Electronic III

Bipolar Transistor Models

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Simplified Bipolar Operation

- When the emitter junction is forward biased, it conducts. It consist of majority carriers from emitter (electrons here) and majority carriers from base (holes here).
- Since emitter is much more heavily doped than base, injected electrons from emitter are many more.
- Assuming collector voltage is high (collector-base is reversed biased) no holes from the base will go to the collector.
- However electrons that travel from the emitter to the base, where they are now minority carriers diffuse away from the base-emitter junction due to the minority carriers concentration gradient in base.

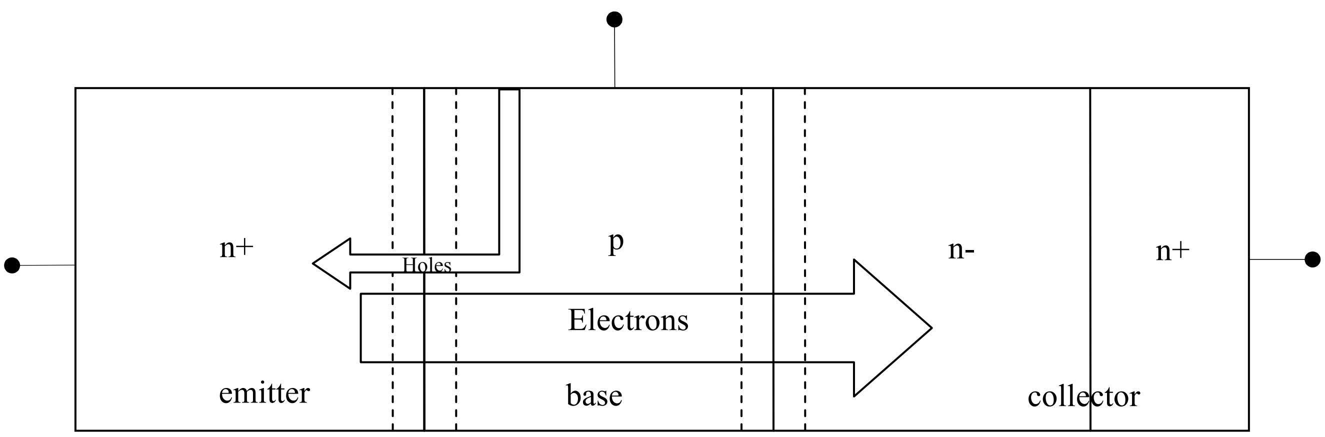

Fig.2.1

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Simplified Bipolar Operation

- Any of these electrons that get close to collector-base junction will immediately be “whisked” across the junction due to the large positive voltage on the collector, which attracts electrons.
- In a properly designed ***vertical bipolar***, the vertical base width W (next page figure) is small, so almost all of electrons that diffuse from the emitter to base reach collector-base junction and are swept across junction.
- So the collector current very closely equals the electron current flowing from the emitter to base.
- The much smaller base current very closely equals the current due to the holes that flow from base to emitter.

Simplified Bipolar Operation

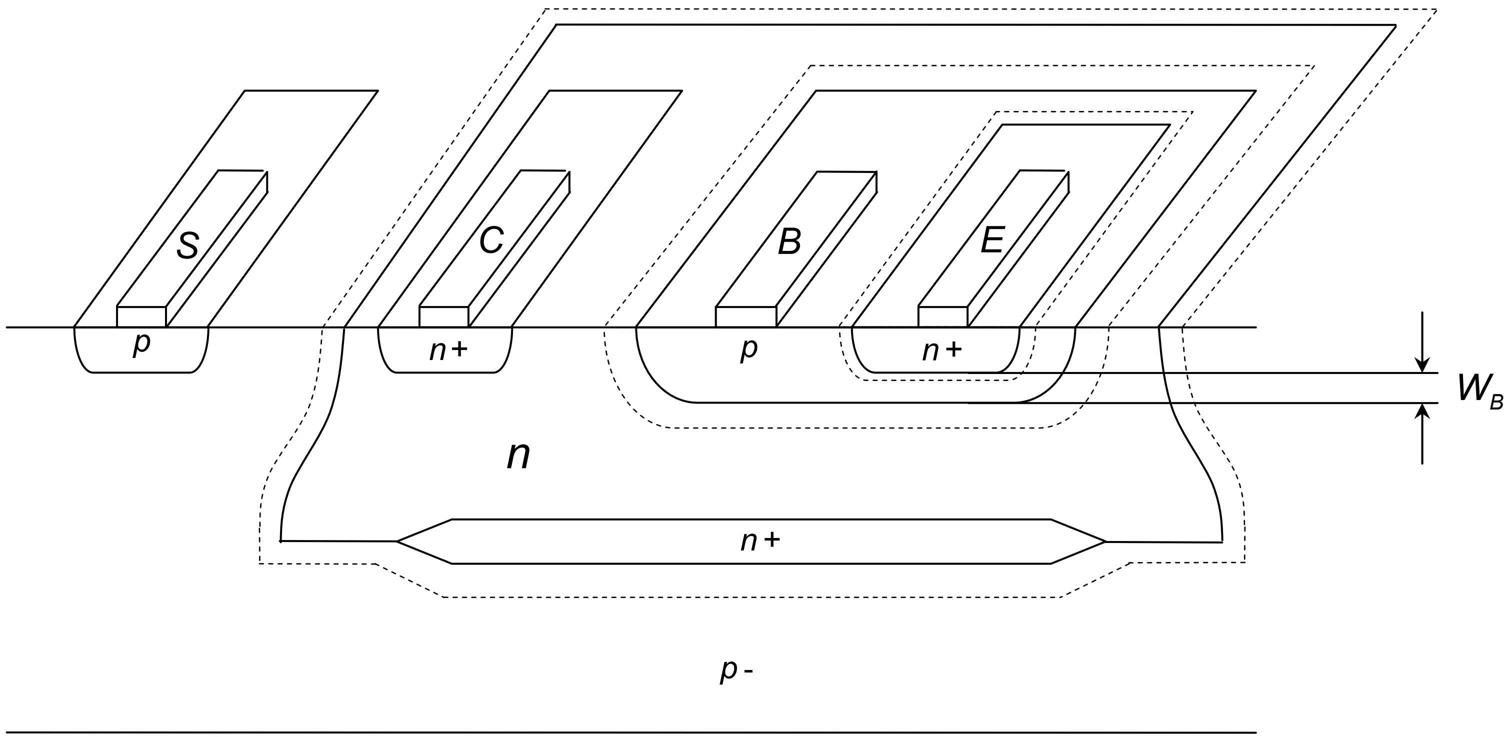


Fig.2.2.The vertical npn bipolar transistor in IC

The Hybrid-π small signal model

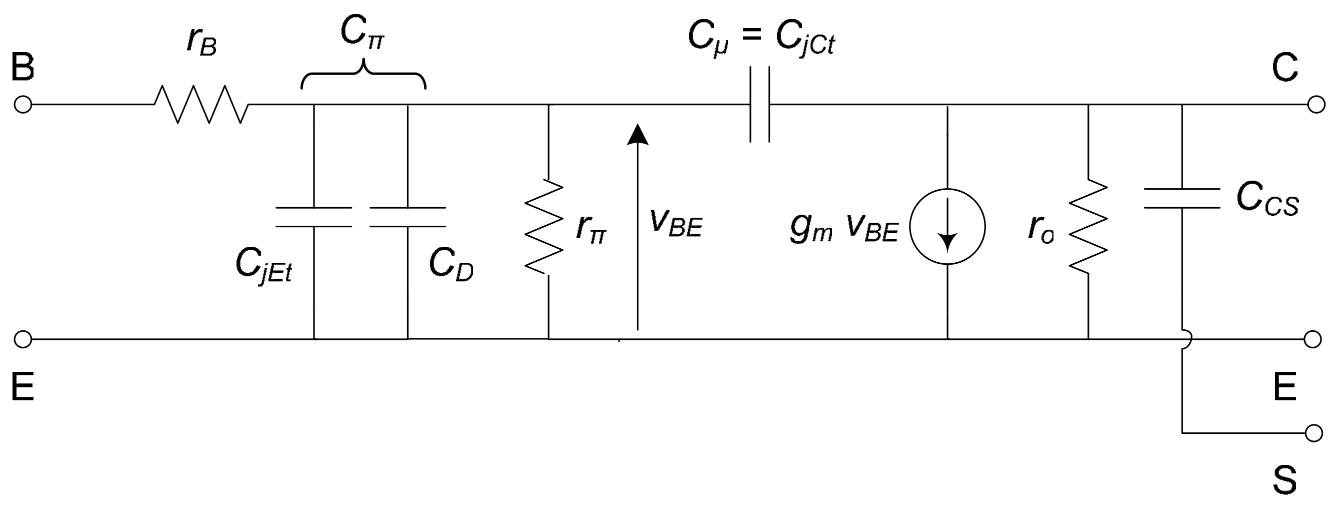


Fig.2.3.The hybrid-π model of bipolar transistor

The Hybrid-π small signal model

$$\begin{cases} I_C = I_S \exp(\dfrac{v_{BE}}{kq/T}) & (2-1) \\ I_S = \dfrac{q\, A_{EB}\, D_n\, n_i^2}{Q_B} & (2-2) \end{cases}$$

- $Q_B = N_B W_B$ is the integrated charge (per cm^2 emitter area) in the base.
  N_B is base doping concentration (m^{-3}).
  W_B is the base width (m).
- A_{EB} is the EB area (m^2).
- D_n is the diffusion constant for electrons (m^2/sec). It is related to mobility by Einstein's relation, given by: $D_n = \mu_n (kT/q)$.
- n_i is intrinsic silicon electron concentration (at a given temperature) (m^{-3}).
  (@ room temperature $n_i = 1.5\times 10^{10} / \mathrm{cm}^3$)

The Hybrid-π small signal model

- **Tranconductance** g_m

$$g_m = \frac{di_C}{dv_{BE}} = \frac{I_C}{kT/q} \qquad (2-3)$$

- Directly proportional with I_C. Doubling the current doubles the Tranconductance.
- $\frac{g_m}{I_C} = \frac{q}{kT} = \frac{1}{26\,\mathrm{mV}}$ @ room temperature larger than the MOST counterpart.

