Electronic III

Frequency Response Of Integrated Circuits

Omid Shoaei, Univ. of Tehran

Single-Stage Bipolar Differential Amp.

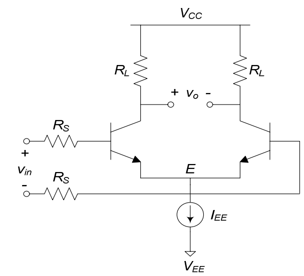

Fig.3.1-a

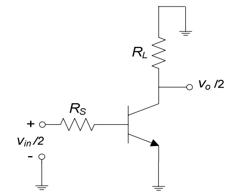

Fig.3.1-b

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Single-Stage Bipolar Differential Amp.

- For simplicity the factor 1/2 is omitted from input and output in small-signal equivalent.
- The effect of C_{CS} has been neglected (which can be added later).

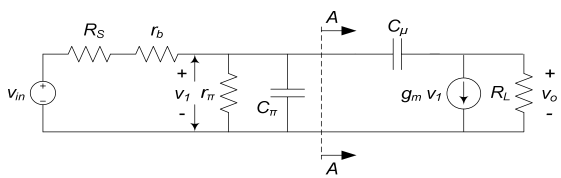

Fig.3.2

- Miller effect approximation can be used by considering the input impedance looking across the plane AA (Fig.3.2):

$$i_1 = (v_1 - v_o) C_\mu s \qquad (3-1a)$$

$$g_m v_1 + \frac{v_o}{R_L} + (v_o - v_1) C_\mu s = 0 \qquad (3-1b)$$

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$$\Rightarrow \quad v_o\left(\frac{1}{R_L} + C_\mu s\right) = -(-C_\mu s + g_m) v_1$$

$$\Rightarrow \quad A_{v1} = \frac{v_o}{v_1} = -\frac{g_m - s C_\mu}{\frac{1}{R_L} + s C_\mu} \qquad (3-2)$$

- The last term in 3-1b is the feedforward current into C_μ.
- Neglecting that compared to the first two terms of 3-1b:

for $f << \min\left(\frac{g_m}{C_\mu}, \frac{1}{R_L C_\mu}\right) \Rightarrow v_o \approx -g_m R_L v_1 \qquad (3-3)$

$(3-3)$ into $(3-1a)$:

$$i_1 = (1 + g_m R_L) C_\mu \quad \Rightarrow \quad \frac{i_1}{v_1} = (1 + g_m R_L) C_\mu s \qquad (3-4)$$

- (3-4) shows the *low frequency* impedance seen across the plane AA:

$$C_M = (1 + g_m R_L) C_\mu \qquad (3-5)$$

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Single-Stage Bipolar Differential Amp.

- Called *Miller capacitance* and can be written as:

  $C_M = (1 + A_{v1} R_L) C_\mu$

- Where A_{v1} is the magnitude of voltage gain from the internal base to the collector.
- $A_{v1} >> 1$, $C_M >> C_\mu$
- **Physical interpretation:** a small signal v_1 produces a large $v_o = -A_{v1} v_1$ of opposite polarity. Thus the voltage across C_M is $(1+A_{v1})C_\mu$ causing large amount of i_1 to flow in this cap.!
- If we solve problem generally in more general manner:

$$v_o = A_{v1} \; v_1$$

$$\text{So in (3-1): } i_1 = (\; v_1 - A_{v1} \; v_1) \; C_\mu \; s = (\; 1 - A_{v1}) \; C_\mu \; s \cdot v_1$$

$$\Rightarrow \frac{i_1}{v_1} = (\; 1 - A_{v1}) \; C_\mu \; s \Rightarrow C_M = (\; 1 - A_{v1}) \; C_\mu$$

Single-Stage Bipolar Differential Amp.

- This approximation yields the same previous result for Miller Cap as long as A_{v1} is negative real (i.e. so $\angle A_v = 180°$ as is for low freq. gain when $A_v = -g_m R_L$)
- If not the impedance is not quite capacitive so a straight C_M can not be defined!

Note: For $\angle A_v = 0$ we see a negative impedance for $|A_{v1}| >> 1$

$$\frac{i_1}{v_1} \approx -|A_{v1}| C_\mu s \Rightarrow C_M \approx -|A_{v1}| C_\mu$$

so we don't use C_C (Compensation Capacitor) for a positive high gain amplifier: However for $|A_{v1}|<1$ for positive A_v (i.e. $\angle A_v = 0$) a $C_M = (1-|A_{v1}|)C_C$ which is smaller than C_C can be defined at input node!

