Chapter 5: Frequency Response

5.1 Nodal Analysis

Fig. 5.1: ECL pair with passive load

Differential mode (Half circuit)

Fig. 5.2: Half circuit and SS model

$$C_m = C_{jc}$$

$$C_p = C_{je} + C_b = C_{je} + g_m t_F$$

Assumptions: Neglect r_m, r_o, C_{cs}

Node 1: $$\frac{v_p - v_i}{R_S + r_b} + \frac{v_p}{r_p} + v_p sC_p + (v_p - v_o)sC_m = 0$$

Node 2 : $$(v_o - v_p)sC_m + g_m v_p + \frac{v_o}{R_L} = 0$$

Solving for v_o/v_i: $$\frac{v_o}{v_i} = A_{vo}\left[\frac{1-\frac{s}{z_1}}{1+as+bs^2}\right]$$

$$A_{vo} = -g_m R_L \frac{r_p}{R_S + r_b + r_p}$$

Neglect $z_1 = g_m/C_m >> w_T = g_m/(C_p + C_m)$;

Typically, $C_p >> C_m$

Coefficient $a = C_m R_L + C_m R_T + C_p R_T + g_m R_L R_T C_m$ where $R_T = (R_S + r_b)||r_p$

Coefficient $b = R_L R_T C_m C_p$

Denominator = $1 + as + bs^2$

$$p_1, p_2 = \frac{-a \pm \sqrt{a^2 - 4b}}{2b} = -\frac{a}{2b}\left[1 \pm \sqrt{1 - \frac{4b}{a^2}}\right]$$

For practical circuits, $b/a^2 << 1$

$$\sqrt{(1-x)} \approx \left(1 - \frac{x}{2}\right) \rightarrow if\ (x << 1)$$

$$p_1, p_2 = -\frac{a}{2b}\left[1 \pm \left(1 - \frac{2b}{a^2}\right)\right]$$

$$p_1, p_2 = -\frac{1}{a}; -\frac{a}{b}\left[1 - \frac{b}{a^2}\right]$$

So $|p_2| >> |p_1|$ where $(|p_2| > w_T)$

Fig. 5.3: Location of poles p_1, p_2

Example:

Fig. 5.4: ECL pair with passive load - example

$$C_m = C_{jc} = 0.5 \text{ pF}$$

$$C_p = C_{je} + g_m t_F = 2 \text{ pF} + (1/52)0.5 \text{ ns} \sim 12 \text{ pF}$$

$$r_p = b/g_m = 100\ (52) = 5.2 \text{ k}\Omega$$

$$p_1 = 1/a = 11 \text{ x } 10^6 \text{ r/s} \sim 2 \text{ MHz}$$

$$p_2 \sim a/b = 2 \text{ x } 10^9 \text{ r/s} \sim 300 \text{ MHz}$$

$$z = g_m/C_m = 40 \text{ x } 10^9 \text{ r/s} = 6 \text{ GHz}$$

$$v_{od}/v_{id} = -g_m R_L (r_p\ /\ (r_p + R_S)) = 160 = 44 \text{ dB}$$

Fig. 5.5: Approximate Bode Plot

- So there are 2 poles and 1 zero where $p_2 >> p_1$ and $z_1 >> p_1$
- Here p_1 predicts the frequency response until $f = p_2$.

5.2 Analysis of CE amplifier using Miller approximation

Fig. 5.6: General Technique

$$i_x = \frac{v_x - Av_x}{Z_x} = \frac{v_x}{Z_y} \Rightarrow Z_y = \frac{Z_x}{1-A}$$

$$i_x = \frac{v_x - Av_x}{Z_x} = -\frac{Av_x}{Z_z} \Rightarrow Z_z = -\frac{AZ_x}{1-A}$$

$$if \cdots Z_x = {}^{1}\!/_{sC}$$

$$Z_y = \frac{1}{s(1-A)C} \Rightarrow (1-A)C$$

$$Z_z = \frac{-A}{s(1-A)C} \Rightarrow \frac{(1-A)}{-A}C$$

An exact answer can be obtained if $A = A(s)$ is known.

