Telecommunications 1

Dr Reza Danesfahani

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Chapter 1

Preamble

1.1 Syllabus

1. Overview and Introduction
   - Transmitter
   - Channel
   - Receiver
2. Signals and Systems
3. Signal Transmission
   - Distortion
   - Power loss
4. Amplitude (Linear) Modulation
   - Double sideband suppressed carrier (DSB-SC)
   - Ordinary amplitude modulation (AM)
   - Single sideband (SSB)
   - Vestigial sideband (VSB)
5. Angle (Exponential) Modulation
   - Frequency modulation (FM)
   - Phase modulation (PM)
6. Random Processes
   - Strict-sense stationary (SSS)
   - Wide-sense stationary (WSS)
   - Statistics

- Random processes and LTI systems
- noise

7. Digital Transmission of Analogue Signals
8. Behaviour of Analogue Systems in the Presence of Noise

1.2 Grading

1. Midterm Exam – Week 8 (20%)
2. Final Exam (80%)
3. Type of Exam: Closed-book.

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[1] S. Haykin, *Communication Systems*, John Wiley and Sons, fourth edition, 2001.

[2] B.P. Lathi, *Modern Digital and Analog Communication Systems*, Oxford University Press, third edition, 1998.

[3] A.B. Carlson, P.B. Crilly and J.C. Rutledge, *Communication Systems, An Introduction to Signals and Noise in Electrical Communication*, McGraw Hill, fourth edition, 2002.

Chapter 2

Overview and Introduction

- Communication describes the general information exchange (Systems, people, media etc.)
- Telecommunication describes the communication with electronic systems. "telecommunication" from the Greek
  1. tele: far away, far off, at a distance
  2. com: with, together
- Modern telecommunications systems : television, radio, telephone, data
- Primitive (though effective) telecommunications systems smoke signals, drums, semaphore
- What sorts of things are communicated? speech, pictures, video, data
- Electrical Energy Systems
  1. waveforms known
  2. design for minimum energy loss
- Communications Systems

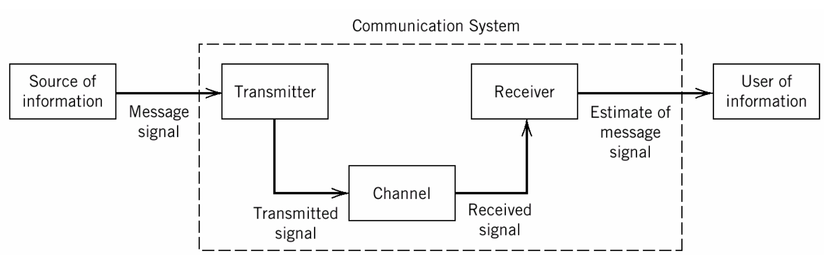


Figure 2.1: Elements of a communication system.

1. waveforms unknown (else no actual information is transmitted)
2. design for efficient information transmission

- Five main concerns for communication system design

  1. Selection of the information bearing waveform
  2. symbol waveform
  3. Bandwidth and power of the waveform
  4. Effect of system noise on the received waveform
  5. Cost of the system, $$$

- Communication systems spend time and frequency band for transferring information.

- In communications systems the transmitted signals tend to be random (unknown). If they were deterministic (known) there would be no point in sending them.

- We use mostly deterministic waveforms in this course as there are lots of insights we can gain through using them.

- In the most fundamental sense, communication involves implicitly the transmission of information from one point to another trough a succession of process:

  1. The generation of a message signal: voice, music, picture, or computer data.
  2. The description of that message signal with a certain measure of precision, by a set of symbols: electrical, aural, or visual.
  3. the encoding of these symbols in a form that is suitable for transmission over a physical medium of interest.
  4. The transmission of the encoded symbols to the desired destination.
  5. the decoding and reproduction of the original signal.
  6. The re-creation of the original message signal, with a definable degradation in quality; the degradation is caused by imperfections in the system.

- Two primary resources are employed

  1. transmitted power, which is the average power of the transmitted signal.
  2. channel bandwidth, which is the band of frequencies allocated for the transmission of the message signal.

- A quantitative way to account for the effect of noise is to introduce signal-to-noise ratio SNR as a system parameter. For example, we may define the SNR at the receiver input as the ratio of the average signal power to the average noise power, both being measured at the same point. Signal-to-noise ratios of 10, 100, 1000 corresponds to 10, 20, and 30 dBs, respectively.

- Low Pass Filtering (LPF) to restrict signal bandwidth avoids wasting signal power on frequencies which will be filtered out by the channel anyway
- Analog to Digital Conversion (ADC) produces a digital word which represents a sample of the analog message waveform
- Signal compression e.g. Huffman or Arithmetic Coding (source coding)
- Add parity bits to aid in correcting bit errors due to channel noise (channel coding)
- Carrier modulation transfers the signal to a frequency band appropriate for the channel, e.g. optical fibre cable baseband signal converted to light frequencies
- General principles of communications apply regardless of the type of channels although different conditioning methods are better suited for different channels.
- Channels always attenuate signals to some degree as well as add noise and, most often, interference.
- Goal of Telecommunications System is "To design the telecommunications system so that the information is transmitted with as little deterioration as possible while satisfying design constraints of allowable transmittable energy and allowable signal bandwidth.
- Common signal deterioration measures:
  1. Digital System: Bit Error Rate (BER) (also called probability of error)
  2. Analog System: Signal to Noise Ratio (SNR) at receiver output