Single-Stage Bipolar Differential Amp.

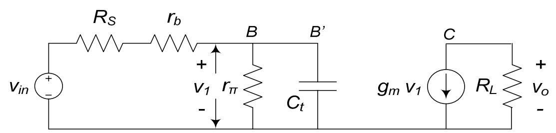


Fig.3.3

- This model is not good for observing the high-frequency reverse transmission or output impedance!!!
- Calculating the gain:

$$\begin{cases} v_1 = \dfrac{\dfrac{r_\pi}{1 + r_\pi C_t s}}{\dfrac{r_\pi}{1 + r_\pi C_t} + R_s + r_b} v_i \quad \text{where } C_t = C_\pi + C_M \;\&\; C_M = (1 + g_m R_L) C_\mu & (3-6) \\ v_o = -g_m R_L v_1 & (3-7) \end{cases}$$

Single-Stage Bipolar Differential Amp.

- (3-6) into (3-7): diff.-mode gain

$$A_{dm} = \frac{v_o}{v_i} = -g_m R_L \frac{r_\pi}{R_S + r_b + r_\pi} \frac{1}{1 + s C_t \dfrac{(R_S + r_b) r_\pi}{R_S + r_b + r_\pi}} \qquad (3-8)$$

$$= \frac{k}{1 - s / p_1} \qquad (3-9)$$

- Where k is the low-frequency gain and p_1 is the pole:

$$\begin{cases} k = -g_m R_L \dfrac{r_\pi}{R_S + r_b + r_\pi} & (3-10a) \\ p_1 = -\dfrac{R_S + r_b + r_\pi}{(R_S + r_b) r_\pi} \dfrac{1}{C_t} & (3-10b) \end{cases}$$

- This analysis shows a single-pole response:

$$\omega_{-3\text{dB}} = \frac{R_S + r_b + r_\pi}{(R_S + r_b) r_\pi} \frac{1}{C_t} = \frac{1}{(R_S + r_b) \| r_\pi} \frac{1}{C_t} = \frac{R_S + r_b + r_\pi}{(R_S + r_b) r_\pi} \frac{1}{C_M + (1 + g_m R_L) C_\mu} \qquad (3-11)$$

Single-Stage Bipolar Differential Amp.

- The larger C_t, the lower -3dB frequency of the amplifier.
- By assuming $R_S >> r_\pi$ and R_L small (so that $C_\pi >> (1+g_mR_L)C_\mu$):

$$|p_1| = \frac{R_S}{R_S\, r_\pi}\frac{1}{C_\pi} = \frac{1}{r_\pi\, C_\pi} = \frac{1}{\beta_0}\frac{g_m}{C_\pi} \approx \frac{\omega_T}{\beta_0}$$

- Upper limit for $|p_1|$.
- Larger R_L will give lower $|p_1|$, smaller R_S will give larger $|p_1|$ but not larger than the above upper limit.

Example:

Calculate -3dB frequency of a common emitter bipolar transistor stage?

$$R_S = 1\ \mathbf{k\Omega}\ ,\ r_b = 200\ \ \Omega\ ,\ I_C = 1\ \ \mathbf{mA}\ ,\ \beta = 100$$

$$f_T = 400\ \ \mathbf{MHz}\ \ (\mathbf{at}\ \ I_C = 1\ \ \mathbf{mA}\ )\ ,\ C_{jE} = 5\ \ \mathbf{pF},\ C_\mu = 0.5\ \ \mathbf{pF}\ ,\ R_L = 5\ \ \mathbf{k\Omega}$$

Single-Stage Bipolar Differential Amp.

Solution:

$$r_\pi = \frac{\beta_0}{g_m} = 100 \times 26\ \ \Omega = 2.6\ \ \mathbf{k\Omega}$$

$$\tau_T = \frac{1}{2\boldsymbol{\pi}\ \ f_T} = 398\ \ \mathbf{p}\sec$$

$$C_\pi + C_\mu = g_m\ \ \tau_T = \frac{1}{26}398\ \ \mathbf{pF} = 15.3\ \ \mathbf{pF} \Rightarrow C_\pi = 14.8\ \ \mathbf{pF}$$

Note that τ_T is different from τ_F:

$$\tau_F = \frac{C_D}{g_m} = \frac{C_b}{g_m}$$

$$\tau_T = \frac{C_\pi + C_\mu}{g_m} = \frac{C_b + C_{je} + C_\mu}{g_m} = \tau_F + \frac{C_{je} + C_\mu}{g_m}$$

Which C_b is represented by C_D in L&S

and C_{je} is represented by $C_{jEt} \approx 2\ \ C_{jE0}$

$$C_M = (\ 1 + g_m\ \ R_L)\ \ C_\mu = (\ 1 + \frac{1}{26}5000\ )\ \ 0.5\ \ \mathbf{pF} = 96.7\ \ \mathbf{pF}$$

Single-Stage Bipolar Differential Amp.

