Multistage Amplifier Frequency Response

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Multistage Amplifier Freq. Response

Dominant-pole Approximation:

$$A(s)=\frac{N(s)}{D(s)}=\frac{a_0+a_1s+a_2s^2+\ldots+a_ms^m}{1+b_1s+b_2s^2+\ldots+b_ns^n} \qquad (7.64)$$

$a_0, a_1, \ldots, a_m$ and $b_1, b_2, \ldots, b_n$ are constant.

For All-pole system (when there is no dominant zero!) :

$$A(s)=\frac{K}{\left(1-\frac{s}{p_1}\right)\left(1-\frac{s}{p_2}\right)\ldots.\left(1-\frac{s}{p_n}\right)} \qquad (7.65)$$

Where K is a constant and $p_1, p_2, \ldots, p_n$ are the poles.

Multistage Amplifier Freq. Response

From (7.64) and (7.65) :

$$b_1 = \sum_{i=1}^{n}\left(-\frac{1}{p_i}\right) \qquad (7.66)$$

An important practical case:

$$|p_1| \ll |p_2|, |p_3|, \ldots \quad so \quad that \quad \left|\frac{1}{p_1}\right| \gg \left|\sum_{i=2}^{n}\left(-\frac{1}{p_i}\right)\right|$$

$$\Rightarrow \quad b_1 = \left|\frac{1}{p_1}\right| \qquad (7.67)$$

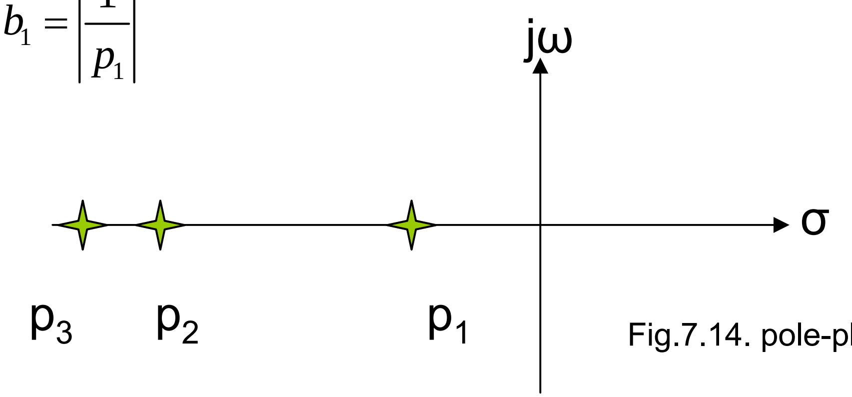


Fig.7.14. pole-plot with dominant pole.

Multistage Amplifier Freq. Response

From (7.65) the gain is:

$$|A(j\omega)| \approx \frac{K}{\sqrt{1+\left(\frac{\omega}{p_1}\right)^2}} \quad ,\omega \le p_1 \qquad (7.69)$$

If a dominant pole exists then:

$$|A(j\omega)| = \frac{K}{\sqrt{\left[1+\left(\frac{\omega}{p_1}\right)^2\right]\left[1+\left(\frac{\omega}{p_2}\right)^2\right]\ldots\left[1+\left(\frac{\omega}{p_n}\right)^2\right]}} \qquad (7.68)$$

This approximation is quite accurate at least until $\omega=|p_1|$, so:

$$\left.\begin{cases} \omega_{-3dB} = |p_1| & (7.70) \\ \omega_{-3dB} \approx \dfrac{1}{b_1} & (7.71) \end{cases}\right\} \rightarrow \text{dominant pole situation!}$$

Zero-Value Time-Constant Analysis

This is an approximate method of analysis that allows an estimate to be made of the dominant-pole freq. (-3dB freq.) of complex circuits.

Zero-Value Time-Constant Analysis

Consider the circuit shown in fig.7.15:

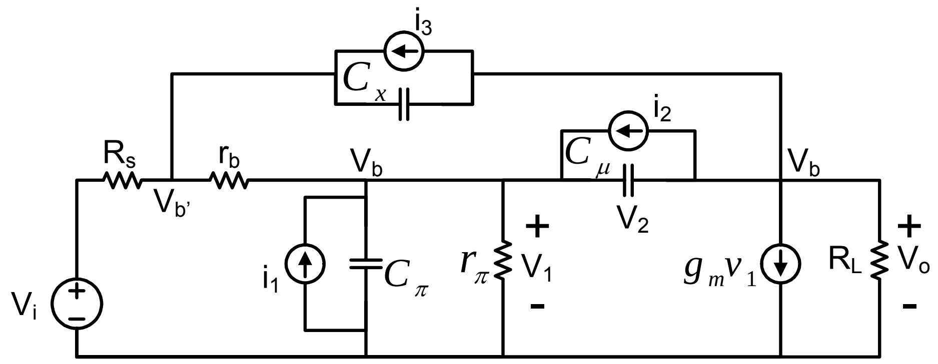


Fig.7.15: A CE circuit with internal Caps.