2.1 Analog and Digital Systems

- Specify whether a system is digital or analog by reference to the possible amplitudes of voltage (and/or current) waveforms.
  1. Analog Information Source produces values defined on a continuum e.g. human voice
  2. Digital Information Source produces a finite set of possible symbols e.g. computer keyboard
- Advantages of digital communication systems
  1. privacy via data encryption
  2. greater dynamic range possible
  3. common channel for different data sources
  4. better immunity to noise
- Disadvantages of digital communication systems

1. greater bandwidth requirements
2. synchronization required

2.2 Some Important Events In Telecommunication

1. 1752 Benjamin Franklin s kite shows that lightning is electricity
2. 1844 Samuel Morse demonstrates a telegraph line
3. 1876 Alexander Graham Bell develops the telephone
4. 1900 Guglielmo Marconi transmits the first transatlantic wireless signal
5. 1920 First scheduled radio broadcasts
6. 1926 Baird and Jenkins demonstrate television
7. 1941 Atanasoff invents the digital computer
8. 1948 Shannon's information theory work first published
9. 1958 First integrated circuits produced
10. 1972 Motorola demonstrates the cellular phone
11. 1976 Personal computers developed
12. 1980 Compact Disks developed
13. 1995 The Internet and the World Wide Web become popular.

2.3 Basic Modes of Communication

1. Broadcasting, which involves the use of a signal powerful transmitter and numerous receivers that are relatively inexpensive to build. Here information-bearing signals flow only in one direction.

2. Point-to-point communication, in which the communication process takes place over a link between a single transmitter and a receiver. In this case, there is usually a bidirectional flow of information-bearing signals, which requires the use of a transmitter and receiver at each end of the link.

2.4 Channel

- Wired channel
  1. twisted pair copper telephone lines
  2. waveguides, coaxial cable
  3. fibre optic cable
- Wireless
  1. air (atmosphere)
  2. vacuum (space)
  3. sea water
- Coaxial cable, support the transmission of digital data at much higher bit rates than twisted pairs. Rates up to 20 Mb/s are feasible using coaxial cables, with 10 Mb/s being the standard.
- Optical fiber
  1. Enormous bandwidth, as high as 2×10^{13} Hz.
  2. Low transmission loss, as low as 0.1 dB/km.
  3. Immunity to electromagnetic interference, which is an inherent characteristic of an optical fiber viewed as a dielectric waveguide.
  4. Small size and weight, characterized by a diameter no greater that that of a human hair.
  5. Ruggedness and flexibility, exemplified by very high tensile strengths and the possibility of being bent or twisted without damage.
- Wireless broadcast channels, supports the transmission of radio and television signals.
- Mobile radio channel.
- Satellite channel.

Communication channels may be classified in the following ways:

1. Linear or nonlinear; a wireless radio channel is linear, whereas a satellite channel is usually (but not always) nonlinear.

2. Time invariant or time varying; an optical fiber is time invariant, whereas a mobile radio channel is typically time varying.

3. Bandwidth limited or power limited; a telephone channel is bandwidth limited, whereas an optical fiber link and a satellite channel are both power limited.

2.5 Receiver

1. Demodulates the received signal so that it is converted back to baseband.
2. Cleans up the demodulated signal using appropriate signal processing techniques.
3. Outputs estimate of original message.

2.6 Modulation/Demodulation

- Modulation: varying some parameter of a carrier wave in accordance with the message signal.
- Demodulation: re-creation of the message signal.
- Classification of the modulation process
  1. Continuous-wave modulation, in which a sinusoidal wave is used as a carrier. Types of continuous-wave modulation are amplitude modulation (AM), and angle modulation. The latter is further divided into frequency modulation (FM) and phase modulation (PM).
  2. Pulse modulation, in which the carrier consists of a periodic sequence of rectangular pulses. Types of pulse modulation are pulse-amplitude modulation (PAM), pulse duration modulation (PDM), pulse position modulation (PPM).
- Pulse code modulation (PCM) is the preferred method of modulation for transmission of analog message signals for the following reasons:
  1. Robustness in noisy environments by regenerating the transmitted signal at regular intervals.
  2. Flexible operation.
  3. Integration of diverse sources of information into a common format.
  4. Security of information in its transmission from source to destination.

2.7 Multiplexing

multiplexing is the process of combining several message signals for their simultaneous transmission over the same channel. Three commonly used methods of multiplexing are

1. FDM, in which CW modulation is used to transmit each message signal to reside in a specific frequency slot inside the passband of the channel by assigning it a distinct carrier frequency.
2. TDM, in which pulse modulation is used to position samples of the different message signals in nonoverlapping time slots.
3. CDM, in which each message signal is identified by a distinctive code.