$$g_m = \frac{2I_{DSQ}}{V_{GSQ} - V_{TH}} \Rightarrow \text{for } \mathrm{V_{eff}} = 0.2 \quad \frac{g_m}{I_{DSQ}} = \frac{2}{0.2} = \frac{1}{100\,\mathrm{mV}}$$

The Hybrid-π small signal model

- **Input Resistance** r_π

The ratio of the AC V_{BE} and the AC I_B is the AC input resistance. It is called r_π as follows:

$$r_\pi = \frac{dV_{BE}}{dI_B} = \frac{dV_{BE}}{dI_C}\frac{dI_C}{dI_B} = \frac{\beta_{AC}}{g_m}$$

Its relationship with β and g_m is depicted in figure.

$$g_m r_\pi = \beta_{AC} \qquad (2-4)$$

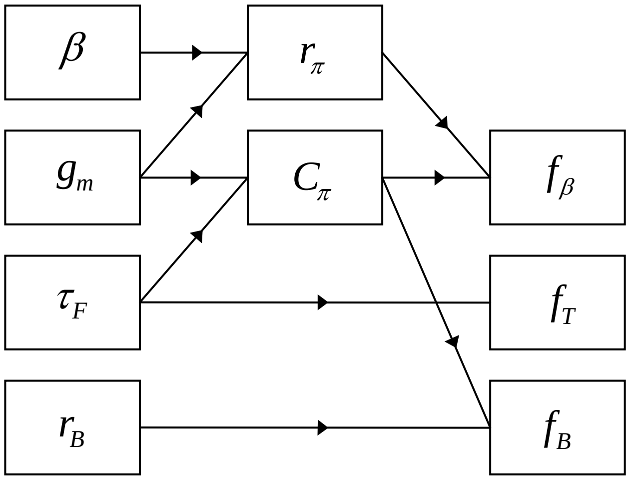


Fig.2.4

The Hybrid-π small signal model

- **Output Resistance** ***r_o***

  r_o is the ratio of the AC V_{CE} to the AC I_C: $r_o = \frac{dV_{CE}}{dI_C}$

$$\begin{cases} I_C = I_S \exp\left(\frac{V_{BE}}{kq/T}\right) \\ I_S = \frac{q \; A_{EB} \; D_n \; n_i^2}{Q_B} \end{cases}$$

$$\Delta V_{CE} = \Delta V_{CB} + \Delta V_{BE} = -\Delta V_{BC} \quad \text{Since} \quad V_{BE} = cte$$

$$r_o = \frac{dV_{CE}}{dI_C} = -\frac{dV_{BC}}{dI_C} = -\frac{dV_{BC}}{dQ_B} \left(\frac{dQ_B}{dI_C}\right) \quad (2-5)$$

Increasing $-V_{BC}$ by $-\Delta V_{BC}$ increases the width of the depletion layer as shown in next figure. The depletion layer charge increases by ΔQ_B.

So less Q_B is left (the base charge is decreased by the same amount ΔQ_B.

The Hybrid-π small signal model

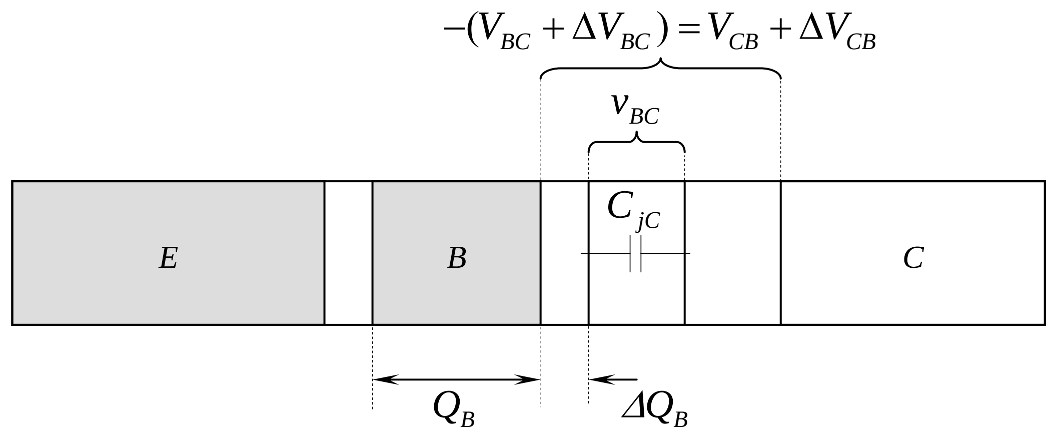


Fig.2.5. Relation between ΔQ_B and ΔV_{BC} through C_{jC}

Also the variation of the depletion layer charge ΔQ_B is linked to the depletion capacitance C_{jC} (in F/cm²) as:

$$q\frac{\Delta Q_B}{\Delta I_C}=C_{jC} \Rightarrow \frac{-dV_{BC}}{-dQ_B}=\frac{q}{C_{jC}}$$

The second term in $r_o = -\frac{dv_{BC}}{dQ_B}\left(\frac{dQ_B}{dI_C}\right)\ldots(2-5)$ is derived from:

$$I_C=\frac{q\,A_{EB}\,D_n\,n_i^2}{Q_B}\exp(V_{EB}/V_T) \quad \Rightarrow \quad \frac{dI_C}{dQ_B}=-\frac{q\,A_{EB}\,D_n\,n_i^2}{Q_B^2}\exp(V_{EB}/V_T)=-\frac{I_C}{Q_B} \qquad (2-7)$$

The Hybrid-π small signal model

$$\Rightarrow r_o = \frac{dV_{CE}}{dI_C} = (-\frac{dV_{BC}}{dQ_B})(\frac{dQ_B}{dI_C}) = (-\frac{q}{C_{jC}})(-\frac{Q_B}{I_C}) = \frac{q \quad Q_B}{C_{jC} \quad I_C}$$

Defining Early voltage as:

$$r_o = \frac{V_A}{I_C} \Rightarrow V_A = \frac{q \quad Q_B}{C_{jC}} = \frac{q \quad A_{BC} \quad Q_B}{C_{jC\,t}} \quad (2-8)$$

$$C_{jC\,t} = A_{BC} \quad C_{jC}$$

A_{BC} is the total base-collector area.

$C_{jC\,t}$ is the total base-collector junction capacitance.

Represented by C_μ in the hybrid-π model

The Hybrid-π small signal model

- **Small-Signal Voltage Gain** A_v

$$A_v = g_m r_o$$

$$g_m = \frac{I_C}{V_T} \; , \; r_o = \frac{V_A}{I_C} \;\; \Rightarrow \;\; A_v = \frac{V_A}{V_T} = \frac{V_A}{kT/q} \qquad (2-9)$$

For $V_A = 50\,\mathrm{V} \Rightarrow \;\; A_v \approx 2000$ V/V or 66 dB

In comparison with BJT gain of MOST stage:

$$A_v = \frac{V_E\, L}{1/2V_{eff}} = \frac{1/(2\lambda)}{V_{eff}}$$

The Hybrid-π small signal model

- **Capacitances**

$$C_\pi = C_{be} = C_j + C_D$$

Where Cj is the depletion capacitance of the base-emitter junction. For forward biased junction:

$$C_{jEt} = \frac{A_E C_{jE0}}{\left(1 - \frac{V_{BE}}{\varphi_{jE0}}\right)^{m_{jE}}} \quad , \quad m_{jE} = 1/3 \qquad (2-10)$$

$$C_{jEt} \approx 2 A_E C_{je0} \qquad (2-11)$$

- ***Diffusion Capacitance C_D***

A variation in base-emitter voltage Δv_{BE} causes a variation in injected charge ΔQ_F.

Q_F, the dynamic charge is not same as Q_B. Q_F, the total integrated charge in the base depends on the forward bias V_{BE}, where Q_B is the charge that is physically present in the base. They don't have same dimension either. Q_B, number of carrier per cm^2, Q_F, total charge in coulombs.

The Hybrid-π small signal model

This is the charge of minority carriers in base (electrons in npn). This variation causes a variation in majority carriers charge in collector (as explained in BJT operation).

$\Delta Q_F/\Delta v_{BE}$ has the dimension of capacitance and is called the diffusion capacitance.

$$C_D = \frac{dQ_F}{dV_{BE}} = \frac{dQ_F}{dI_C}\frac{dI_C}{dV_{BE}} = \tau_F \quad g_m \quad (2-12)$$

- ***Base Transit Time τ_F***

  We can write the current in BJT as follows:

- The more injected minority charge into base, the more current to collector.
- The faster injected charge reach collector (smaller τ_F), the higher collector current is.

  As it will be seen: $\tau_F = \dfrac{W_B}{v_{sat}}$, v_{sat} is the saturation velocity.

The Hybrid-π small signal model

- This leads to the physical interpretation that τ_F is the average time in which the electrons diffuse through the base from the emitter side to the collector side.

  It is a measure of the max. frequency: $$f_{T\max} = \frac{1}{2\pi\tau_F} \qquad (2-13)$$

Note: because of existing channel length modulation, the MOST has v_{sat} and max. frequency:

$$f_{\max} = \frac{1}{2\pi}\frac{v_{sat}}{L_{eff}}$$

$$C_D = \tau_F g_m = \frac{W_B^2}{2 \quad D_n} g_m = \frac{W_B^2}{2 \quad \mu_n \quad kT/q} \frac{I_C}{kT/q} = \frac{W_B^2 \quad I_C}{2 \quad \mu_n \quad (kT/q)^2} \quad (2-14)$$

So physically C_D is directly proportional with W_B^2 (width of base region squared) and the collector current, μ_n and kT/q are constant.

The Hybrid-π small signal model

- The diffusion capacitance is much larger than the base-emitter junction capacitance. It increases exponentially with V_{BE}, whereas C_{jEt} increases only with the square root of V_{BE}.