For analysis, the cap voltages, V_1, V_2, ad V_3 are chosen as variables. The external input V_i is removed and the circuit excited with three independent current sources i_1, i_2 and i_3 across the caps:

$$\begin{cases} i_1 = \left(g_{11} + sC_\pi\right)v_1 + g_{12}v_2 + g_{13}v_3 & (7.72) \\ i_2 = \left(g_{21}\right)v_1 + \left(g_{22} + sC_\mu\right)v_2 + g_{23}v_3 & (7.73) \\ i_3 = g_{31}v_1 + g_{32}v_2 + \left(g_{33} + sC_x\right) & (7.74) \end{cases}$$

Zero-Value Time-Constant Analysis

The g terms are conductions. Note that terms involving s contributed by the caps are associated only with their respective cap voltage and appear only on the diagonal of the system determinant.

$$\Delta(s) = k_3 s^3 + k_2 s^2 + k_1 s + k_0 = k_0\left(1 + b_1 s + b_2 s^2 + b_3 s^3\right) \qquad (7.75),(7.76)$$

we showed previously that if there are no dominant zeros, and if there is a dominant pole, then:

$$\omega_{-3dB} \approx \frac{1}{b_1}$$

Zero-Value Time-Constant Analysis

Poles of the transfer function are zeros of determinant Δ:

$$\Delta(s) = k_3 s^3 + k_2 s^2 + k_1 s + k_0 \qquad (7.75)$$

$$from\,(7.75):$$

$$\Delta(s) = k_0\left(1 + b_1 s + b_2 s^2 + b_3 s^3\right) \qquad (7.76)$$

$$b_1 k_0 = k_1 \Rightarrow b_1 = \frac{k_1}{k_0}$$

This form is like (7.64).

Note that this is a 3rd-order determinant because there are three caps in the circuit.

Zero-Value Time-Constant Analysis

K_0 in (7.75) is the value of $\Delta(s)$ if all caps (C_x=C_μ=C_π=0):

$$\Rightarrow \textit{from} \quad (7.72),(7.73),(7.74):$$

$$\left\{ \begin{aligned} k_0 &= \Delta\Big|_{C_\pi = C\mu = C_x = 0} \\ k_0 &\triangleq \Delta_0 \end{aligned} \right\} \qquad (7.77)$$

Consider now the term $k_1 s$ in (7.75) from (7.72)-(7.74).

It is apparent that s only occurs when associated with a capacitance.

$$\Rightarrow k_1 s = h_1 s C_\pi + h_2 s C_\mu + h_3 s C_x \qquad (7.78)$$

Where h terms are constants.

Zero-Value Time-Constant Analysis

h_1 can be evaluated by expanding the determinant of (7.72) to (7.74) about the first row:

$$\Delta(s) = (g_{11} + sC_\pi)\Delta_{11} + g_{12}\Delta_{12} + g_{13}\Delta_{13} \qquad (7.79)$$

Where Δ_{11}, Δ_{12} and Δ_{13} are cofactors of determinant.

Which includes C_μ and C_x terms! (no C_π)

(7.72) to (7.74) show that C_π only occurs in the first term of (7.79), Δ_{12} has only C_x and Δ_{13} has C_μ. In Δ_{11} both C_x and C_μ exists.

Therefore the coefficient of sC_π in (7.78) is found by evaluating Δ_{11} with C_μ=C_x=0 which eliminates the other capacitive terms in Δ_{11}.

We need to set C_μ=C_x=0. Otherwise the order of coefficient of C_π will be higher than s^1.

Zero-Value Time-Constant Analysis

The coefficient of sC_π is h_1 in (7.78):

$$h_1 = \Delta_{11}\Big|_{C_\mu = C_x = 0} \qquad (7.80)$$

Now we expand the determinant about the second row:

$$\Delta(s) = g_{21}\Delta_{21} + \left(g_{22} + sC_\mu\right)\Delta_{22} + g_{23}\Delta_{23} \qquad (7.81)$$

C_μ occurs only in 2nd term in (7.81). [Δ_{21} has only C_x and Δ_{23} has only C_π]

Zero-Value Time-Constant Analysis

The coefficient of sC_μ is found by evaluating Δ_{22} when $C_\pi=C_x=0$, which eliminate the other Cap terms in Δ_{22} (which includes both C_π and C_x terms!).