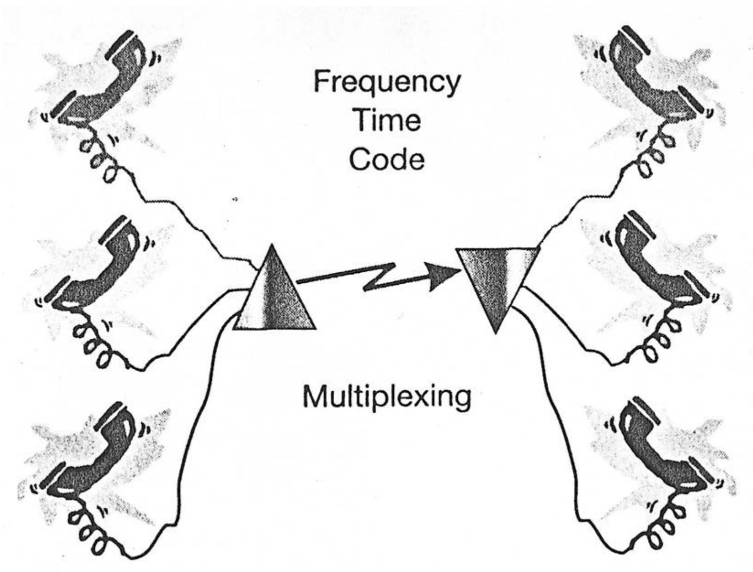


Figure 2.2: Multiplexing.

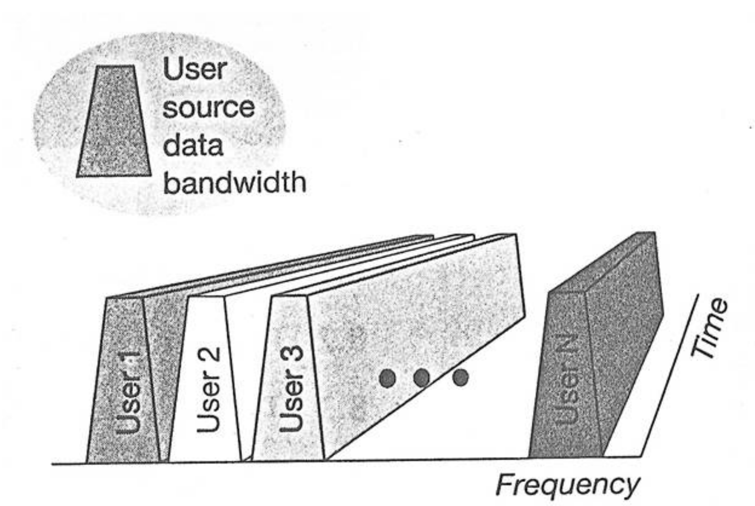


Figure 2.3: Frequency division multiplexing.

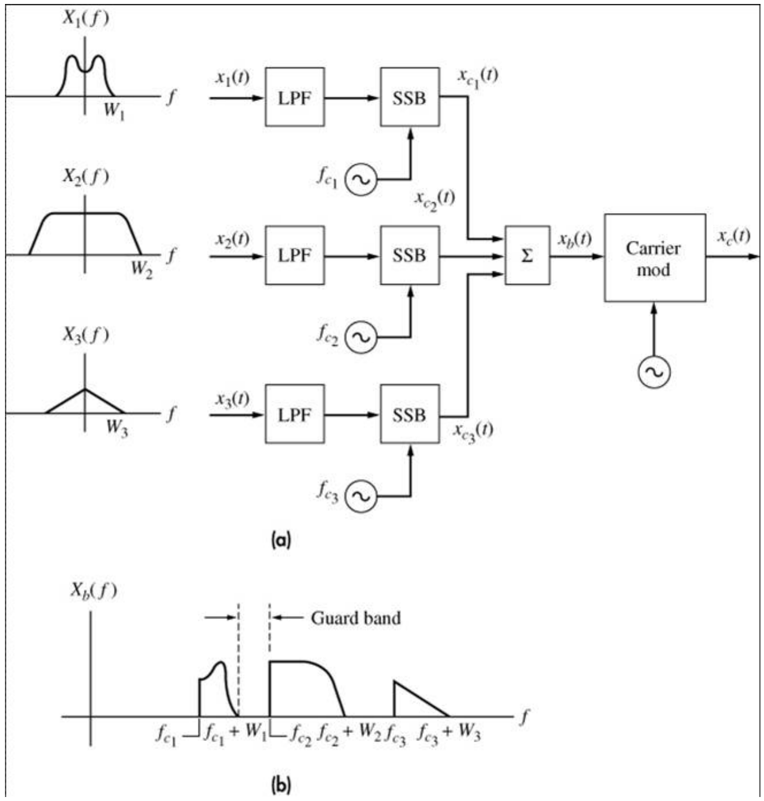


Figure 2.4: FDM transmitter (a) input spectra & block diagram (b) baseband FDM spectrum