C_M is much greater than C_π and dominates the frequency response:

$$f_{-3\mathrm{dB}} = \frac{1}{2\pi}\frac{R_S + r_b + r_\pi}{(R_S + r_b)\,r_\pi}\frac{1}{C_t} = \frac{1}{2\pi}\frac{1000 + 200 + 2600}{(1000 + 200)\,2600}\frac{1}{(96.7 + 14.8)\,\mathrm{pF}} = 1.74\ \mathrm{MHz}$$

Low freq. gain :

$$A_{dm}\big|_{\omega=0} = -g_m R_L \frac{r_\pi}{R_S + r_b + r_\pi} = -\frac{5000}{26}\frac{2.6}{5 + 0.2 + 2.6} = -64.1 \approx 36.1\,\mathrm{dB}$$

Single-Stage Bipolar Differential Amp.

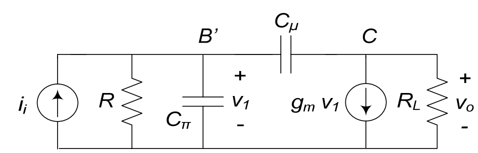


Fig.3.4.
small signal equivalent of Fig.3.2 using *Norton* equivalent

- **Exact Calculation**

$$R = (R_S + r_b)\,\|\, r_\pi \qquad (3-12)$$

$$i_i = \frac{v_i}{R_S + r_b} \qquad (3-13)$$

B′: $$i_i = \frac{v_1}{R} + v_1\, C_\pi\, s + (v_1 - v_o)\, C_\mu\, s \qquad (3-14)$$

C: $$\frac{v_o}{R_L} + g_m\, v_1 + (v_1 - v_o)\, C_\mu\, s = 0 \qquad (3-15)$$

Single-Stage Bipolar Differential Amp.

$$v_1\,(g_m - C_\mu s) = -v_o\left(\frac{1}{R_L} + C_\mu s\right) \;\Rightarrow\; v_1 = -v_o\,\frac{\frac{1}{R_L} + C_\mu s}{g_m - C_\mu s} \qquad (3-16)$$

$(3-16)$ into $(3-14)$:

$$i_i = -\left(\frac{1}{R_L} + C_\pi s + C_\mu s\right)\frac{\frac{1}{R_L} + C_\mu s}{g_m - C_\mu s}\,v_o - C_\mu s\,v_o$$

$$\Rightarrow\quad \frac{v_o}{i_i} = \frac{R R_L\,(g_m - C_\mu s)}{1 + s\,(C_\mu R_L + C_\mu R + C_\pi R + g_m R_L R C_\mu) + s^2 R_L R C_\pi C_\mu} \qquad (3-17)$$

i_i from $(3-13)$ into $(3-17)$:

$$\frac{v_o}{v_i} = -\frac{g_m R_L R}{R_S + r_b}\,\frac{1 - \frac{C_\mu}{g_m}s}{1 + s\,(C_\mu R_L + C_\mu R + C_\pi R + g_m R_L R C_\mu) + s^2 R_L R C_\pi C_\mu} \qquad (3-18)$$

Single-Stage Bipolar Differential Amp.

R from $(3-13)$ into $(3-18)$:

$$\left.\frac{v_o}{v_i}\right|_{\omega=0} = -g_m R_L\,\frac{r_\pi}{R_S + r_b + r_\pi} \qquad (3-19)$$

- (3-18) shows a positive zero with magnitude g_m/C_μ (at high frequency) and two poles:

$$\mathrm{D}(s) = \left(1 - \frac{s}{p_1}\right)\left(1 - \frac{s}{p_2}\right) \;\Rightarrow\; \mathrm{D}(s) = 1 - s\left(\frac{1}{p_1} + \frac{1}{p_2}\right) + s^2\,\frac{1}{p_1 p_2} \qquad (3-20)$$

For $|p_2| >> |p_1| \;\Rightarrow\; \mathrm{D}(s) \approx 1 - \frac{s}{p_1} + \frac{s^2}{p_1\,p_2} \qquad (3-21)$

Comparing $(3-22)$ with (7-18):

$$p_1 = -\frac{1}{RC_\pi + C_\mu\,(R_L + R + g_m R R_L)} = -\frac{1}{R}\,\frac{1}{C_\pi + C_\mu[(1 + g_m R_L) + R_L/R]} \qquad (3-22)$$

Dominant Pole

Single-Stage Bipolar Differential Amp.