This coefficient of sC_μ is h_2 in (7.78):

$$h_2 = \Delta_{22}\big|_{C_\pi=C_x=0} \qquad (7.82)$$

Similarly by expanding about 3rd-row:

$$h_3 = \Delta_{33}\big|_{C_\pi=C_\mu=0} \qquad (7.83)$$

Zero-Value Time-Constant Analysis

Combining (7.78) with [(7.80),(7.82) and (7.83)] we have:

$$k_1 = \left(\Delta_{11\left|_{C_\mu = C_x = 0}\right.} \times C_\pi \right) + \left(\Delta_{22\left|_{C_\pi = C_x = 0}\right.} \times C_\mu \right) + \left(\Delta_{33\left|_{C_\mu = C_\pi = 0}\right.} \times C_x \right) \qquad (7.84)$$

$$\Rightarrow$$

$$b_1 = \frac{k_1}{k_0} = \left(\frac{\Delta_{11\left|_{C_\mu = C_x = 0}\right.}}{\Delta_0} \right) \times C_\pi + \left(\frac{\Delta_{22\left|_{C_\pi = C_x = 0}\right.}}{\Delta_0} \right) \times C_\mu + \left(\frac{\Delta_{33\left|_{C_\mu = C_\pi = 0}\right.}}{\Delta_0} \right) \times C_x \qquad (7.85)$$

Now consider i_2=i_3=0 in fig. 7.15.

Solving Eqns. (7.72)-(7.74) for v_1 gives:

$$v_1 = \frac{\Delta_{11}}{\Delta(s)} i_1 \Rightarrow \frac{v_1}{i_1} = \frac{\Delta_{11}}{\Delta(s)} \qquad (7.86)$$

Zero-Value Time-Constant Analysis

This is the driving point impedance at C_π node pair and,

$$\frac{\Delta_{11}\big|_{C_\mu = C_x = 0}}{\Delta_0}$$

Is the driving-point resistance at the node pair when all caps are equal to zero because:

$$\frac{\Delta_{11}\big|_{C_\mu = C_x = 0}}{\Delta_0} = \frac{\Delta_{11}}{\Delta}\bigg|_{C_\mu = C_x = C_\pi = 0} \qquad (7.87)$$

We define:

$$R_{\pi 0} = \frac{\Delta_{11}}{\Delta_0}\bigg|_{C_\mu = C_x = 0} \qquad (7.88)$$

Zero-Value Time-Constant Analysis

Similarly:

$$R_{\mu 0} = \frac{\Delta_{22}\big|_{C_\pi = C_x = 0}}{\Delta_0}$$

Is the driving point resistance at the C_μ node pair with all capacitors put equal to zero.

therefore from (7.85) we can write:

$$b_1 = R_{\pi 0} C_\pi + R_{\mu 0} C_\mu + R_{x0} C_x \qquad (7.89)$$

The time constant in (7.89) are called "zero-value time constant" because all caps are put equal to zero to perform the calculation.

Zero-Value Time-Constant Analysis

Thus:

$$\omega_{-3dB} \approx \frac{1}{\sum T_o} \qquad (7.90)$$

Where $\sum T_0$ is the sum of the Zero-Value time constants.

for circuit of fig.7.15 one can show:

$$b_1 = R_{\pi 0} C_\pi + R_{\mu 0} C_\mu + R_{x0} C_x \qquad (7.89)$$

Where $R_{\pi 0}$ is the driving-point resistance at C_π node pair with all capacitors equal to zero.

Zero-Value Time-Constant Analysis

$R_{\mu 0}$ is the driving-point resistance at C_μ node pair with all Caps equal to zero, and R_{x0} the same for C_x.

Although derived in terms of a specific example, this result is true in any circuit for which the assumptions made in this analysis are valid! (such as no dominant zero and a dominant pole!)

Zero-Value Time-Constant Analysis (Ex.)

For C_π, by inspection:

$$R_{\pi 0} = r_\pi \parallel \left(R_s + r_b\right) \qquad (7.91)$$

For C_μ: we apply a test current i at the C_μ terminals as shown in fig.7.16.