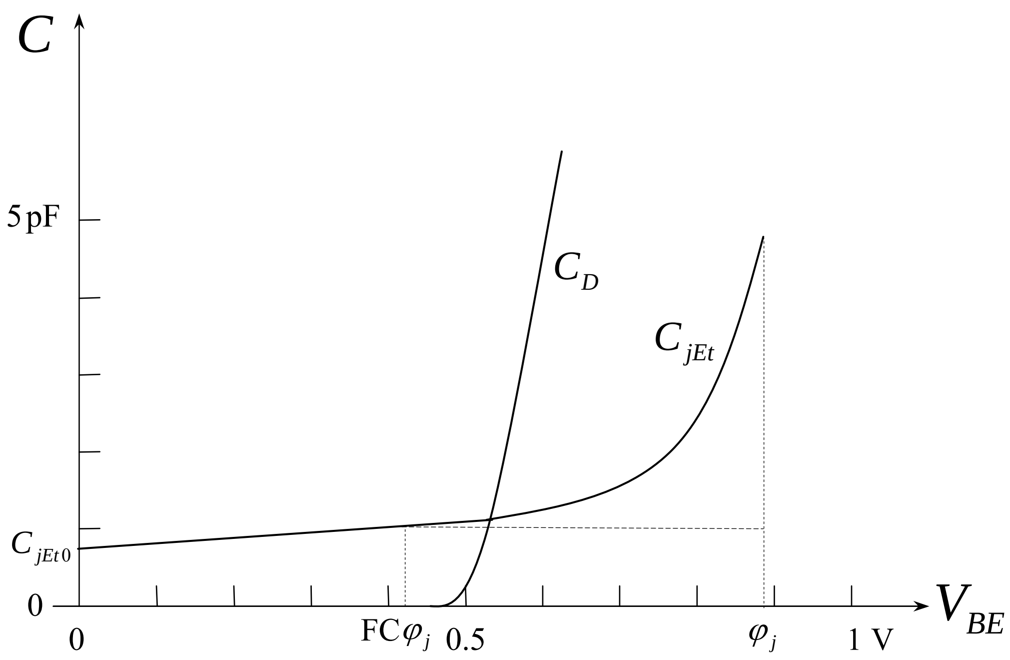


Fig.2.6

The Hybrid-π small signal model

- ***Collector Junction Capacitance C_μ***

  C_μ models the depletion capacitance of the collector-base junction (normally in reverse bias). Since this is a graded junction:

$$C_\mu = C_{jCt} = C_{jCb} = \frac{A_C\, C_{jC0}}{(1+\frac{V_{CB}}{\varphi_{C0}})^{1/3}} \qquad (2-15)$$

Where A_C is the effective area of the collector-base interface.

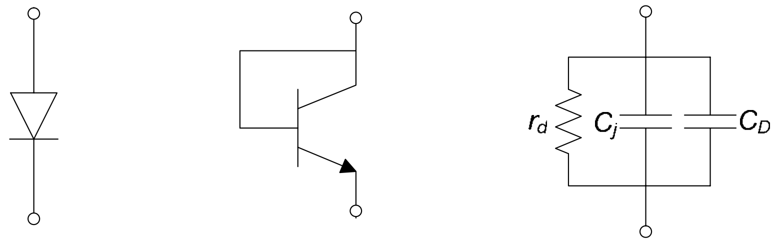
Junction Potentials:

$$\varphi_{EB} = \frac{kT}{q}\ln(\frac{N_E\, N_B}{n_i^2}) \approx 0.95\,\mathrm{V}$$

$$\varphi_{CB} = \frac{kT}{q}\ln(\frac{N_C\, N_B}{n_i^2}) \approx 0.73\,\mathrm{V}$$

$$N_E = 10^{19}\,/\,\mathrm{cm}^3 \quad , N_B = 2\times 10^{17}\,/\,\mathrm{cm}^3 \quad , N_C = 2\times 10^{15}\,/\,\mathrm{cm}^3$$

Small Signal Model for Forward-Biased Diode



$$r_d = \frac{V_T}{I_D} \quad , C_t = C_D + C_j \qquad (2-16)$$

$$C_D = \tau_F \frac{I_D}{V_T} \quad , C_j \approx 2 C_{j0} \qquad (2-17)$$

Where:

$$C_{j0} = \sqrt{\frac{q\,\varepsilon_{si}}{2\varphi_0} \frac{N_D N_A}{N_A + N_D}}\Bigg|_{N_A >> N_D} = \sqrt{\frac{q\,\varepsilon_{si} N_D}{2\varphi_0}}$$

and $\tau_F = \dfrac{W_n^2}{2 D_n}$

The Hybrid-π small signal model

Base Resistance r_B

The active region of the bipolar transistor is located directly underneath the emitter. The base region is contacted by means of ohmic regions that add series resistance as well as additional capacitance.

For example, a series base resistance r_B is present between the base contact metal and the active base. It is the most important parasitic series resistance and is included in the hybrid-π model.

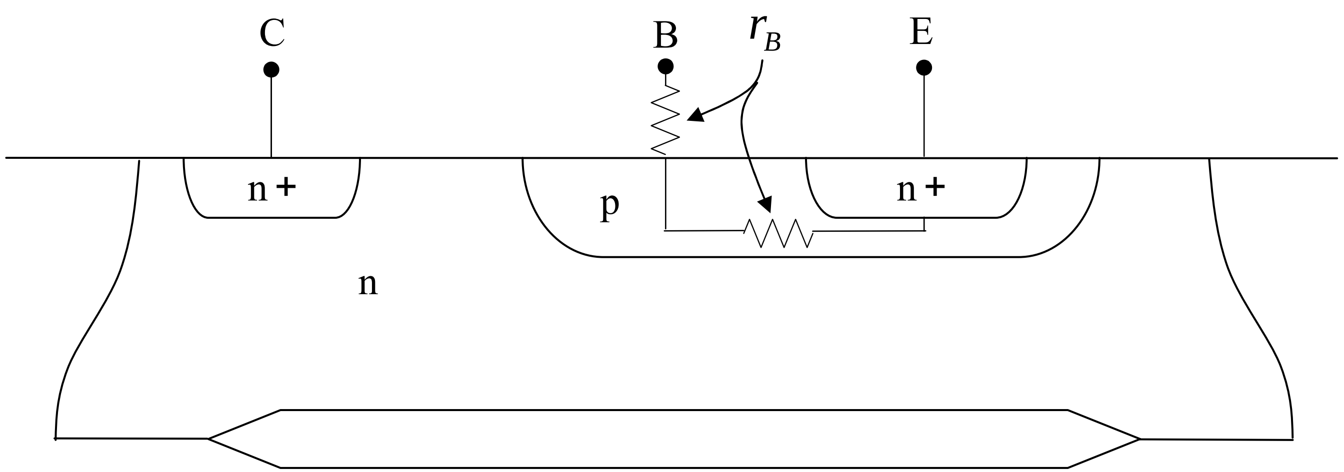


Fig.2.7

A Simple High-Freq. Model for CE with Current Drive

- In first example we want to realize a *current-gain amplifier.* For this purpose a BJT amplifier, which has a large input source resistance R_S.
- The AC signal is amplified, then is short circuited by a large cap. C_∞ to ground, through which we can measure i_{Out}.

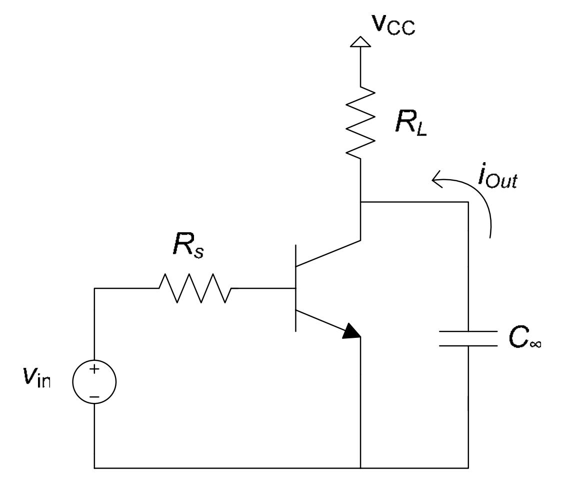


Fig.2.8-a

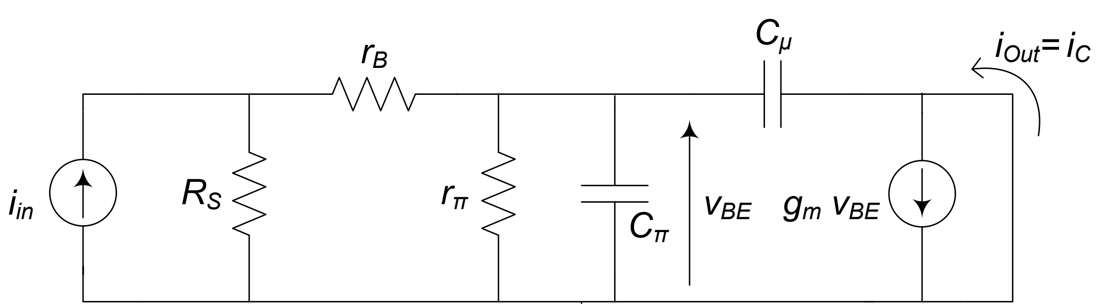


Fig.2.8-b

A Simple High-Freq. Model for CE with Voltage Drive

- In 2nd example we want to realize a *voltage-to-current amplifier.* For this purpose a BJT amplifier, which has a small input source resistance R_S.
- The AC signal is amplified, then is short circuited by a large cap. C_∞ to ground, through which we can measure i_{Out}.

