least square algorithm.

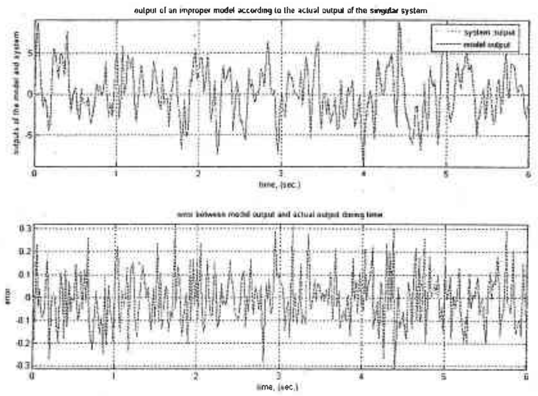


Fig 4: upper: performance of the quasi-static method in modeling a proper linear singular system; lower: modeling error

Fig. 5 shows the convergence rate of mean square error for models based on these two methods. It is obvious that the ordinary least square method could not get to the proper model. On the other hand, the performance of the quasi-static method according to the convergence of the mean square error to zero is much better.

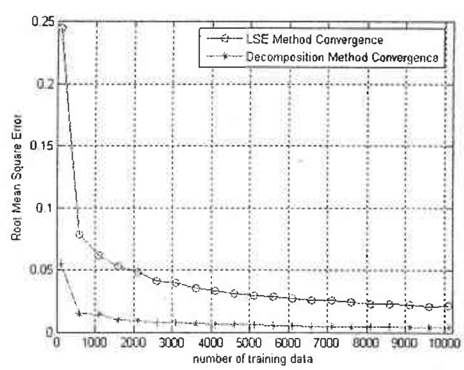


Fig. 5: The convergence of the mean square error of ordinary least square method and the quasi-static method by increasing the number of training data.

4.2. Case study 2

Consider transfer function of a linear singular system with index two as:

$$G(s) = 1 + s + s^2 + \frac{3s+5}{s^2+6s+5} \qquad (17)$$

The discrete form of this transfer function by Euler approximation is:

$$G_T(z) = \frac{\frac{1}{T^2}z^2 + \frac{7T-4}{T^2}z + \frac{12T^2-21T+4}{T^2}}{1+(6T-2)z^{-1}+(5T^2-6T+1)z^{-2}} + \frac{\frac{14T^3-24T^2+21T-4}{T^2}z^{-1}}{1+(6T-2)z^{-1}+(5T^2-6T+1)z^{-2}} + \frac{\frac{10T^4-14T^3+12T^2-7T+1}{T^2}z^{-2}}{1+(6T-2)z^{-1}+(5T^2-6T+1)z^{-2}} \qquad (18)$$

This system is stable if $0<T<0.34$. We use $T=0.3$ and like former case study we consider Gaussian white noise for input signal and white noise as a measurement noise. Table 2 shows the performance of the quasi-static method compare to the ordinary least square method in approximating the parameters of this linear singular system:

Table 2. Performance of the quasi-static method in estimating the parameters of proper singular system in comparison with the ordinary least square method

	Real Parameters	LSE Method	Quasi-static Method
a_0	0.2	-0.001	0.1825
a_1	0.35	-0.004	0.3209
p_0	11.11	11.27	11.11
p_1	-21.11	-18.76	-20.78
p_2	8.667	8.77	8.47
b_0	5.755	0.837	5.46
B_1	-3.52	-0.1504	-3.27
RMSE	------	0.135	0.0124

Fig. 6 shows the performance of the model which is obtained by quasi-static method in comparison with the singular system's output. It is obvious that the performance of this method is much better in approximating the parameters of this singular system in comparison with the performance of the ordinary least square algorithm.

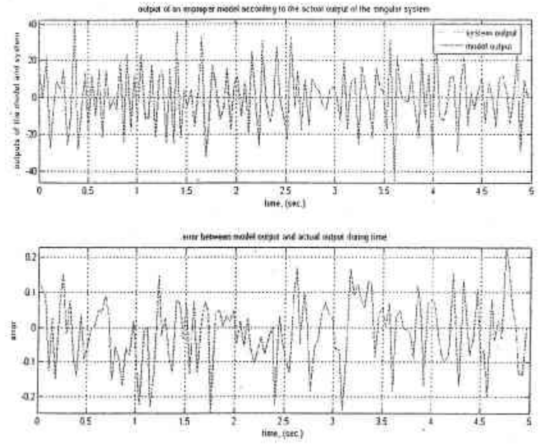


Fig 6: upper: performance of the quasi-static method in modeling a proper linear singular system; lower: modeling error

5 Conclusions

In this paper, the problem of system identification of linear time invariant singular systems is considered. It has to be said that the literature on identification of singular system is not as rich as the literature on optimization or Kalman filtering of singular systems. To do so, this paper proposes a new learning method as a quasi static algorithm to tune the parameters of the transfer function of singular system. The great performance of this quasi static algorithm in tuning parameters of linear singular systems in comparison with an important algorithm in system identification which is called ordinary least square algorithm depicts its potential in this subject.

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