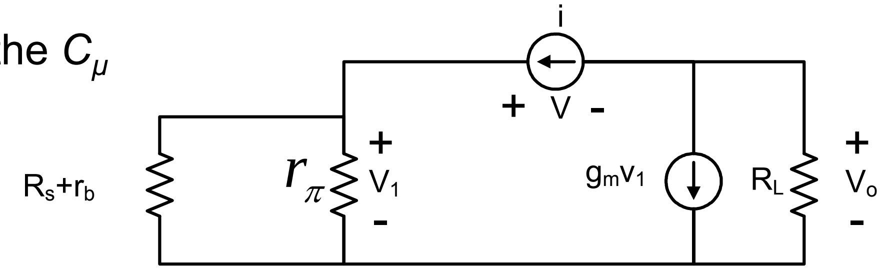


Fig. 7.16

$$v_1 = R_{\pi 0} i \qquad (7.92)$$

$$v_o = -\left(i + g_m v_1\right) R_L \qquad (7.93)$$

(7.92) into (7.93): $$v_o = -\left(i + g_m R_{\pi 0} i\right) R_L \qquad (7.94)$$

$$R_{\mu 0} = \frac{v}{i} \quad \text{and} \quad R_{\mu 0} = \frac{v_i - v_o}{i} \qquad (7.95)$$

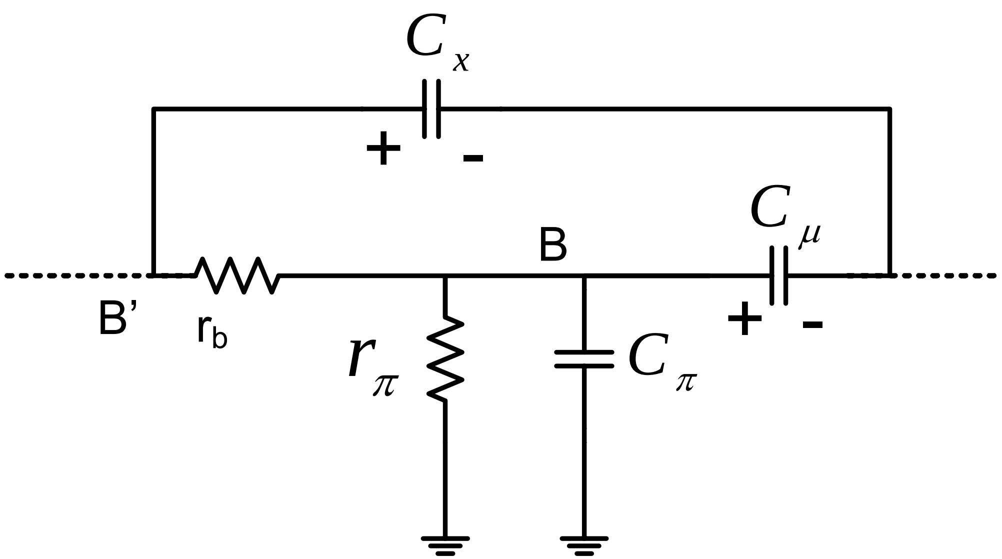
(7.92) and (7.94) in (7.95):

$$R_{\mu 0} = R_{\pi 0} + R_L + g_m R_L R_{\pi 0} \qquad (7.96)$$

Emitter-Follower (Cont'd)

Why $R_b << r_\pi$ results in $R_{x0} \approx R_{\mu 0}$?

The negative plates of Cμ and Cx are tied together. So if we have their positive plates tied too then Cμ will be in parallel with Cx having identical driving-point resistance (i.e. $R_{x0} \approx R_{\mu 0}$).



Emitter-Follower (Cont'd)

It is obvious at least in DC that if $R_b << r_\pi$ then:

$$\frac{v_B}{v_{B'}} = \left.\frac{r_\pi}{r_\pi + r_b}\right|_{r_b \ll r_\pi} \approx 1$$

So the approximation $R_b << r_\pi$ leads to $R_{x0} \simeq R_{\mu 0}$!

Zero-Value Time-Constant Analysis (Ex.)

R_{x0} can be calculated in a similar fashion. It is obvious that

$$R_{x0} \approx R_{\mu 0} \quad if \quad r_b \ll r_\pi$$

C_x can be lumped in with C_μ if r_b is small:

$$\omega_{-3dB} = \frac{1}{R_{\pi 0}C_x + R_{\mu 0}C_\mu} \qquad (7.97)$$

$$\omega_{-3dB} = \frac{1}{R_{\pi 0}\left\{C_\pi + C_\mu[(1+g_m R_L) + \frac{R_L}{R_{\pi 0}}]\right\}} \qquad (7.98)$$

Zero-Value Time-Constant Analysis (Ex.)

This is the same as what obtained in (7.24) & (7.25) by exact analysis; R in (7.25) is equal to $R_{\pi 0}$ in (7.98).

Zero-value time constant gives the result with much less effort. However it does not give any information on the non-dominant poles!