- The $|p_1|$ from exact calculation is almost ω_{-3dB} obtained from the Miller approximation (3-11); The only difference is the last term (i.e. R_L/R). Miller-effect is like neglecting the high frequency poles.
- Non-dominant pole : By comparing the coefficient of s^2 in (3-21) with that in (3-18) we have:

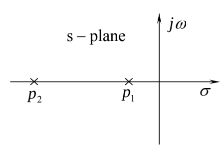
$$p_2 = \frac{1}{p_1}\,\frac{1}{R R_L C_\mu C_\pi} \qquad (3-23)$$

p_1 from $(3-21)$ into $(3-22)$:

$$p_2 = -\left(\frac{1}{R_L C_\mu} + \frac{1}{RC_\pi} + \frac{1}{R_L C_\pi} + \frac{g_m}{C_\pi}\right) \qquad (3-24)$$

- The last term is $\frac{g_m}{C_\pi} = \omega_T$ and thus $|p_2| > \omega_T$, always $|p_1| << |p_2|$

Current unity-gain frequency



Single-Stage Bipolar Differential Amp.

$$p_2 = -\left(\frac{1}{R_L C_\mu} + \frac{1}{RC_\pi} + \frac{1}{R_L C_\pi} + \frac{g_m}{C_\pi}\right)$$

$C_\mu \to \infty$ (if be shorted) $\Rightarrow$ remaining time constant is: $C_\pi\,(R \,\|\, R_L \,\|\, g_m)$

$C_\pi \to \infty$ (if be shorted) $\Rightarrow$ remaining time constant is: $C_\mu R_L$

Example:

Calculate non-dominant pole for previous example, from (3-24).

Solution:

$$p_2 = -\left(\frac{1}{R_L C_\mu} + \frac{1}{R_L C_\pi} + \frac{1}{RC_\pi} + \omega_T\right)$$

$$R = (R_S + r_b) \,\|\, r_\pi = 1200 \,\|\, 2600 = 821\Omega$$

$$p_2 = -\left(\frac{10^{12}}{5000\times 0.5} + \frac{10^{12}}{821\times 14.8} + \frac{10^{12}}{5000\times 14.8} + 2\pi\times 400\times 10^6\right)\mathrm{rad/sec}$$

$$= -(4\times 10^8 + 0.8\times 10^8 + 0.1\times 10^8 + 25\times 10^8)\,\mathrm{rad/sec} = -30\times 10^8\,\mathrm{rad/sec}$$

$$= -476\,\mathrm{MHz}$$

CMOS Common-Source (CS) Freq. Response

- Razavi's Book:
  - 6.2
  - 6.3
  - 6.4
  - 6.5

CM Response Single-Stage BJT Diff. Amp.

- **Common-mode gain of diff. pair**
- R_E and C_E are the equivalent output resistance and capacitance of the current source, I_{EE}.
- In half circuit impedances common to the two devices are doubled (such that in parallel result in the actual value) so R_E and C_E become $2R_E$ and $C_E/2$.

- **Approximate analysis**
- $R_E \sim r_o$ (where at low bias ≈ 5 MΩ)
- $C_E \approx C_{CS} \approx 2$ pF

$$\Rightarrow \quad R_E\,C_E = 10\,\mu\text{sec} \quad \Rightarrow \quad \frac{1}{2\pi\,R_E\,C_E} = 16\,\text{kHz}$$

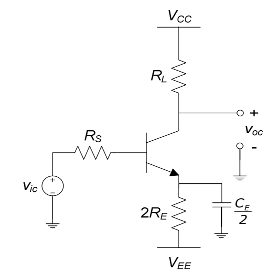


Fig.3.5. CM half circuit for Fig.3.1-a

CM Response Single-Stage BJT Diff. Amp.