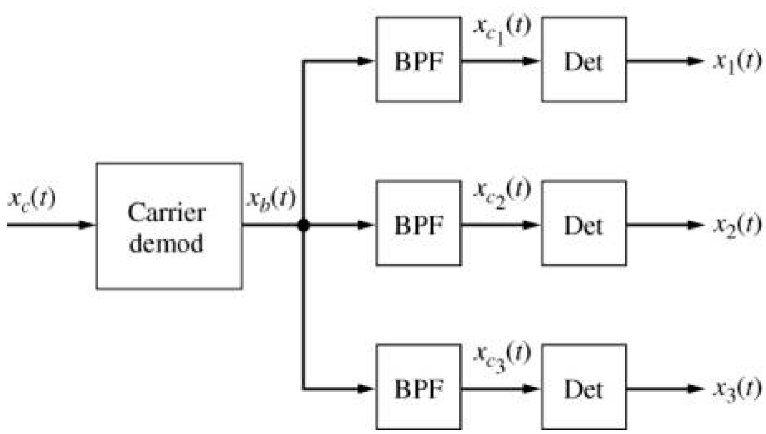


Figure 2.5: FDM receiver

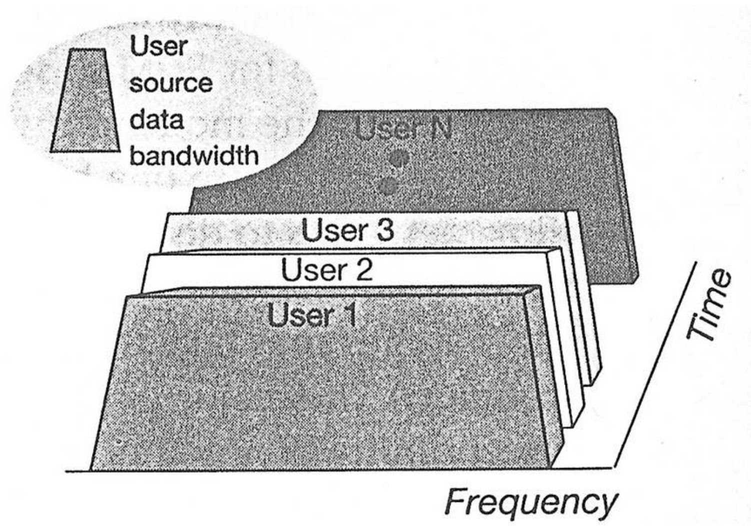


Figure 2.6: Time division multiplexing.

2.8 Radio Spectrum

1. Extremely low frequency (ELF): 3 - 30 Hz
2. Super low frequency (SLF): 30 - 300 Hz
3. Ultra low frequency (ULF): 0.3 - 3 kHz
4. Very low frequency (VLF): 3 - 30 kHz
5. Low frequency (LF): 30 - 300 kHz
6. Medium frequency (MF): 300 - 3000 kHz
7. High frequency (HF): 3 - 30 MHz
8. Very high frequency (VHF): 30 - 300 MHz
9. Ultra high frequency (UHF): 0.3 - 3 GHz
10. Super high frequency (SHF): 3 - 30 GHz
11. Extremely high frequency (EHF): 30 - 300 GHz

2.9 Microwave Frequency Bands

1. L band: 1 to 2 GHz
2. S band: 2 to 4 GHz
3. C band: 4 to 8 GHz

4. X band: 8 to 12 GHz
5. Ku band: 12 to 18 GHz
6. K band: 18 to 26.5 GHz
7. Ka band: 26.5 to 40 GHz
8. V band: 50 to 75 GHz
9. W band: 75 to 110 GHz

2.10 Wavelength Designation

1. longwave (LW): 153 - 279 kHz
2. mediumwave (MW): 531 - 1620 kHz
3. shortwave (SW): 2310 kHz - 30 MHz
4. microwave: 300 MHz and 300 GHz

Chapter 3

Signals and Systems

3.1 Waveform

Waveform of interest in communications systems is usually either voltage, $v(t)$, or current, $i(t)$ as a function of time. Mathematical representation of signals does not always lead to a physically realisable signal e.g.

1. negative frequencies
2. infinite length (time or frequency)
3. discontinuities

3.2 Periodic Waveform

A waveform $w(t)$ is periodic with period T_0 if

$$w(t) = w(t + T_0) \qquad \text{for all } t \tag{3.1}$$

where T_0 is the smallest positive number that satisfies this relationship.

3.3 DC value of a Waveform

The DC value of a waveform $w(t)$ is given by its time average. Thus

$$W_{\text{dc}} = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} w(t)\, dt \tag{3.2}$$

3.4 Instantaneous Power

Let $v(t)$ denote the voltage across a set of circuit terminals and let $i(t)$ denote the current into the terminal. The instantaneous power associated with the circuit is given by

$$p(t) = v(t)i(t) \tag{3.3}$$

3.5 Root Mean Square Value

The root mean square (rms) value of $w(t)$ is given by

$$W_{\text{rms}} = \sqrt{\langle w^2(t) \rangle} \tag{3.4}$$

3.6 Average Power

If a load is resistive (i.e., unity power factor), the average power is

$$\begin{aligned} P &= \frac{\langle v^2(t) \rangle}{R} = \left\langle i^2(t) \right\rangle R = \frac{v_{\text{rms}}^2}{R} \\ &= I_{\text{rms}}^2 R = V_{\text{rms}} I_{rms} \end{aligned} \tag{3.5}$$

where R is the value of the resistive load.

3.7 Average Normalized Power

The average normalized power is given by

$$P = \left\langle w^2(t) \right\rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} w^2(t)\, dt \tag{3.6}$$

where $w(t)$ represents a real voltage or current waveform.

3.8 Power Waveform

$w(t)$ is a power waveform if and only if the normalized average power, P, is finite and nonzero (i.e., $0 < P < \infty$).

3.9 Normalized Energy

The total normalized energy is given by

$$E = \lim_{T\to\infty} \int_{-T/2}^{T/2} w^2(t)\, dt \tag{3.7}$$

3.10 Energy Waveform

$w(t)$ is an energy waveform if and only if the total normalized energy is finite and nonzero (i.e., $0 < E < \infty$).