Zero-value time constant

Emitter-Follower

Now see zero-value time constant approach for emitter-follower of Fig. 7.9 where only C_π has been included. $R_{\pi 0}$ can be calculated by inserting a current source i:

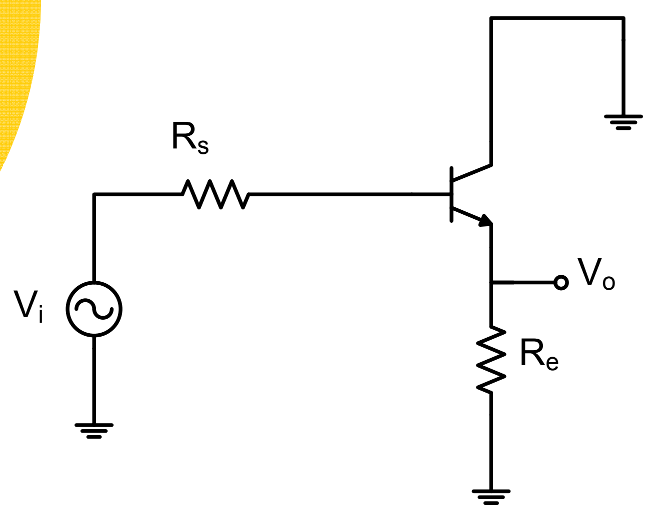


Fig. 7.9

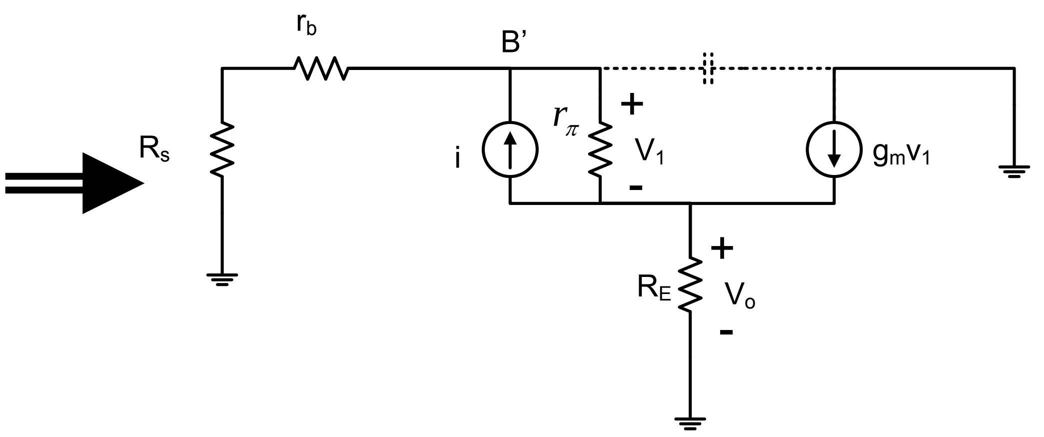


Fig. 7.17

Emitter-Follower (Cont'd)

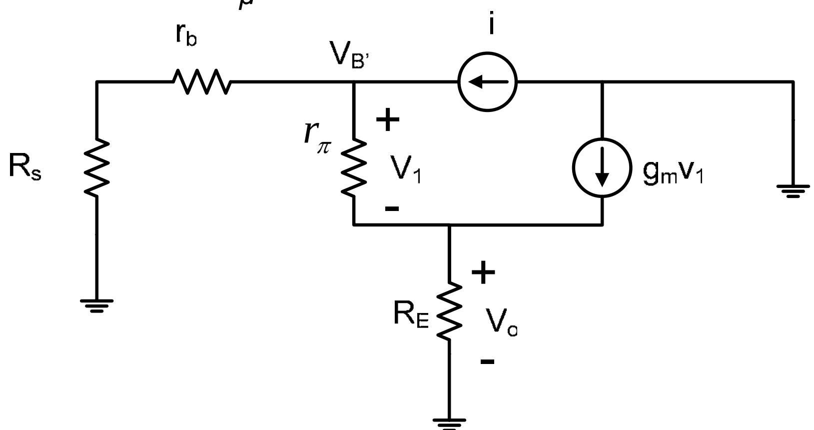
$$\left.\begin{array}{ll} i = \dfrac{v_1}{r_\pi} + \dfrac{v_1 + v_o}{R_s + r_b} & (7.99) \\ \dfrac{v_1}{r_\pi} - i + g_m v_1 = \dfrac{v_o}{R_E} & (7.100) \end{array}\right\} \Rightarrow i = \frac{v_1}{r_\pi} + \frac{v_1}{R_s + r_b} + \frac{R_E}{R_s + r_b}\left(\frac{v_1}{r_\pi} + g_m v_1 - i\right)$$