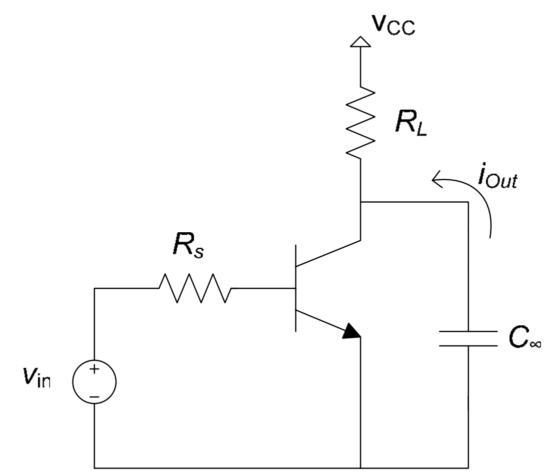


Fig.2.8-a

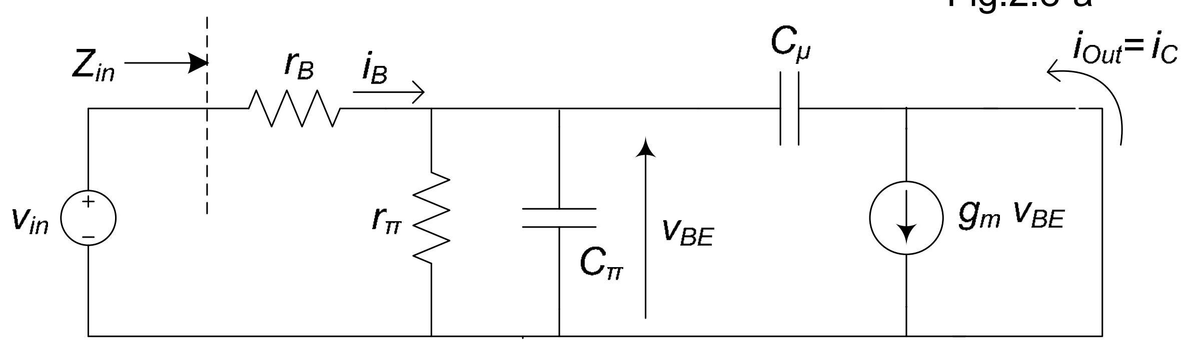


Fig.2.9

A Simple High-Freq. Model for CE with Current Drive

- Since output is short-circuited r_o can be left out. A current $i_C = i_{Out}$ flows in the output short circuit.
- $R_S >> r_\pi + r_B$ therefore the transistor is current driven. Its input current is approximately $i_{in} = v_{in} / R_S$.
- $R_S = \infty$, so from Fig.2.8-b one could say since the input impedance of device is very small compared to R_S so the input current is almost equal to v_{in} / R_S .

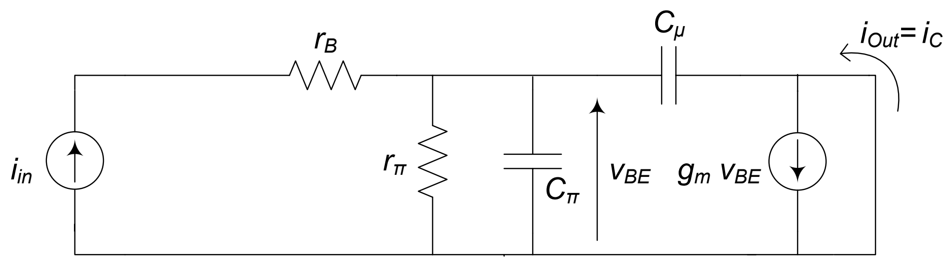


Fig.2.8-c

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A Simple High-Freq. Model for CE with Current Drive

- A common indicator for the speed of a BJT is the frequency at which the transistor current gain drops to unity, when the collector is connected to a small-signal ground: f_T = Unity-Gain Frequency
- So for $R_S >> r_\pi + r_B$:

$$v_{BE} = i_B [r_\pi + \frac{1}{s(C_\pi + C_\mu)}] = i_B \frac{\frac{r_\pi}{s(C_\pi + C_\mu)}}{r_\pi + \frac{1}{s(C_\pi + C_\mu)}} = i_B \frac{r_\pi}{1 + s r_\pi (C_\pi + C_\mu)}$$

and $\quad i_C = g_m v_{BE} \quad \Rightarrow \quad \frac{i_C}{i_B} = \frac{g_m r_\pi}{1 + s r_\pi (C_\pi + C_\mu)}$

This is true for the frequencies near f_T if: $f_T << g_m / C_\mu$

$$\frac{i_C}{i_B} = \frac{\beta_{AC}}{1 + j f / f_\beta} \quad \text{where} \quad f_\beta = \frac{1}{2\pi r_\pi (C_\pi + C_\mu)} \quad \text{and} \quad \beta_{AC} = g_m r_\pi$$

Note: $\left| \frac{i_C}{i_B}(\omega) \right| = \frac{g_m r_\pi}{\omega (C_\pi + C_\mu) r_\pi}$ for $f >> f_\beta$

for $\left| \frac{i_C}{i_B}(\omega) \right| = 1 \Rightarrow \omega_T = \frac{g_m}{C_\pi + C_\mu}$ or $f_T = \frac{g_m}{2\pi (C_\pi + C_\mu)} \qquad (2-18)$

A Simple High-Freq. Model for CE with Current Drive

$$\beta(j\omega) = A_I = \frac{i_{Out}}{i_{in}} = \frac{\beta_{AC}}{1 + j\dfrac{f}{f_\beta}} \qquad (2-19)$$

$$, f_\beta = \frac{f_T}{\beta_{AC}} \qquad (2-20)$$

$$, f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)}$$ Unity-Gain Frequency

$\Rightarrow \quad f_\beta = f_{-3\text{dB}}$ for current gain A_I

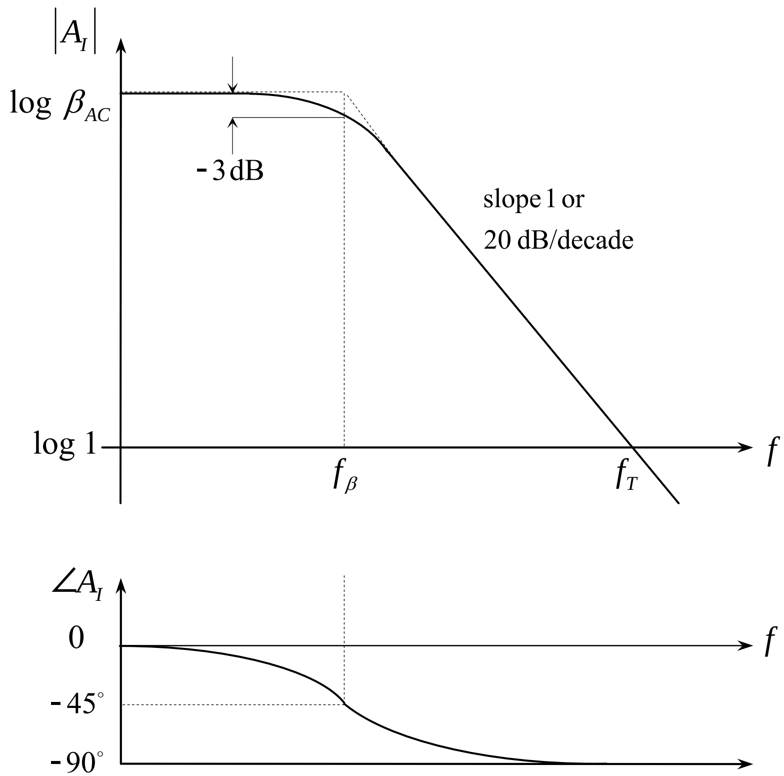


Fig.2.10

A Simple High-Freq. Model for CE with Current Drive

$$f_T = \frac{g_m}{2\pi(C_\pi + C_\mu)} = \frac{g_m}{2\pi(C_D + C_{jEt} + C_\mu)}$$

and $C_D = g_m \tau_F$

$$\Rightarrow \quad f_T = \frac{g_m}{2\pi(g_m \tau_F + C_{jEt} + C_\mu)}$$

$$= \frac{1}{2\pi\tau_F} \frac{1}{1 + \frac{C_{jEt} + C_\mu}{g_m \tau_F}} \quad \Rightarrow \quad f_T = \frac{1}{2\pi\tau_F} \frac{I_C}{I_C + \frac{I_C(C_{jEt} + C_\mu)}{g_m \tau_F}}$$

$$= \frac{1}{2\pi\tau_F} \frac{I_C}{I_C + (C_{jEt} + C_\mu)\frac{V_T}{\tau_F}}$$

$$\begin{cases} f_T = \dfrac{1}{2\pi\tau_F} \dfrac{I_C}{I_C + I_{CfT}} & (2-21) \\ I_{CfT} = (C_{jEt} + C_\mu)\dfrac{kT/q}{\tau_F} & (2-22) \end{cases}$$

A Simple High-Freq. Model for CE with Current Drive

- f_T is reached to its max. value at medium and high currents. The transition current at which this occurs is denoted by I_{CfT} :

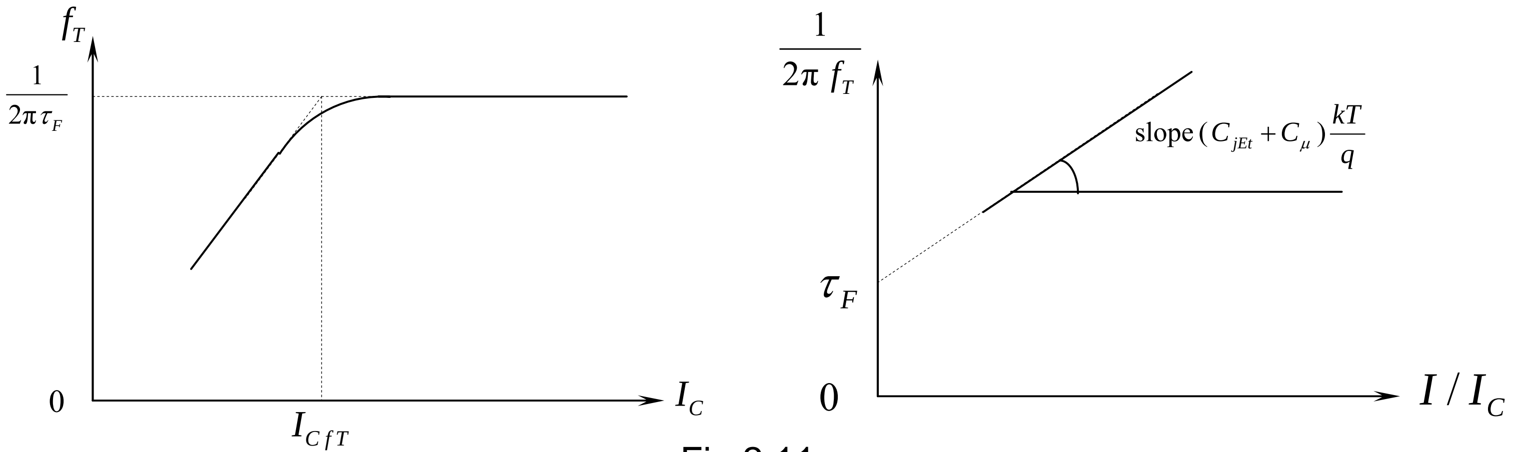


Fig.2.11

- Finally, f_T is specified for a bipolar transistor with a short-circuited (for AC signal) collector. If an ohmic series resistance r_C is present, the output can be shorted but there is still some collector resistance that remains.

$$\frac{1}{2\pi f_T}=\tau_F+(C_{jEt}+C_\mu)\frac{kT/q}{I_C}+r_C C_\mu \qquad (2-23)$$

A Simple High-Freq. Model for CE with Current Drive

Example:

Calculate f_β and f_T for I_C=0.01 mA, 0.1 mA & 1 mA ?