3.11 Gain

The decibel gain of a circuit is given by

$$\mathrm{dB} = 10\log\left(\frac{\text{average power out}}{\text{average power in}}\right) = 10\log\left(\frac{P_{\text{out}}}{P_{\text{in}}}\right) \tag{3.8}$$

3.12 Signal-to-Noise Ration

The decibel signal-to-noise ratio is

$$(S/N)_{\text{dB}} = 10\log\left(\frac{P_{\text{signal}}}{P_{\text{noise}}}\right) = 10\log\left(\frac{\langle s^2(t)\rangle}{\langle n^2(t)\rangle}\right) \tag{3.9}$$

3.13 Fourier Transform

The Fourier transform (FT) of a waveform $w(t)$ is

$$W(f) = \int_{-\infty}^{\infty} w(t)e^{-j2\pi ft}\,dt \tag{3.10}$$

The energy spectral density (ESD) is defined for energy waveforms by $|W(f)|^2$, where $W(f)$ is the FT of w(t). The energy spectral density has units of joules per hertz.

3.14 Dirac Delta Function

The Dirac delta function $\delta(x)$ is defined by

$$\int_{-\infty}^{\infty} w(x)\delta(x) = w(0) \tag{3.11}$$

where $w(x)$ is any function that is continuous at $x = 0$.

3.15 Unit Step Function

Unit step function

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases} \tag{3.12}$$

3.16 Rectangular Pulse

Rectangular pulse

$$\Pi(\frac{t}{T}) = \begin{cases} 1, & |t| \le \dfrac{T}{2} \\ \\ 0, & |t| > \dfrac{T}{2} \end{cases} \tag{3.13}$$

3.17 Triangular Pulse

Triangular pulse

$$\Lambda(\frac{t}{T}) = \begin{cases} 1 - \frac{|t|}{T}, & |t| \leq T \\ 0, & |t| > T \end{cases} \tag{3.14}$$

3.18 Convolution

The convolution of a waveform $w_1(t)$ with a waveform $w_2(t)$ to produce a third waveform $w_3(t)$ is

$$w_3(t) = w_1(t) * w_2(t) = \int_{-\infty}^{\infty} w_1(\lambda) w_2(t - \lambda)\, d\lambda \tag{3.15}$$

where $w_1(t) * w_2(t)$ is shorthand notation that delineates this integral operation and $*$ is read "convolve with."

3.19 Power Spectral Density

The power spectral density (PSD) for a deterministic power waveform is

$$P_w(f) = \lim_{T\to\infty} \left(\frac{|W_T(f)|^2}{T} \right) \tag{3.16}$$

where $w_T(t) \leftrightarrow W_T(f)$ and $P_w(f)$ has units of watts per hertz.

3.20 Autocorrelation

The autocorrelation of a real (physical) waveform is

$$R_w(\tau) = \langle w(t) w(t + \tau) \rangle = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} w(t) w(t + \tau)\, dt \tag{3.17}$$

3.20.1 Example

$$w(t) = A \sin \omega_0 t \tag{3.18}$$

The autocorrelation function is

$$R_w(\tau) = \frac{A^2}{2} \cos \omega_0 \tau \tag{3.19}$$

The power spectral density is

$$P_w(f) = \frac{A^2}{4} [\delta(f - f_0) + \delta(f + f_0)] \tag{3.20}$$

3.21 Fourier Series

A physical waveform (i.e., finite energy) may be represented over the interval $a < t < a + T_0$ by the complex exponential Fourier series

$$w(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t} \tag{3.21}$$

where the complex (phasor) Fourier coefficients c_n are given by

$$c_n = \frac{1}{T_0} \int_a^{a+T_0} w(t) e^{-jn\omega_0 t}\, dt \tag{3.22}$$

where $\omega_0 = 2\pi f_0 = 2\pi/T_0$.

3.22 Power Spectral Density

For a periodic waveform, the power spectral density (PSD) is given by

$$P(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0) \tag{3.23}$$

where $T_0 = 1/f_0$ is the period of the waveform and the $|c_n|$ are the corresponding Fourier coefficients for the waveform.

3.23 Bandlimited Waveform

A waveform $w(t)$ is said to be (absolutely) bandlimited to B Hertz if

$$W(f) = 0 \qquad |f| \geq B \tag{3.24}$$

3.24 Time Limited Waveform

A waveform $w(t)$ said to be (absolutely) time limited to T seconds if

$$w(t) = 0 \qquad |t| > T \tag{3.25}$$

An absolutely bandlimited waveform cannot be absolutely time limited and vice versa.

3.25 Absolute Bandwidth

Absolute bandwidth is $f_2 - f_1$, where the spectrum is zero outside the interval $f_1 < f < f_2$ along the positive axis.

3.26 3-dB Bandwidth

3-dB bandwidth (or half-power bandwidth) is $f_2 - f_1$, where for frequencies inside the band $f_1 < f < f_2$, the magnitude spectra, say $|H(f)|$, fall no lower than $1/\sqrt{2}$ times the maximum value of $|H(f)|$, and the maximum value at a frequency inside the band.

3.27 Null-to-Null Bandwidth

Null-to-null bandwidth (or zero-crossing bandwidth) is $f_2 - f_1$, where f_2 is the first null in the envelope of the magnitude spectrum above f_0 and, for bandpass systems, f_1 is the first null in the envelope below f_0, where f_0 is the frequency where the magnitude spectrum is a maximum. For baseband systems f_1 is usually zero.

