A Hybrid Method for Predicting Chaotic Tent Map

Foroogh Motallebzadeh [a,*] , **Caro Lucas** [a,b]

[a] Electrical and Computing Engineering Department, Tehran University, Tehran, Iran
[b] School of Cognitive Sciences, Institute for studies in theoretical Physics and Mathematics (IPM), Tehran, Iran
f.motallebzadeh@ece.ut.ac.ir

Abstract: *This paper presents Genetic Algorithm based Cellular Automata for predicting chaotic time series. The simple and interesting Tent map,* $x_{n+1} = a(1-2|1/2 - x_n|)$, *is used here. The values are first coded into binary form and the first N of them are given as initial condition to Cellular Automata (CA). Genetic Algorithm evolves CA rules, and the second N values of the map are the CA output. One of the important advantages of this work in compare with others is its ability for parallel computing.*

Keywords: Cellular Automata, chaotic time series, Genetic Algorithm,

1 Introduction

The systematic study of chaos has originated in the 1960s. Because linear methods which were used widely before, couldn't afford studding chaotic behaviors with acceptable precision.
Chaotic dynamics is explained by sensitivity to initial conditions and exponential divergence of close orbits.
Chaotic systems often have a control parameter that determines the strength of the chaos. A simple model system with such a control parameter is the well-known tent map. The identification and modeling of nonlinear systems is still a very active field of research at present.
The Tent Map, defined by the equation:
$x_{n+1} = a(1-2|1/2 - x_n|)$ f: [0,1] -> [0,1]
is studied in the mathematics of dynamical systems. Because of its simple shape, the Tent Map's shape under iteration is very well understood.
And despite its simple shape, it has several interesting properties.
The Tent map shows different behaviors for different values of the parameter *a*. Depending on the value of *a*, it demonstrates a range of dynamical behavior ranging from predictable to chaotic. For *a*>1/2 it finds chaotic behavior and for a=1, it shows a fully chaotic behavior.
Bifurcation diagram for the tent map is shown below. Higher density indicates increased probability of the x variable acquiring that value for the given μ parameter (μ=2*a).

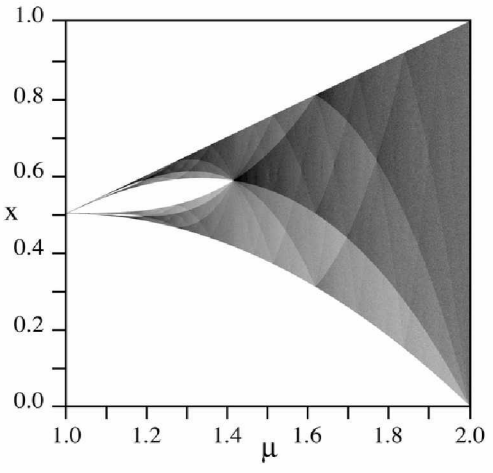

Figure 1: Bifurcation diagram for the tent map

Most of the times it is not so easy to have precise and acceptable prediction of chaotic time series.
Hybrid systems' development is one of the most intensively evolving areas of connectionist science. Hybrid systems utilize various soft computing methods like artificial neural networks, fuzzy

engines, evolutionary computation, and Cellular Automata.
In this work Cellular Automata, the one-dimensional one, with the help of Genetic Algorithm, is chosen to predict chaotic Tent map. It has been frequently observed that the simultaneous execution of a single relatively simple rule at many local sites leads to the emergence of interesting and complex global behavior (Langton 1989). Cellular automata are an abstract way of studying and analyzing the simultaneous execution of local rules (Burks 1970; Farmer, Toffoli, and Wolfram 1983; Wolfram 1986; Gutowitz 1991; Wuensche and Lesser 1992).
A CA begins with an initial state of its cells. Then it evolves following a set of rules. But it can be difficult to program CA. In this work Genetic algorithm is used to evolve CA rule.
Results show that the proposed method can provide CA rules that could be considered acceptable.

2 Methods:

At the first step, the Temp map with control parameter, *a*, equal to 0.9 was used:

$$x_{n+1} = 0.9(1 - 2|1/2 - x_n|)$$

The first N points were used as initial values of the cells, after they were coded to binary digits.
Cellular Automata was to calculate or predict the second N points.
Genetic algorithm gave the proper rules for CA. Each chromosome of GA was in fact a set of rules for CA. after each set of rule is produced, CA would evolve. The states of the cell would change according to the rules. The neighborhood of each cell consists of three cells, and the state of each cell would change according to its current state and the state of its left and right neighbors.
After the determined iterative stages, the output of the CA was the second 10 points of the map.
The last work was to convert these binary digits into decimal form.
In order to evolve the rules, MATLAB GA toolbox was used. As crossover operator, the single-point operator was used, and mutation operator was in the uniform form. Number of bits that were correctly predicted was used as fitness function (GA was suppose to minimize the number of incorrect predicted bits).