If r_C=30 Ω, what is the $f_{T\max}$?

What is the value of transition current (I_{CfT}) ?

$\beta = 100\,,\ \tau_F = 0.25\,\text{ns}\,,\ C_{jEt} = 5\,\text{pF}\,,\ C_\mu = 1\,\text{pF}$

Solution:

$$\frac{1}{2\pi\, f_T} = \tau_F + (C_{jEt} + C_\mu)\frac{kT/q}{I_C}$$

$$I_C = 0.01\,\text{mA} \quad \Rightarrow \quad \frac{1}{2\pi\, f_T} = 0.25\times10^{-9} + (5+1)\times10^{-12}\times\frac{25\,\text{mV}}{0.01\ \text{mA}} = 15.25\times10^{-9}$$

$$f_T = 10.6\,\text{MHz}$$

for $\quad I_C = 0.1\,\text{mA} \quad \Rightarrow \quad f_T = 88\ \text{MHz}$

$\quad I_C = 1\,\text{mA} \quad \Rightarrow \quad f_T = 393\ \text{MHz}$

$$f_{T\max} = \frac{1}{2\pi\,\tau_F} = \frac{1}{2\pi\times0.25\times10^{-9}} = 636\ \text{MHz}$$

A Simple High-Freq. Model for CE with Current Drive

Addition of r_C $\Rightarrow$

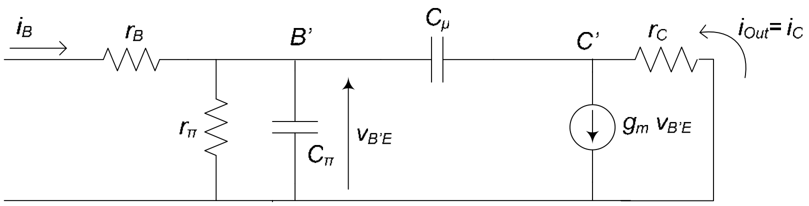
$$\frac{1}{2\pi f_{T\max}} = \tau_F + r_C C_\mu$$

$$\Rightarrow \quad f_{T\max} = \frac{1}{2\pi(\tau_F + r_C C_\mu)} = \frac{1}{2\pi(0.25\times10^{-9} + 30\Omega\times1\times10^{-12})} = 568\,\text{MHz}$$

$$I_{C\,fT} = (C_{jEt} + C_\mu)\frac{kT/q}{\tau_F} = (5+1)\times10^{-12}\times\frac{25\,\text{mV}}{0.25\times10^{-9}} \cong 0.6\text{ mA}$$

$$f_T\Big|_{I_C=0.6\text{ mA}} = 318\text{ MHz} = 1/2\, f_{T\max}$$

A Simple High-Freq. Model for CE with Current Drive



An incorrect approach:

$$r_\pi \,\|\left(\frac{1}{s\ C_\pi}\right)\|\left(\frac{1}{s\ C_\mu}+r_C\right)=r_\pi\,\|\,\frac{\frac{1}{s\ C_\pi}\left(\frac{1}{s\ C_\mu}+r_C\right)}{\frac{1}{s\ C_\pi}+\frac{1}{s\ C_\mu}+r_C}=r_\pi\,\|\,\frac{1+r_C\ C_\mu\ s}{s\ (\ C_\pi+C_\mu\ )+s^2\ r_C\ C_\pi\ C_\mu}$$

$$\frac{v_{B'E}}{i_B}=r_\pi\,\|\,\frac{1+r_C\ C_\mu\ s}{s\ (\ C_\pi+C_\mu\ )+s^2\ r_C\ C_\pi\ C_\mu}$$

$$r_B=0\Rightarrow\frac{i_C}{i_B}=\frac{g_m\ r_\pi\ (1+r_C\ C_\mu\ s)}{1+s\ [r_C\ C_\mu+r_\pi\ (\ C_\mu+C_\pi\ )]+s^2\ r_\pi\ r_C\ C_\pi\ C_\mu}$$

Note: for $\omega<<\dfrac{1}{r_C\ C_\mu}$ or $r_C\ C_\mu\ s<<1$

One can see $s^2\ r_\pi\ r_C\ C_\pi\ C_\mu=(\ r_C\ C_\mu\ s\ )(\ r_\pi\ C_\pi\ s\ )<<(\ 1\ )(\ r_C\ C_\mu\ s\ )$

A Simple High-Freq. Model for CE with Current Drive

So we can ignore the s^2 term compared
s term in dominant as well as
$r_C\, C_\mu\, s$ term in numerator:

for $\omega << \dfrac{1}{r_C\, C_\mu} \Rightarrow \dfrac{i_C}{i_B}(s) \approx \dfrac{g_m\, r_\pi}{1 + s[r_\pi\, C_\mu + r_\pi\,(C_\mu + C_\pi)]} \Rightarrow$

$$f_\beta = \frac{1}{2\pi[(C_\mu + C_\pi)\, r_\pi + C_\mu\, r_C]} \qquad (2-24)$$

<u>Much more accurate approach:</u>

The voltage across C_μ:

$V_{B'C'} \approx V_{be'} - (-g_m\, r_C V_{be'}) = V_{be'}(1 + g_m\, r_C)$

So one can simply replace C_μ with $(1 + g_m r_C) C_\mu$ in previous equation for f_T (2-18)

$$f_T = \frac{g_m}{2\pi\ (C_\pi + C_\mu)} \rightarrow f_T = \frac{g_m}{2\pi\ (C_\pi + C_\mu + g_m\, r_C C_\mu)}$$

$$\Rightarrow f_T = \frac{1}{2\pi\ \left(\dfrac{C_\pi + C_\mu}{g_m} + r_C C_\mu\right)} \qquad (2\text{-}24)$$

CE Configuration with Voltage Drive

- **<u>Tranconductance</u>** $\boldsymbol{A_G = i_{out} / v_{in}}$
  The configuration is the same as below figure but R_S is much smaller than r_π . The small-signal model is shown below (for zero R_S).

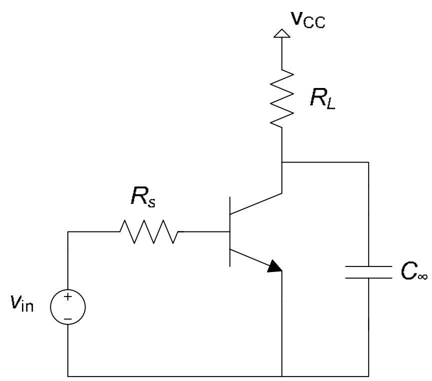


Fig.2.12-a

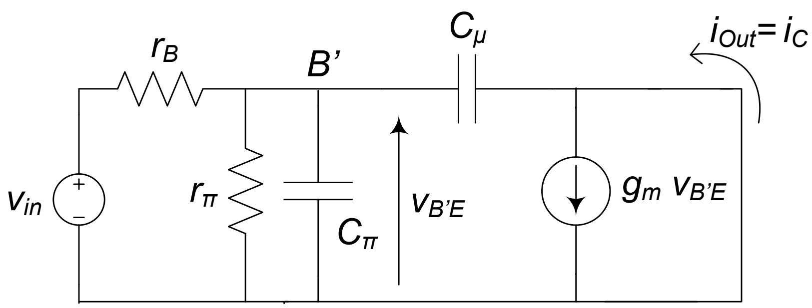


Fig.2.12-b

Low-Frequency:
Neglect all capacitances.

$$\text{for } r_B << r_\pi \quad \Rightarrow \quad \frac{i_{Out}}{v_{in}} = g_m$$

CE Configuration with Voltage Drive

High-Frequency

$$v'_{BE} = \frac{g_B}{g_B + [g_\pi + s(C_\mu + C_\pi)]} v_{in}$$

$i_{Out} \approx g_m v'_{BE}$ neglecting C_μ

$$\Rightarrow \quad \frac{i_{Out}}{v_{in}} = \frac{i_{Out}}{v'_{BE}} \qquad \frac{v'_{BE}}{v_{in}} = g_m \frac{g_B}{g_B + [g_\pi + s(C_\mu + C_\pi)]} = \frac{\dfrac{g_m g_B}{(g_m + g_B)}}{1 + s\dfrac{C_\mu + C_\pi}{g_m + g_B}}$$

for $g_B >> g_\pi$ or $r_B << r_\pi \quad \Rightarrow \quad g_B + g_\pi \cong g_B$

$$\frac{i_{Out}}{v_{in}} \approx \frac{g_m}{1 + s\dfrac{C_\mu + C_\pi}{g_B}} \quad \Rightarrow \quad A_G = \frac{i_{Out}}{v_{in}} = \frac{g_m}{1 + j\, f / f_B} \qquad (2-25)$$

where $f_B = \dfrac{1}{2\pi r_B (C_\mu + C_\pi)}$ $\qquad (2-26)$

CE Configuration with Voltage Drive

$\Rightarrow \quad f_B = f_{-3\text{dB}}$ for transconductance current-gain

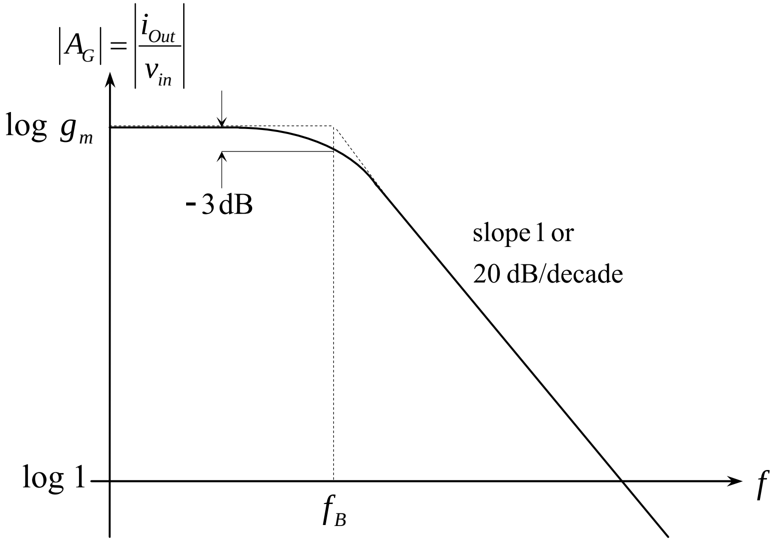


Fig.2.13

Unity-gain frequency for A_G (Tranconductance) is meaningless, i.e. we are not interested to know where g_m=1 A/V!!