3.28 Bounded Spectrum Bandwidth

Bounded spectrum bandwidth is $f_2 - f_1$ such that outside the band $f_1 < f < f_2$, the PSD, which is proportional to $"|H(f)"|^2$, must be down by at least a certain amount, say 50 dB, below the maximum value of the power spectral density.

3.29 Power Bandwidth

Power bandwidth is $f_2 - f_1$, where $f_1 < f < f_2$ defines the frequency band in which 99% of the total power resides. This is similar to the FCC definition of occupied bandwidth, which states that the power above the upper band edge f_2 is 0.5% and the power below the lower band edge is 0.5%, leaving 99% of the total power within the occupied band.

3.30 Equivalent Noise Bandwidth

Equivalent noise bandwidth is the width of a fictitious rectangular spectrum such that the power in that rectangular band is equal to the power associated with the actual spectrum over positive frequencies. Let f_0 be the frequency where the magnitude spectrum has a maximum; then the power in the equivalent rectangular band is proportional to equivalent power $=$ $B_e q|H(f_0)|^2\,df$ where B_{eq} is the equivalent bandwidth that is to be determined. The actual power for positive frequencies is proportional to actual power $= \int_0^\infty |H(f)|^2\,df$ Setting equivalent power equal to actual power, we have the formula that gives the equivalent noise bandwidth $B_{\text{eq}} = \frac{1}{|H(f_0)|^2} \int_0^\infty |H(f)|^2\,df$

3.31 FCC Bandwidth

FCC bandwidth is an authorized bandwidth parameter assigned by the FCC to specify the spectrum allowed in communication systems. When the FCC bandwidth parameter is substituted into the FCC formula, the minimum attenuation is given for the power level allowed in a 4-kHz band at the band edge with respect to the total average signal power. Quoting Sec. 21.106 of the FCC Rules and Regulations: "For operating frequencies below 15 GHz, in any 4 kHz band, the center frequency of which is removed from the assigned frequency by more than 50 percent up to and including 250 percent of the authorized bandwidth, as specified by $A = 35 + 0.8(P - 50) + 10\log_{10}(B)$ where A is attenuation (in decibels) below the mean output power level, P is percent removed from the carrier frequency, and B is authorized bandwidth in megahertz.

Chapter 4

Signal Transmission

4.1 Undistorted Transmission

The output is undistorted if it differs from the input only by a multiplying constant and a finite time delay. That is we have distortionless transmission if

$$y(t) = Kx(t - t_d) \tag{4.1}$$

where K and t_d are constant.

$$H(f) = Ke^{-j\omega t_d} \tag{4.2}$$

A system giving distortionless transmission

$$|H(f)| = |K| \qquad \arg H(f) = -2\pi t_d f \pm m180^\circ \tag{4.3}$$

1. Amplitude distortion occurs when

$$|H(f)| \neq |K| \tag{4.4}$$

2. Delay distortion occurs when

$$\arg H(f) \neq -2\pi t_d f \pm m180^\circ \tag{4.5}$$

Clearly, delay distortion can be critical in pulse transmission, and much labor is spent equalizing transmission delay for digital data systems and the like. On the other hand, the human ear is curiously insensitive to delay distortion.

4.2 Example

Consider the simplest equalizer with taps c_{-1}, c_0 and c_1. The delay line is 2Δ. Thus the output is

$$y(t) = c_{-1}x(t) + c_0x(t - \Delta) + c_1x(t - 2\Delta) \tag{4.6}$$

or

$$H_{eq}(f) = \left(c_{-1}e^{+j\omega\Delta} + c_0 + c_1e^{-j\omega\Delta}\right) e^{-j\omega\Delta} \tag{4.7}$$