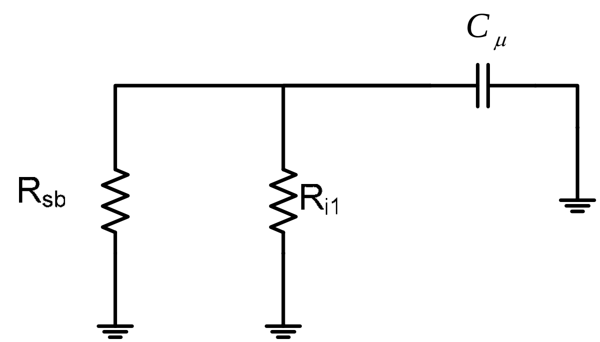
$$\Rightarrow i = \frac{v_1}{r_\pi} + v_1 \frac{1 + g_m R_E}{R_s + r_b + R_E} \Rightarrow R_{\pi 0} = \frac{v_1}{i} = r_\pi \parallel \left(\frac{R_s + r_b + R_E}{1 + g_m R_E}\right) \qquad (7.101)$$

Zero-Value Time-Constant Analysis

Emitter follower (Cont'd):

For C_μ we have:





$$R_{i1} = r_\pi \left(1 + g_m R_E\right)$$

$$i = \frac{v_1 + v_o}{R_S + r_b} + \frac{v_1}{r_\pi} \Rightarrow i = v_1 \left[\frac{1}{R_S + r_b} + \frac{1}{r_\pi} + \frac{R_E}{R_S + r_b} \left(g_m + \frac{1}{r_\pi} \right) \right]$$

$$\frac{v_o}{R_E} = g_m v_1 + \frac{v_1}{r_\pi} = \left(g_m + \frac{1}{r_\pi} \right) v_1 = \frac{1 + g_m r_\pi}{r_\pi} v_1$$

$$R_{\mu 0} = \frac{v_1 + v_o}{i} = \left[\left(R_S + r_b\right) \| r_\pi \| \left(\frac{R_S + r_b}{R_E} \cdot \frac{r_\pi}{1 + g_m r_\pi} \right) \right] \left[1 + \frac{R_E}{r_\pi} \left(1 + g_m r_\pi\right) \right]$$

Zero-Value Time-Constant Analysis

$$for \quad g_m r_\pi \gg 1$$

$$R_{\mu 0} = \left\{ (R_S + r_b) \| r_\pi \| \frac{R_S + r_b}{g_m R_E} \right\} (1 + g_m R_E)$$

$$\approx \left\{ \left[(R_S + r_b) \| r_\pi \right] (1 + g_m R_E) \right\} \| (R_S + r_b)$$

Note:

1- For a large R_S (output resistance of a preceding CE stage)

& a small resistive R_E : $\quad (R_S + r_b) \| r_\pi \approx r_\pi$

$$\Rightarrow R_{\mu 0} \approx r_\pi (1 + g_m R_E) \| (R_S + r_b)$$

2- For a large R_S and a large R_E (like a current source), then:

$$R_{\mu 0} \approx r_\pi (1 + g_m R_E) \| (R_S + r_b) \approx (g_m r_\pi R_E) \| (R_S + r_b) \approx (R_S + r_b)$$

Note: higher R_E results in a lower $R_{\mu 0}$ (time-constant), so a higher BW!

Emitter-Follower (Cont'd)

Thus:

$$\omega_{-3\text{dB}} = \frac{1}{R_{\pi 0}C_{\pi} + R_{\mu 0}C_{\mu}} \qquad (7.102)$$

Interesting to note that performing the KCL Nodal Analysis while ignoring C_μ results in:

$$\omega_{-3\text{dB}} = \frac{1}{R_{\pi 0}C_{\pi}} \qquad (7.41)$$

Emitter-Follower (Cont’d)

Which one is the dominant pole? (7.102) is not in agreement with (7.41) obtained by KCL analysis ignoring C_μ (to be obtained by students!). Zero-value time constant tells us nothing about the dominant zero showed in the nodal analysis (ignoring C_μ):

$$z_1 = \frac{-g_m}{C_\pi} \approx -\omega_T$$

$$p_1 = -\frac{1}{C_\pi R_1} \qquad where \qquad R_1 = r_\pi \parallel \frac{r_b + R_E}{1 + g_m R_E}$$

Because of the dominant zero in the results obtained by the nodal analysis, the pole frequency obtained there in (7.41) can not the -3dB frequency.

If there is a major capacitance between input and output such as C_π (in source follower) zero-value time constant can not predict ω_{-3}dB (BW) very well!