CE Configuration with Voltage Drive

$$r_B\ (C_\pi + C_\mu) = r_B\ (C_D + C_{jEt} + C_\mu) = r_B\ (C_{jEt} + C_\mu)\left(1 + \frac{g_m\ \tau_F}{C_{jEt} + C_\mu}\right)$$

Defining: $\tau_B = r_B\ (C_{jEt} + C_\mu)\quad (2-27)$

$$r_B\ (C_{jEt} + C_\mu) = \tau_B\left(1 + \frac{g_m\ \tau_F}{C_{jEt} + C_\mu}\right)\ (\ \mathrm{I}\ )$$

Besides: $1 + \dfrac{g_m\ \tau_F}{C_{jEt} + C_\mu} = 1 + \dfrac{I_C\ \tau_F}{\dfrac{kT}{q}\ (C_{jEt} + C_\mu)}$

and $I_{C\,f\,T} = (C_{jEt} + C_\mu)\dfrac{kT/q}{\tau_F}$

$$1 + \frac{g_m\ \tau_F}{C_{jEt} + C_\mu} = 1 + \frac{I_C}{I_{C\,f\,T}} = \frac{I_{C\,f\,T} + I_C}{I_C}\ (\ \mathrm{II}\ )$$

from (I) & (II) we can rewrite f_B as follows:

$$f_B = \frac{1}{2\boldsymbol{\pi}\ r_B(C_\pi + C_\mu)} = \frac{I_{C\,f\,T}}{2\boldsymbol{\pi}\ \tau_B\ (I_C + I_{C\,f\,T})}\quad (2-28)$$

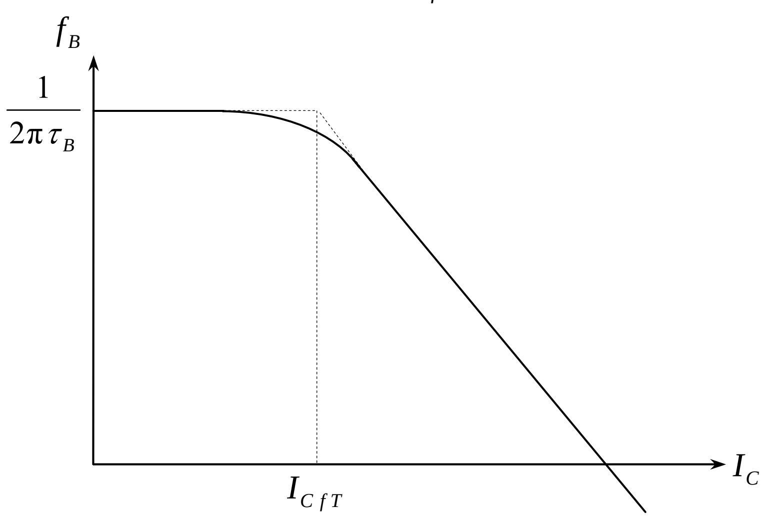
CE Configuration with Voltage Drive

Note: Can be modified as follows too (by adding collector resistor related time constant):

$$\tau_B = r_B \left(C_{jEt} + C_\mu \right) + r_C C_\mu$$

$$f_{B\max} = f_B \big|_{I_C=0} = \frac{1}{2\pi \tau_B} = \frac{1}{2\pi \left[r_B \left(C_{jEt} + C_\mu \right) + r_C C_\mu \right]}$$

from $f_B = \dfrac{I_{C\,f\,T}}{2\pi \tau_B \left(I_C + I_{C\,f\,T} \right)}$ plot of f_B vs. I_C is as follows:



Decreasing I_C results increasing of corner frequency of f_B (Pole of transconductance) but reduces the value of $g_m = I_C / V_T$.

Fig.2.14

CE Configuration with Voltage Drive

$$\frac{f_B}{f_\beta}=\frac{\dfrac{1}{2\pi\ r_B\ (C_\pi+C_\mu)}}{\dfrac{1}{2\pi\ r_\pi\ (C_\pi+C_\mu)}}=\frac{r_\pi}{r_B}\quad \text{so } f_B > f_\beta \text{ if } r_\pi > r_B$$

So voltage drive BW is larger than current drive BW for $r_\pi > r_B$, which is almost always the case; i.e. always $r_\pi > r_B$

So $f_B > f_T$ if $r_B\ g_m < 1$ or $r_B < 1/g_m$

r_B could be smaller than $1\ /\ g_m$ for small g_m (or small I_C) values.

Definition: for $I_C = I_{CTB} = \dfrac{kT/q}{r_B}$ we have $f_T = f_B$ because $r_B = 1\ /\ g_m$.

The max. value of f_B is reached for current smaller than $I_{C\ f\ T}$.

for $I_C \to 0$ $\quad f_{B\max} = \dfrac{1}{2\pi\ \tau_B} = \dfrac{1}{2\pi\ [\ r_B\ (\ C_{jE\ t} + C_\mu\ ) + r_C\ \ C_\mu\ ]}$

CE Configuration with Voltage Drive

- This maximum depends only the base resistance and both collector and emitter junction capacitances, and as can be noted it is independent of the base transit time τ_F!
- In general, it can be verified that only r_B limits the high frequency performance of bipolar junction transistor. For r_B=0 infinite BW can be achieved!!!

CE Configuration with Voltage Drive

Example:

Calculate f_T, f_B, I_{CTB} for $I_C = 0.01\ \ \mathbf{mA},\ 0.1\ \ \mathbf{mA},\ 1\ \ \mathbf{mA}$?

$f_{T\max}$ if $r_C = 30\ \ \Omega$?

$\beta = 100,\ \tau_F = 0.25\ \ \mathbf{ns},\ C_{jE\,t} + C_{jC\,t} = 6\ \ \mathbf{pF},\ r_B = 100\ \ \Omega$

from previous example: $f_T = 10\ \ \mathbf{MHz},\ 88\ \ \mathbf{MHz},\ 393\ \ \mathbf{MHz}$ respectively for $I_C = 0.01\ \ \mathbf{mA},\ 0.1\ \ \mathbf{mA},\ 1\ \ \mathbf{mA}$.

$$\tau_B = r_B\ \ (\ C_{jE\,t} + C_\mu\ \ ) + r_C\ \ C_\mu = 100\ \ (\ 6\times10^{-12}) + 30\times1\times10^{-12} = 630\times10^{-12}\ \ \text{sec}$$

$$\Rightarrow\ f_B = \frac{1}{2\boldsymbol{\pi}\ \ \tau_B}\frac{I_{C\,f\,T}}{I_C + I_{C\,f\,T}}$$

$f_B = 248\ \ \mathbf{MHz},\ 216\ \ \mathbf{MHz},\ 95\ \ \mathbf{MHz}$ respectively for $I_C = 0.01\ \ \mathbf{mA},\ 0.1\ \ \mathbf{mA},\ 1\ \ \mathbf{mA}$.

$$f_{B\max} = \frac{1}{2\boldsymbol{\pi}\ \ \tau_B} = 252.6\ \ \mathbf{MHz}$$

CE Configuration with Voltage Drive

$$I_C = I_{C\,f\,T} = 0.6\ \ \mathbf{mA} \Rightarrow f_B = \frac{0.6}{1.2}\frac{1}{2\boldsymbol{\pi}\ \times 630\times 10^{-12}} = 126.3\ \ \mathbf{MHz}$$

The frequency that the f_B reduce to one half of its DC value

$$I_{CTB} = \frac{kT/q}{r_B} = \frac{25\ \ \mathbf{mV}}{100} = 0.25\ \ \mathbf{mA}$$

The current I_C that the $f_B = f_T$

$$f_B(\ I_C = 0.25\ \ \mathbf{mA}\ ) = \frac{0.6\ \ \mathbf{mA}}{0.25\ \ \mathbf{mA} + 0.6\ \ \mathbf{mA}}\frac{1}{2\boldsymbol{\pi}\times 630\times 10^{-12}} \approx 177\ \ \mathbf{MHz}$$

Conclusion

- f_β, f_T and f_B are the most important "frequency" parameters of a bipolar junction transistor (BJT). They all depend on the parameters shown in Fig.2.6:

i.e. $\beta,\ g_m,\ \tau_F,\ r_B$ and Junction capacitances $C_{jE\,t},\ C_\mu$.

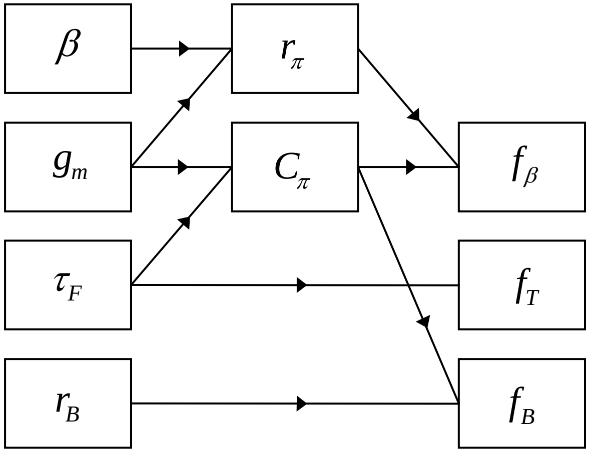


Fig.2.4

β and τ_F are determined by technology.

g_m can be varied by varying the current I_C.

r_B can be varied only by taking different (larger) layouts.