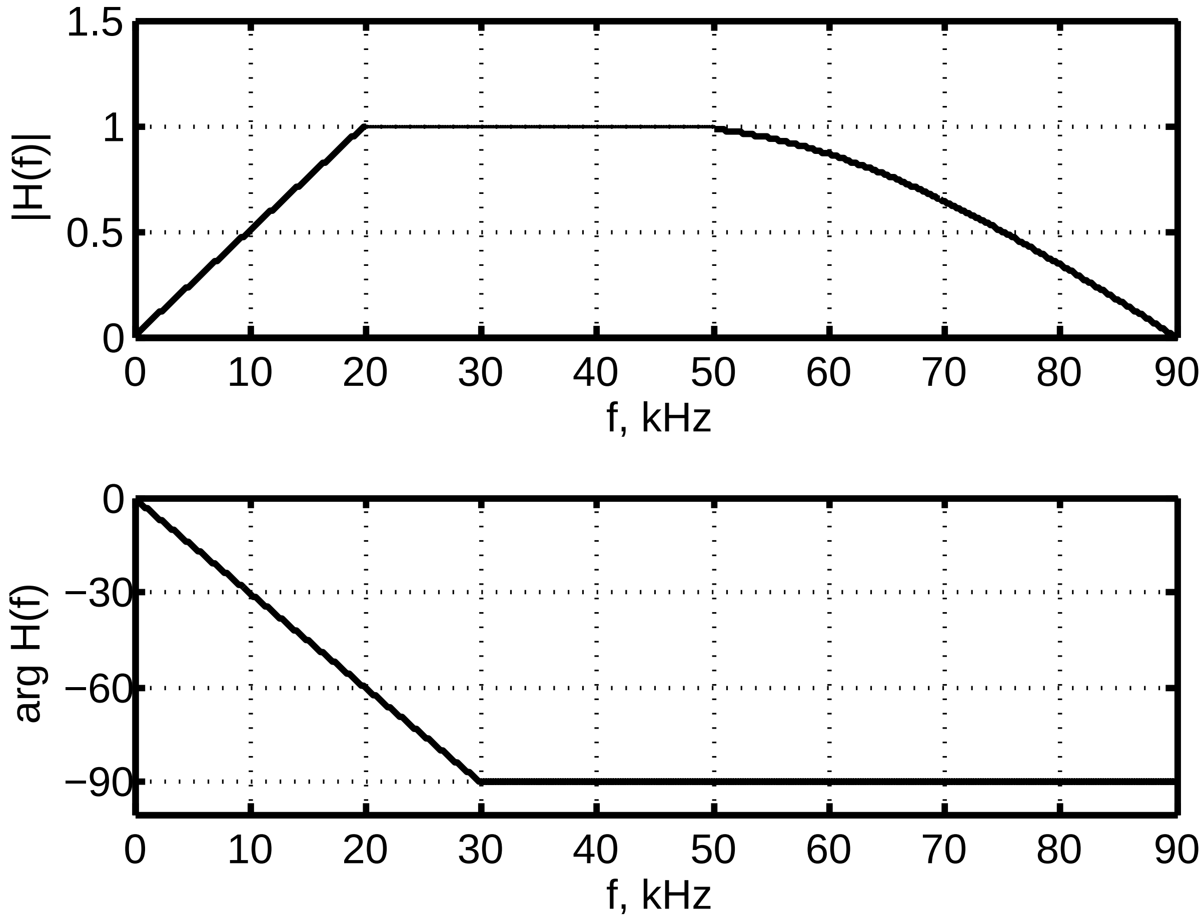


Figure 4.1: Amplitude and phase distortion.

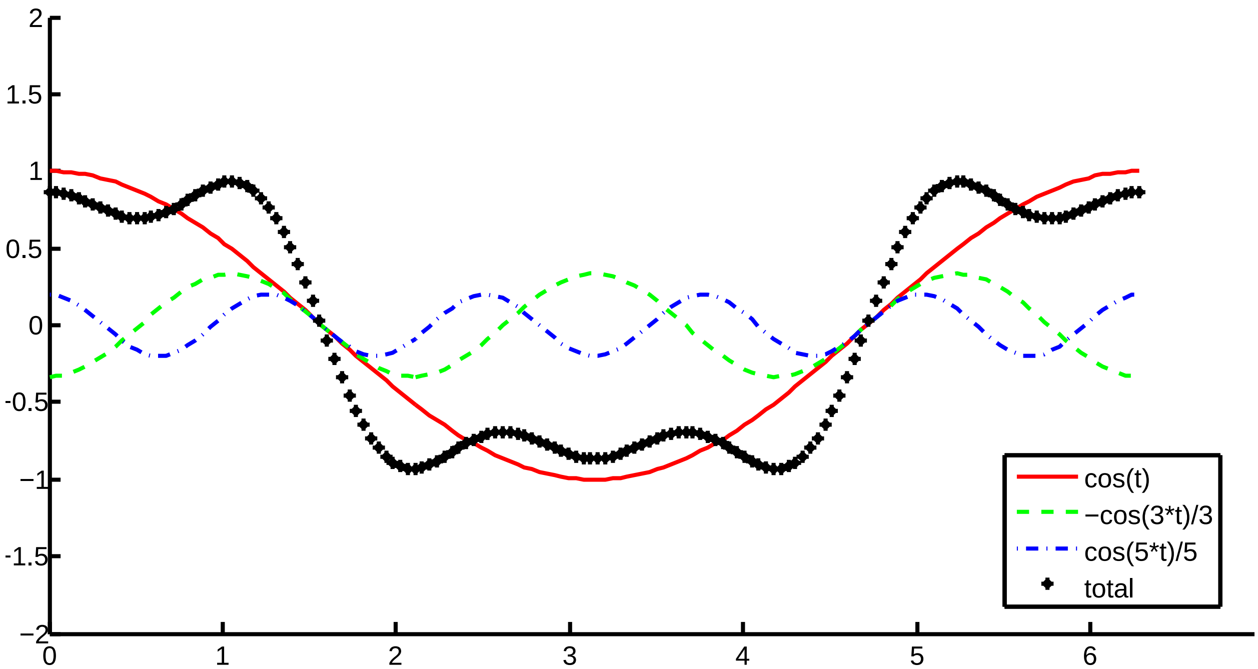


Figure 4.2: Test signal.