Emitter-Follower (Cont'd)

$$p_2 = -\left(\frac{1}{[(r_b + R_s) \| R_E] C_\mu} + \frac{1}{r_\pi C_\pi} + \frac{1}{R_E C_\pi} + \frac{g_m}{C_\pi} \right) \approx \frac{g_m}{C_\pi} \quad \text{for both large } R_s \text{ and } R_E$$

$C_\mu \to \infty$ (if be shorted) $\Rightarrow$ remaining time constant is : $C_\pi (r_\pi \| R_E \| g_m)$

$C_\pi \to \infty$ (if be shorted) $\Rightarrow$ remaining time constant is : $C_\mu [(r_b + R_s) \| R_E]$

There is a pole-zero cancellation! (p_2 and z_1)

Example:

Calculate poles & zero by different analyses: C_μ=1pF, C_π=10pF, R_E=2kΩ, R_s=50Ω, r_b=150Ω, β=100, I_c=1mA.

Solution:

1. Nodal Analysis (ignoring C_μ):
   In reality, not ignoring C_μ, there are two poles such that the above p_1 is the 2nd pole

$$p_1 = 2\pi(568.2\text{MHz})$$

$$z_1 = f_T = 612\text{MHz}$$

2. Zero-Value Time Constant Analysis:
   To be checked by Students!
   Also No ideas about Zero or 2nd Pole

$$p_1 = \frac{1}{60\Omega(10\text{pF}) + 148\Omega(1\text{pF})} = 2\pi(212.8\text{MHz})$$

Zero-value time constant;

Cascaded Common-Emitter Freq. Response

Zero-value time constant is advantageous for circuits with more than one device:

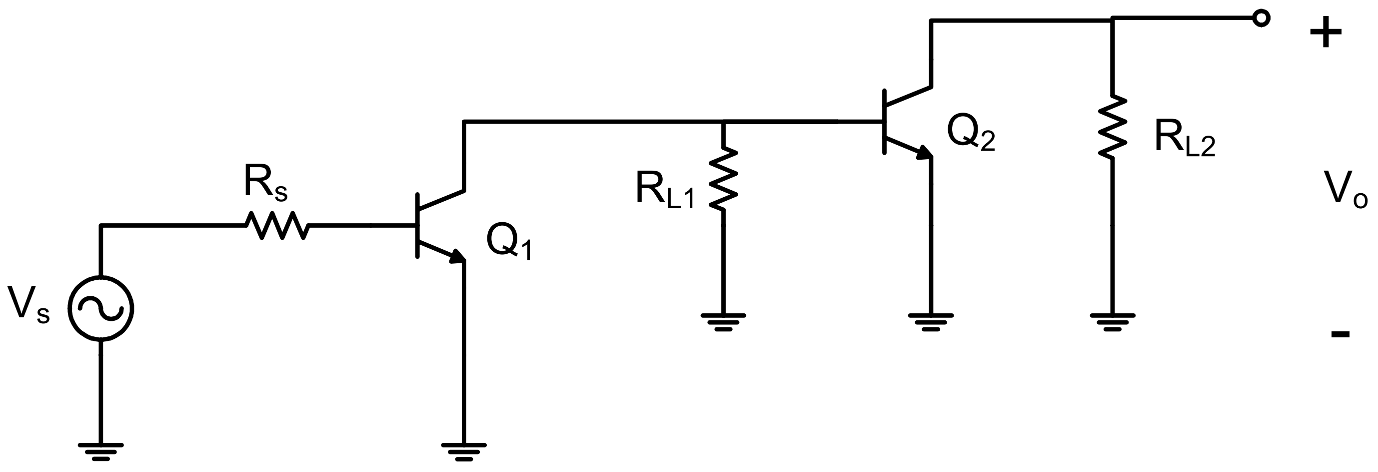


Fig.7.18: Single-ended or a differential half circuit

Example:
Cascaded Common-Emitter Freq. Response

Find ω_{-3dB} of Fig.7.18 With:

$$R_s = 10k\Omega \qquad r_{b1} = r_{b2} = 400\Omega \qquad r_{\pi 1} = 20k\Omega \qquad C_{\pi 1} = 5pF$$

$$C_{\pi 2} = 10pF \qquad C_{\mu 1} = C_{\mu 2} = 1pF \qquad R_{L1} = 10k\Omega \qquad C_{cs1} = C_{cs2} = 2pF$$

$$r_{\pi 2} = 10k\Omega \qquad R_{L2} = 5k\Omega \qquad g_{m1} = 3mA/V \qquad g_{m2} = 6mA/V$$

Cascaded Common-Emitter Freq. Response

Small signal:

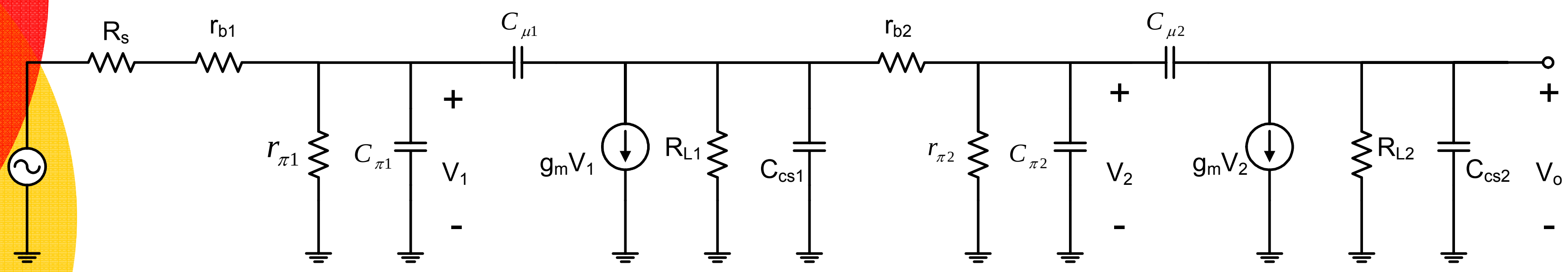


Fig. 7.19

For $C_{\mu 1}$ and $C_{\mu 2}$ Eqn. 7.96 can be applied:

$$\begin{cases} C_{\mu 1} R_{\mu 01} = C_{\mu 1} R_{\pi 01} \left(1 + g_{m1} R_{L1eff} + \dfrac{R_{L1eff}}{R_{\pi 01}}\right) & (7.103) \\[2ex] C_{\mu 2} R_{\mu 02} = C_{\mu 2} R_{\pi 02} \left(1 + g_{m2} R_{L2eff} + \dfrac{R_{L2eff}}{R_{\pi 02}}\right) & (7.104) \end{cases}$$

Cascaded Common-Emitter Freq. Response

Where:
$$R_{L1eff} = R_{L1} \| (r_{b2} + r_{\pi 2}) = 5.1k\,\Omega$$
$$R_{L2eff} = R_{L2} = 5k\,\Omega$$
$$R_{\pi 01} = r_{\pi 1} \| (r_{b1} + R_{s1}) = 20k\,\Omega \| (10.4k\,\Omega) = 6.84k\,\Omega$$
$$R_{\pi 01} = r_{\pi 1} \| (r_{b1} + R_{s1}) \qquad where: \qquad R_{s1} = R_{L1} = 10k\,\Omega$$
$$R_{\pi 02} = 10k\,\Omega \| (10.4k\,\Omega) = 5.1k\,\Omega$$

And for $C_{\pi 1}$, $C_{\pi 2}$:
$$C_{\pi 1} R_{\pi 01} = 5 \times 6.84ns = 34.2n\sec$$
$$C_{\pi 2} R_{\pi 02} = 10 \times 5.1ns = 51n\sec$$

This substituting in (7.103) and (7.104):
$$\begin{cases} C_{\mu 1} R_{\mu 01} = 1 \times 6.84(1 + 3 \times 5.1 + \dfrac{5.1}{6.84})ns = 116.6n\sec \\ C_{\mu 2} R_{\mu 02} = 1 \times 5.1(1 + 6 \times 5 + \dfrac{5}{5.1})ns = 163.2n\sec \end{cases}$$

Cascaded Common-Emitter Freq. Response

For C_{cs1}, C_{cs2}:

$$\begin{cases} C_{cs1}R_{cs01} = C_{cs1}R_{L1eff} = 2\times 5.1n\sec = 10.2n\sec \\ C_{cs2}R_{cs02} = C_{cs2}R_{L2eff} = 2\times 5n\sec = 10n\sec \end{cases}$$

- Assuming the circuit has a dominant pole, the -3dB freq. can be estimated as

$$\omega_{-3dB} = \frac{1}{\sum T_0} = \frac{10^9}{34.2+51+116.6+136.2+10.2+10} rad/\sec$$

$$= \frac{10^9}{385.2} rad/\sec = 2.6\times 10^6 rad/\sec$$

$$\Rightarrow f_{-3dB} = 413kHz$$

Cascaded Common-Emitter Freq. Response

A computer simulation gave:

$$f_{p1} = -463kHz$$
$$f_{p2} = -4.37MHz$$
$$f_{p3} = -41.06MHz$$
$$f_{p4} = -212MHz$$

$$f_{z1} = -478MHz$$
$$f_{z2} = -955MHz$$

$$\Rightarrow f_{-3dB} = -456KHz \qquad \text{only } 10\% \text{ error}$$