CC and CB Configurations

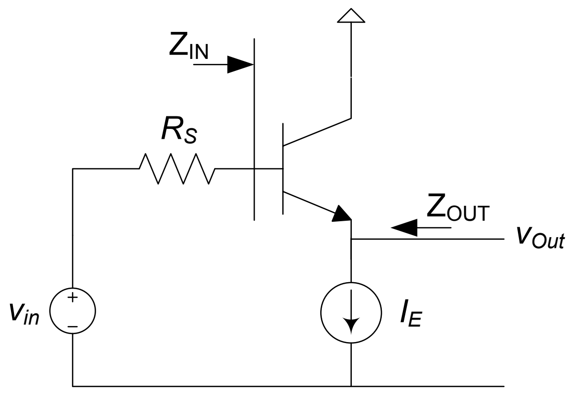


Fig.2.15-a

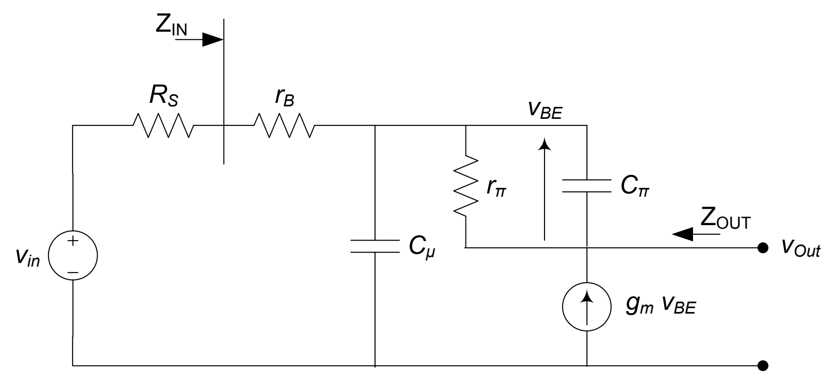


Fig.2.15-b

CC: two bias sources: 1- bias voltage at base
2- emitter current source

low-frequency

- I_E with $1/\beta$ margin error is equal to I_C. Having a fixed I_C requires a constant V_{BE} , so only a DC shift from base to emitter and $v_{Out} = v_{in}$ and $Z_{IN} = \infty$.

CC and CB Configurations

Impedance is converted from high to low:

$$R_{Out\,LF} = \frac{R_S + r_B + r_\pi}{1 + \beta_{AC}} \qquad (2-29)$$

for $\beta_{AC} >> 1$ we have:

$$R_{OUT\,LF} \approx \frac{1}{g_m} + \frac{R_S + r_B}{\beta_{AC}}$$

Note: For MOST $R_{OUT\,LF} = \dfrac{1}{g_m + g_{mb} + g_o} \approx \dfrac{1}{g_m}$

i.e. source follower

CC and CB Configurations

High-Frequency

- At high frequencies caps have to be taken into accounts:

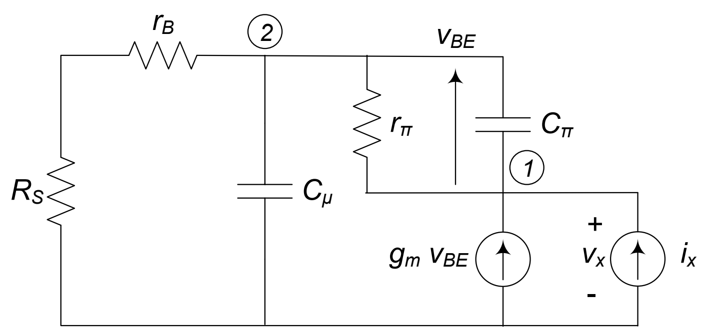


Fig.2.16

$$(1)\ i_x + g_m\ v_{BE} + (g_\pi + s\ C_\pi)\ v_{BE} = 0$$

$$(2)\ (g_\pi + s\ C_\pi)\ v_{BE} + (g + s\ C_\mu)\ (v_{BE} + v_x) = 0 \text{ where } g = \frac{1}{r_B + R_S}$$

$$(2) \rightarrow v_{BE}\left[g_\pi + g + s\ (C_\mu + C_\pi)\right] = -\ v_x\ (s\ C_\mu + g)$$

$$\Rightarrow v_{BE} = \frac{-\ (s\ C_\mu + g)}{g_\pi + g + s\ (C_\pi + C_\mu)}\ v_x \rightarrow \text{into (1)}$$

CC and CB Configurations

$$\Rightarrow i_x = -\ v_{BE}\ (g_m + g_\pi + s\ C_\pi) = -\frac{(g_m + g_\pi + s\ C_\pi)\ (s\ C_\mu + g)}{g_\pi + g + s\ (C_\mu + C_\pi)}\ v_x$$

$$\Rightarrow Z_{OUT} = \frac{v_x}{i_x} = -\frac{g_\pi + g\ +\ s\ (C_\mu + C_\pi)}{(g_\pi + g_m + s\ C_\pi)\ (g + s\ C_\mu)}$$

$$= -\frac{g_\pi + g}{(g_m + g_\pi)\ g}\ \frac{1 + s\ \dfrac{C_\pi + C_\mu}{g_\pi + g}}{\left(1 + s\dfrac{C_\pi}{g_m + g_\pi}\right)\left(1 + \dfrac{C_\mu}{g}\right)}$$

$$= \frac{\left(\dfrac{1}{r_\pi} + \dfrac{1}{R_S + r_B}\right)}{\left(g_m + \dfrac{1}{r_\pi}\right)\left(\dfrac{1}{R_S + r_B}\right)}\ \frac{1 + j\ \ f / f_Z}{(1 + j\ \ f / f_{P1})\ (1 + j\ \ f / f_{P2})}$$

$$f_Z = -\frac{1}{2\boldsymbol{\pi}\ (C_\pi + C_\mu)\ [\ r_\pi\ \|\ (r_B + R_S)\ ]}\quad \text{for } R_S = 0\ \ \&\ \ r_B << r_\pi\quad f_Z \approx f_B$$

CC and CB Configurations

$$f_{P1} = -\frac{g_m + g_\pi}{2\pi\ C_\pi} \approx -\frac{g_m}{2\pi\ C_\pi} \approx -\ f_T$$

$$f_{P2} = -\frac{g}{2\pi\ C_\mu} = -\frac{1}{2\pi\ (R_S + r_B)\ C_\mu} \Rightarrow \text{Very high frequency, that can be ignored.}$$

$$\frac{\frac{1}{r_\pi} + \frac{1}{R_S + r_B}}{\left(g_m + \frac{1}{r_\pi}\right)\left(\frac{1}{R_S + r_B}\right)} = \frac{R_S + r_B + r_\pi}{1 + \beta} \approx \frac{1}{g_m} + \frac{R_S + r_B}{\beta} = R_{OUT\ LF}$$

So Provided $g_m(R_S + r_B)C_\mu \ll C_\pi$ i.e. $f_{p1} \ll f_{p2}$:

$$Z_{OUT} = R_{OUT\ LF}\frac{1 + j\ \ f/f_B}{1 + i\ \ f/f_T}, \begin{cases} f_B = \dfrac{1}{2\pi\ r_B\ (C_\pi + C_\mu)} = \dfrac{1}{2\pi\ \tau_B}\dfrac{I_{C\ f\ T}}{I_C + I_{C\ f\ T}} \\ \tau_B = r_B\ (C_{jE\ t} + C_\mu) + r_C\ C_\mu \\ f_T = \dfrac{1}{2\pi\ \tau_F}\dfrac{I_C}{I_C + I_{C\ f\ T}} \qquad (2-30) \\ I_{C\ f\ T} = (C_{jE\ t} + C_\mu)\dfrac{kT/q}{\tau_F},\ \dfrac{1}{2\pi\ f_T} = \tau_F + (C_{jE\ t} + C_\mu)\dfrac{kT/q}{I_C} + r_C\ C_\mu \end{cases}$$

CC and CB Configurations

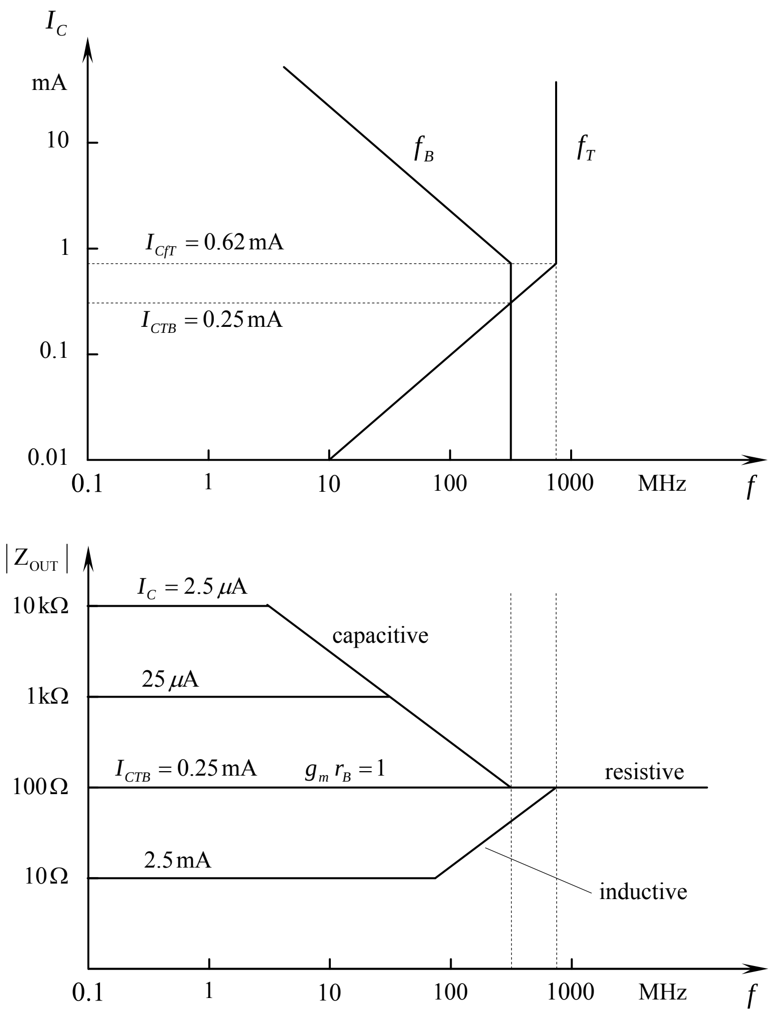


Fig.2.17.
Position of pole and zero & bode diagram of Z_{OUT} of emitter follower for β =100, r_B = 100 Ω, τ_F = 0.25 ns ,C_{jEt} + C_μ= 6 pF.