Miller Approximation:

Since we do not know $A(s)$, we can use $A = A\ (s = 0)$ as a way to predict the first pole because $A(s)$ is quite close to $A(0)$. However, results for the second pole may be quite inaccurate.

Applying the Miller Approximation to CE analysis:

Fig. 5.7: SS circuit for CE amplifier

Since input and output loop are not independent, analysis is a bit tricky. With Miller approximation, we can split C_m

Fig. 5.8: SS circuits for CE amplifier – Miller Approx.

Miller Approximation: Use $A \sim A_{dc} = -g_m R_L$

So then, from the left half of the circuit, the first pole can be estimated,

$$|p_1| = \frac{1}{\left[(R_S + r_b) \parallel r_p\right]\left[C_p + C_m (1 + g_m R_L)\right]}$$

Compared to the first pole obtained by nodal analysis in the previous section,

$$|p_1| = -\frac{1}{a} = \frac{1}{\left[(R_S + r_b) \parallel r_p\right]\left[C_p + C_m (1 + g_m R_L) + C_m \frac{R_L}{R_T}\right]}$$

The second pole, however, is quite inaccurate in the Miller approximation,

$$|p_2| = \frac{1}{R_L C_m \left[{(1 + g_m R_L)} \big/ {g_m R_L} \right]} \sim \frac{1}{R_L C_m}$$

p_2 = 30 MHz in example compared to 300 MHz from the exact analysis

There is no zero provided by the Miller approximation.

The Gain Bandwidth (GBW) product then,

$$GBW = g_m R_L \frac{r_p}{r_p + r_b + R_S} \frac{1}{\left[(R_S + r_b) \parallel r_p\right]\left[C_p + C_m (1 + g_m R_L)\right]} \approx \frac{1}{(R_S + r_b) C_m}$$

Note that the GBW is independent of $R_L = R_C$ which indicates the tradeoff between gain and BW. Maximum GBW achieved for minimum values of R_S, r_b, and C_m

5.3 Frequency Response of multistage amplifiers

If we assume no dominant zeroes:

$$\frac{v_o}{v_i} = \frac{K}{b_0 + b_1 s + ... + b_n s^n} = \frac{K_1}{1 + \frac{b_1}{b_0} s + ... + \frac{b_n}{b_0} s^n}$$

where n = number of an independent energy storage element

$$\frac{v_o}{v_i} = \frac{K_1}{\left(1 + \frac{s}{p_1}\right) .. \left(1 + \frac{s}{p_n}\right)}$$

If we compare the co-efficients of the above two equations,

$$\frac{b_1}{b_0} = \frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} + ... + \frac{1}{p_n} \cong \frac{1}{p_1}$$

since, in most practical multistage integrated circuits, there is one dominant pole.

Fig. 5.20: Pole location in multi-stage amplifiers

If the dominant pole is on the negative real axis,

Fig. 5.21: Bode plot showing effect of dominant pole

We need to find b_1/b_0!

Here, we introduce zero value time constant analysis that gives b_0 and b_1 exactly even for complicated networks

$$f_{-3dB} \cong \frac{1}{2p\left(\frac{b_1}{b_0}\right)}$$

Consider an example of the CE amplifier using ZVTC analysis,

Fig. 5.23: CE amplifier circuit

Fig. 5.24: CE amplifier SS circuit

Start by open circuiting capacitors and turning off independent sources,

$\boldsymbol{R_{pEQ}}$

$$R_{pEQ} = (R_S + r_b)||r_p$$

Fig. 5.25: SS cct to extract equivalent resistance for C_p

R_{mEQ}

Fig. 5.26: SS cct to extract equivalent resistance for C_m

Node 1:

$$v_p = i_t R_{pEQ}$$

Node 2:

$$i_t + g_m\, i_t R_{pEQ} + (i_t R_{pEQ} - v_t)/R_L = 0$$

$$R_{mEQ} = v_t/i_t = \quad R_{pEQ} + (1 + g_m R_{pEQ})R_L$$

$$R_{mEQ} = R_{pEQ}\,(1 + g_m R_L) + R_L$$

$$w_{-3dB} = \frac{1}{\sum T_0} = \frac{1}{C_p R_{pEQ} + C_m R_{mEQ}}$$

If R_S = 10 kΩ, R_L = 10 kΩ, I_c = 100 μA, C_{je} = 1 pF, C_{jc} = 0.5 pF, t_F = 0.5ns, b = 100

g_m = 1/260Ω; r_p = 26kΩ; C_p = 1 pF + $g_m t_F$ = 3 pF; w_{-3dB} = $(22\text{ ns} + 150\text{ ns})^{-1}$ ~ 6 MHz

5.3.1 Limitations of ZVTC analysis

- Does not predict zeroes or work with dominant zeroes

Example: emitter follower

- Does not work with complex poles

Fig. 5.27: Bode plots

- When there isn't one dominant pole, the predicted w_{-3dB} is less accurate.

For example, in a cascade of identical amplifier stages,

Fig. 5.28: Cascaded amplifiers

Here,

$$\frac{v_o}{v_i} = \frac{K_1}{\left(1+\frac{s}{p_1}\right)\ldots\left(1+\frac{s}{p_n}\right)} = \frac{K_1}{\left(1+\frac{s}{p_1}\right)^n};\, p_1 = \frac{1}{RC};\, n = \#\, stages$$

when $w = w_{-3dB}$,
$$\frac{v_o}{v_i} = \frac{1}{\left(1+\left(\frac{w}{p_1}\right)^2\right)^{\frac{n}{2}}} = \frac{1}{\sqrt{2}}$$

and
$$w_{-3dB} = p_1\sqrt{2^{1/n} - 1}$$

n	w_{-3dB} (exact)	w_{-3dB} (ZVTC)	% error
1	p_1	p_1	0
2	0.64 p_1	0.5 p_1	22
3	0.51 p_1	0.33 p_1	35

Fig. 5.29: Comparison of ZVTC and exact analysis for more than one dominant pole

5.4 Frequency response of the 741 OPAMP

$$w_{-3dB} = \frac{1}{\sum T_0}$$

Fig. 5.30: Simplified 741 opamp schematic for frequency response calculations

Look for the node with the highest resistance and capacitance! For 741, an internal compensating capacitor of 30 pF has been added so this part is easy.

Consider the output of stage 1:

$$R_1 = R_{o1} \| R_{i2} = 6.8\ \text{M}\Omega \ \|\ 5.5\ \text{M}\Omega = 3\ \text{M}\Omega$$

Consider the output of stage 2:

$$R_2 = R_{o2} \| R_{i3} = 83\ \text{k}\Omega \ \|\ 9.0\ \text{M}\Omega = 82\ \text{k}\Omega$$

Fig. 5.31: Second stage of 741 opamp

$G_m = 1/147\Omega$;

$R_{EQ} = R_1 + R_2 + G_m R_1 R_2 = 3\ \text{M}\Omega + 82\ \text{k}\Omega + (1/147)(3\text{M})(82\text{k}) \sim 1.7 \times 10^9\ \Omega$

Therefore, $f_{-3dB} \sim (2\pi R_{EQ} C_{comp})^{-1} \sim 3.1$ Hz (in reality, if SPICE simulations are performed, you will calculate this value to be around 5 Hz)

Using the Miller method since C_{comp} is a series capacitor,

$C_M = (1 + G_m R_2) C_{comp} = (1 + (1/147)82\text{k})(30\text{pF}) \sim 16{,}765$ pF

Therefore, $f_{-3dB} \sim (2\pi R_1 C_{comp})^{-1} \sim 3.2$ Hz

- There are the other non-dominant poles namely the Q_{23} PNP emitter follower, active load and the PNP level shifter (Q_2, Q_4)

- Adding the internal compensating capacitor had the beneficial effect of moving the dominant pole far away from the other poles so one gets a stable, well-defined gain function.

- However, this comes at a loss in gain at higher frequencies since the pole introduces a -20 dB/decade slope in the gain function at the very low frequency of 5 Hz.