3 Simulated results:

Initial point of the tent map was considered 0.1.
Each row of the CA is filled with a sequence of 13 bit representing the decimal number.
The first 7 numbers were used as CA input and the second 7 numbers were the outputs.
The rule which was produced by the GA with the crossover probability of about 0.85 and the mutation rate of 0.32 is (00111101), while the GA employed 50 individuals in each generation. It indicated that for example when a cell was off and its two neighbors were off too, then it would remain still off.
The result of the GA is shown below:

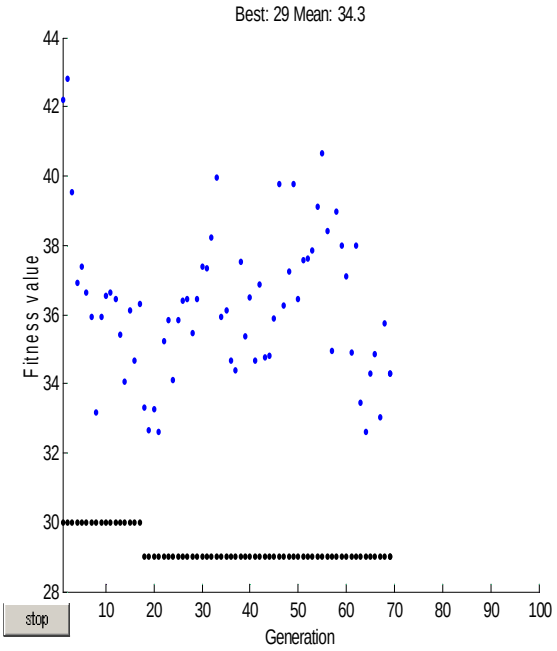

Figure 2: The result of GA

Then the rules were given to CA and it would reach the answer in 10 evolutionary steps.

The results are shown below:

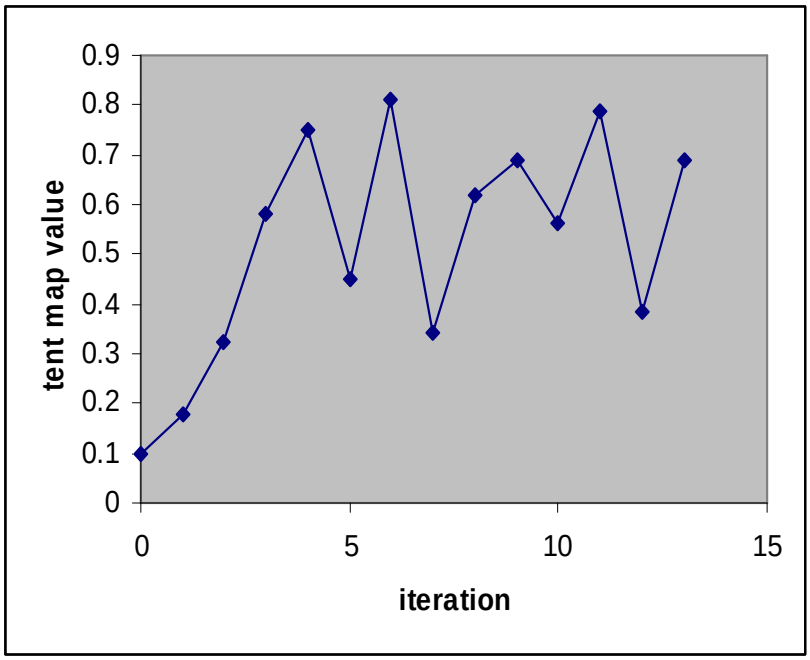

Figure 3: Real values

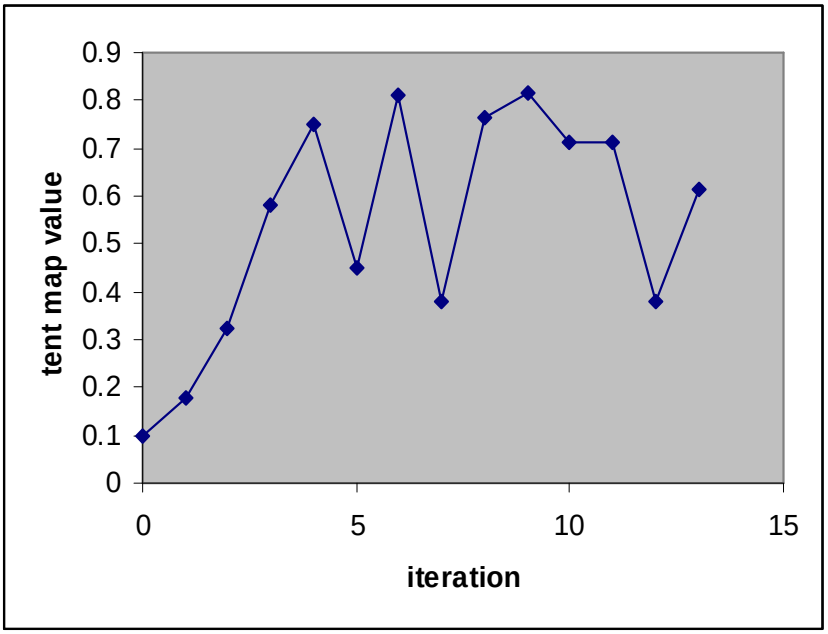

Figure 4: Predicted values

It can be seen that the difference is not so much and an acceptable result is achieved.

4 Conclusion:

In this work a method which was a combination of genetic algorithm and Cellular Automata, was proposed for the prediction of a chaotic time

series. The rules of the CA were evolved by Genetic Algorithm. The method was tested on the Tent map which shows chaotic behavior for some values of its control parameter. The result showed acceptable prediction. The most important advantage of the method is its ability of parallel computing and its high speed.

5 References:

[1] D. kugiumtzis, B. Lillekjendlie, N. christophersen. *Chaotic Time Series*
Part I: Estimation of Invariant Properties in State Space. 1994

[2] Suresh Subbiah and Dean J. Driebe. *Spectral Decomposition of the Tent Map with Varing Height*, 1998

[3] Mariusz Sozanski and Jan zabrowski. *On the application of DFA to the Analysis of Unimodal Maps*. Acta Physica Polonica, 2005

[4] A.V. Adamopoulos, N.G Pavlidis and M.N. Vrahatis. *Genetic Algorithm Evolution of Cellular Automata Rules for complex Binary Sequence Prediction*, Lecture Series on Computer and Computational Sciences,2005