CC and CB Configurations

$$Z_{\mathrm{OUT}}(jf) = R_{OUT\,LF}\frac{1+jf/f_B}{1+jf/f_T}$$

neglecting $\dfrac{R_S + r_B}{\beta} \Rightarrow \left|Z_{\mathrm{OUT}}(jf)\right|_{f\to\infty} = \left(\dfrac{1}{g_m}\right)\times\dfrac{f_T}{f_B}$

$$= \left(\frac{1}{g_m}\right)\frac{\dfrac{g_m}{2\pi(C_\pi + C_\mu)}}{\dfrac{1}{2\pi r_B(C_\pi + C_\mu)}} \Rightarrow$$

$$\left|Z_{\mathrm{OUT}}(jf)\right|_{f\to\infty} = \left(\frac{1}{g_m}\right)(g_m r_B) = r_B$$

- So for every I_C the output impedance of the emitter follower at very high frequency is equal to r_B.
- So note that at I_{CBT} we have $r_B = \left.\dfrac{1}{g_m}\right|_{I_C = I_{CTB}} = \dfrac{V_T}{I_{CTB}}$.

CC and CB Configurations

- In previous figure, the asymptotic values of f_B and f_T are plotted versus I_C. This plot gives the positions of the zero and the pole with I_C as so is called pole-zero position plot.
- @ I_{CTB}, $g_m r_B = 1$: $f_T = f_B$ so a pure resistance results, i.e. r_B.
- At lower I_C, f_T (pole) < f_B (zero) so the output impedance rolls off vs. frequency (*capacitive*).
- At higher I_C, f_B < f_T. so there is a region in which output impedance increases with frequency. This region is called an *inductive region*.
- This inductance could cause instability if combined with parasitic cap. at output terminal, so it is safer to reduce the biasing current.

CC and CB Configurations

- Note previous results obtained for

$$R_S << r_B << r_\pi \Rightarrow f_Z \approx f_B$$

- However, for (perhaps more practical case)

$$r_B << r_\pi << R_S \Rightarrow f_Z \approx f_\beta = \frac{1}{2\boldsymbol{\pi}\ r_\pi\ (C_\pi + C_\mu)}$$

moves to a lower frequency

$$f_{p1} = f_T \text{ unchanged; and } f_{p2} = \frac{C_\mu}{g} \approx R_s C_\mu$$

moves to a higher frequency

CB Configurations

- The input impedance of CB is exactly the same as the output impedance of the Emitter-Follower. So the same pole-zero position plot can used for CB input impedance.
- Particularly the previous assumption for the source impedance could be more practical: $R_S << r_B << r_\pi \Rightarrow f_Z \approx f_B$

Comparison between MOSTs and Bipolar transistors

- **Maximum Frequency of Operation:**

  f_T is assumed as the parameter (unity-gain of amplifier can be discussed too!).

  from bipolar: $f_{T\max} = \dfrac{1}{2\pi \tau_F}$ where $\tau_F = \dfrac{W_B}{v_{\text{sat}}}$

  for MOST, it can be shown that:

  $$\tau_F = \frac{L_{eff}}{v_{\text{sat}}}$$

- τ_F (transit time) for bipolar is likely to be smaller for a bipolar transistor than for a MOST because the vertical W_B is easier to make smaller than the lateral L_{eff}. [for 0.1 µm, this time constant is about 1 ps <=> $f_T \approx$160 GHz]

Comparison between MOSTs and Bipolar transistors

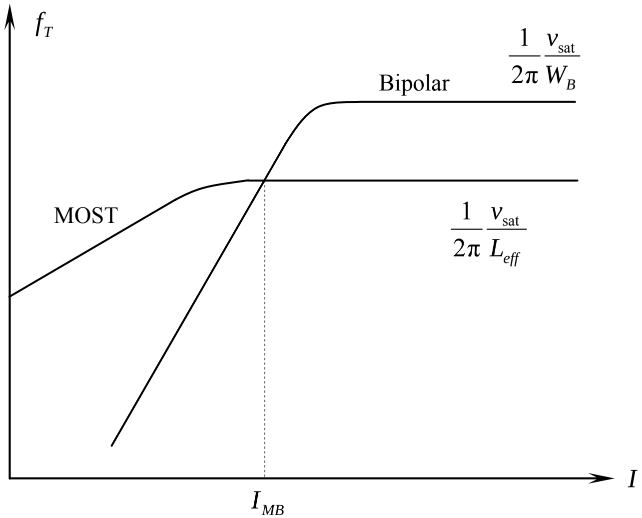


Fig.2.18

- When saturation velocity and so the drain saturation current happens well before pinch-off!

$$I_D = v_{\text{sat}}\, Q_d = v_{\text{sat}}\, W\, C_{ox}\,(V_{GS} - V_{TH}) \qquad (2-31)$$

$$\Rightarrow \quad g_{m,\text{sat}} = W\, C_{ox}\, v_{\text{sat}} \qquad (2-32)$$

$$f_{T\max} = \frac{1}{2\pi}\frac{g_{m,\text{sat}}}{C_{GS}} \approx \frac{1}{2\pi}\frac{W\, C_{ox}\, v_{\text{sat}}}{W\, L_{eff}\, C_{ox}} = \frac{1}{2\pi}\frac{v_{\text{sat}}}{L_{eff}} \qquad (2-33)$$

Comparison between MOSTs and Bipolar transistors

- In long-channel since the quadratic relations already exist:

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})^2 \quad \Rightarrow \quad g_m = \mu_n C_{ox} \frac{W}{L_{eff}} (V_{GS} - V_{TH})$$

$$\Rightarrow \quad f_{T\max} = \frac{1}{2\pi} \frac{g_m}{C_{GS}} = \frac{1}{2\pi} \frac{\mu_n C_{ox} W / L_{eff} (V_{GS} - V_{TH})}{W L_{eff} C_{ox}} = \frac{1}{2\pi} \frac{\mu_n}{L_{eff}^2} (V_{GS} - V_{TH}) \qquad (2-34)$$

- In short-channel considering short-channel effects (i.e. mobility degradation

$$I_D = \frac{1}{2}\mu_n C_{ox} \frac{W}{L} \frac{(V_{GS} - V_{TH})^2}{1 + \theta (V_{GS} - V_{TH})} \quad \text{where} \quad \theta = \frac{1}{L_{eff} E_C} \qquad (2-35)$$

- As shown in the Fig.2.18 increasing I beyond some point makes f_T saturated. This can be somehow explained by equations 2-32 & 2-35. Increasing I_D is done by increasing (V_{GS}-V_{TH}). For very large I_D and so (V_{GS}-V_{TH}) if θ(V_{GS}-V_{TH})>>1 then 2-35 becomes:

$$I_D = \frac{\frac{1}{2}\mu_0 C_{ox} \frac{W}{L}}{\theta} (V_{GS} - V_{TH}) = \frac{1}{2}\mu_n E_C W C_{ox} (V_{GS} - V_{TH}), \mu_0 = \frac{v_{sat}}{E_C} \qquad (2-36)$$

Comparison between MOSTs and Bipolar transistors

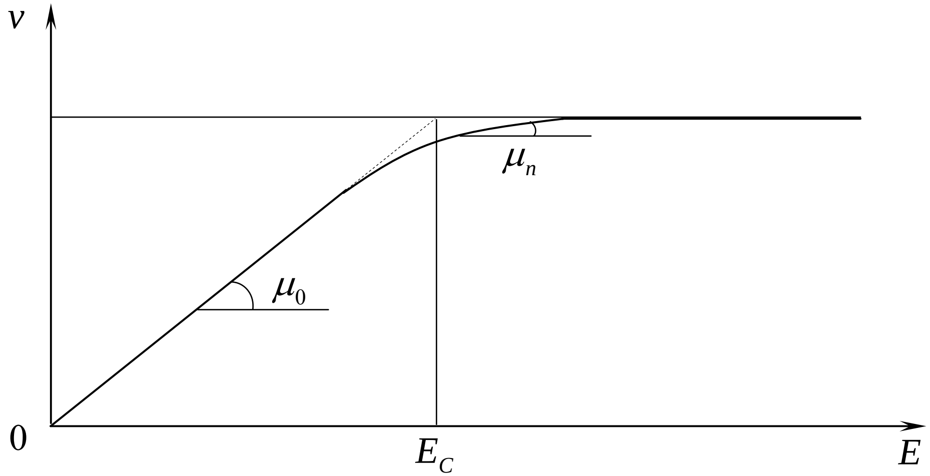


Fig.2.19

Comparison between MOSTs and Bipolar transistors

➢ **Tranconductance-Current Ratio:**

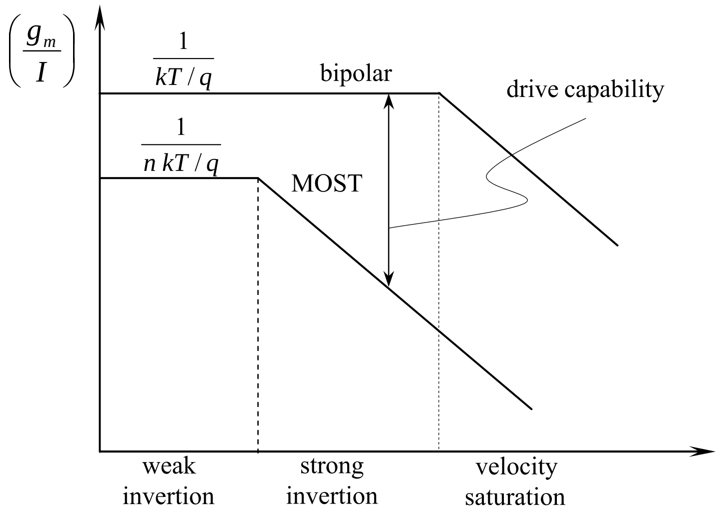


Fig.2.20

- Bipolar offers a better current drive capability. Less input voltage is required to drive a larger output current!