- Below this freq. $Z_E \approx R_E$, and above this freq. C_E dominates. thus as freq. is increased the emitter impedance shows freq. variation well before the rest of the circuit.

$$v_{oc} \approx -g_m\,R_L\,v_1$$

$$v_{ic} \approx v_1 + (g_m + g_\pi)Z_E\,v_1 = \left[1 + (g_m + g_\pi)Z_E\right]v_1$$

$$\Rightarrow \quad \frac{v_{oc}}{v_{ic}} = -\frac{g_m\,R_L}{1 + (g_m + g_\pi)Z_E} = -\frac{R_L}{1/g_m + (1 + 1/\beta)Z_E}$$

$$\text{for } |Z_E| >> \frac{1}{g_m} \quad \Rightarrow \quad \frac{v_{oc}}{v_{ic}} \approx -\frac{R_L}{Z_E} \qquad (3-25)$$

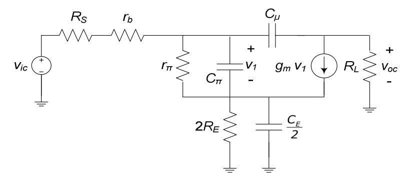


Fig.3.6.
small signal equivalent

CM Response Single-Stage BJT Diff. Amp.

$$\text{Where} \quad Z_E = \frac{2\,R_E}{1 + s\,C_E\,R_E} \qquad (3-26)$$

$(3-26)$ into $(3-25)$:

$$A_{CM} = \frac{v_{oc}}{v_{ic}}(s) \approx -\frac{R_L}{2\,R_E}(1 + s\,C_E\,R_E) \qquad (3-27)$$

$$\text{or} \quad A_{CM}(j\omega) \approx -\frac{R_L}{2\,R_E}(1 + j\omega C_E\,R_E) \qquad (3-28)$$

- So CM gain has a zero $\omega_z = \frac{1}{R_E\,C_E}$ causing CM gain to rise. This is undesirable!
- CM gain can not increase continuously as other caps become important.
- A_{dm} begins to roll of at

$$f = \frac{1}{2\pi\,R\,C_t} \quad \left[R = (R_S + r_b) \,\|\, r_\pi\right]$$

- Important parameter *common-mode rejection ratio* (CMRR):

$$\text{CMRR} = \left|\frac{A_{dm}}{A_{CM}}\right| \qquad (3-29)$$

CM Response Single-Stage BJT Diff. Amp.

- The decrease of CMRR further increases when $|A_{dm}|$ begins to fall with frequency.
- Thus diff. pairs are far less able to reject CM as the freq. of CM signals increases.
- On the other hand, according to the equation (3-25), $A_{CM}=-R_L/Z_E$ and it shows the CM gain of both leg of diff. pair (i.e. single-ended). If the matching was complete, the output signal wouldn't have the CM gain. In fact, if the output was completely differential, the exact A_v is defined by (6-72) Razavi's Book equation.
- Any way, the main goal is to have a low gain in both leg of differential pair, and fortunately in the low frequency that goals become possible, but in the high frequency, the node of emitter which is common, practically connected to ground by the C_{CS}, and the gain of single-ended in any legs decreases and their difference decreases, too.

CM Response Single-Stage BJT Diff. Amp.

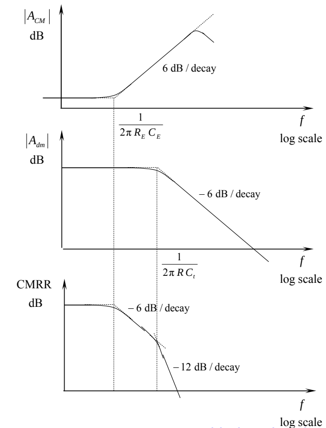


Fig.3.7. Frequency response of:
(a) common-mode gain, (b) differential gain,
(c) Common Mode Rejection Ratio

CM Response Single-Stage CMOS Diff. Amp.

Razavi's Book: Section 6.6

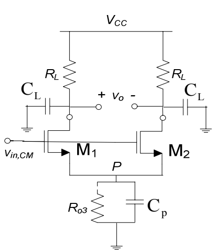


Fig.3.8

$$A_{CM}(s) = -\frac{\Delta g_m [R_L \parallel (\frac{1}{C_L s})]}{(g_{m1}+g_{m2})[r_{o3} \parallel (\frac{1}{C_P s})]+1}$$

$$\omega_z = -\frac{1}{r_{o3}C_P}$$

$$\omega_{p1} = -\frac{1}{R_L C_L}$$

$$\omega_{p2} = -\frac{g_{m1}+g_{m2}}{C_P}$$

Diff. Pair with Active Current Mirror

- Fig. 6.30 (Razavi), p.189 (last paragraph)- p.192