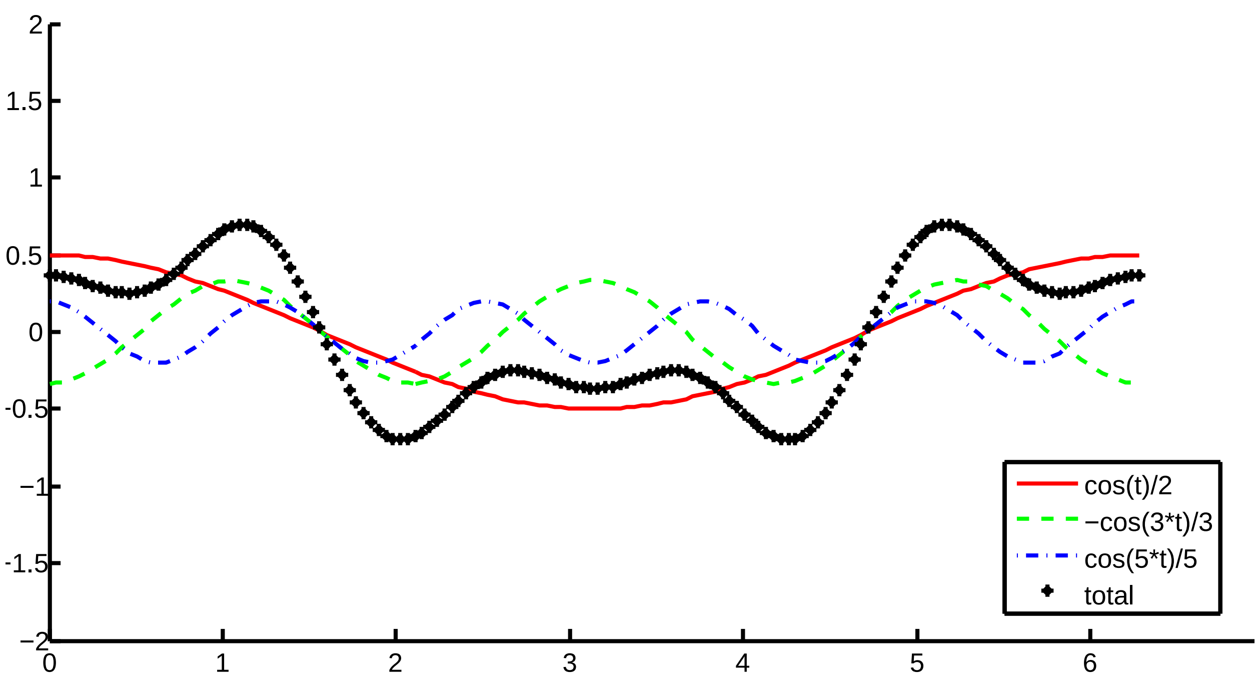


Figure 4.3: Test signal with amplitude distortion (a) low frequency attenuated.

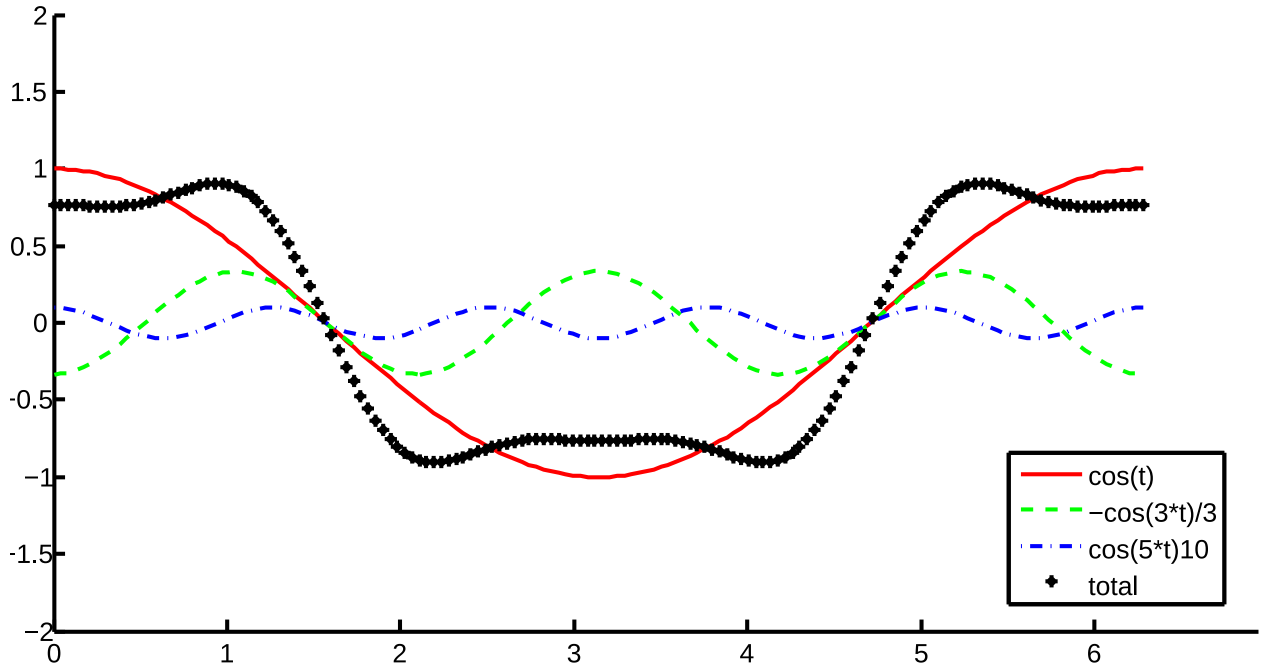


Figure 4.4: Test signal with amplitude distortion (b) high frequency attenuated.

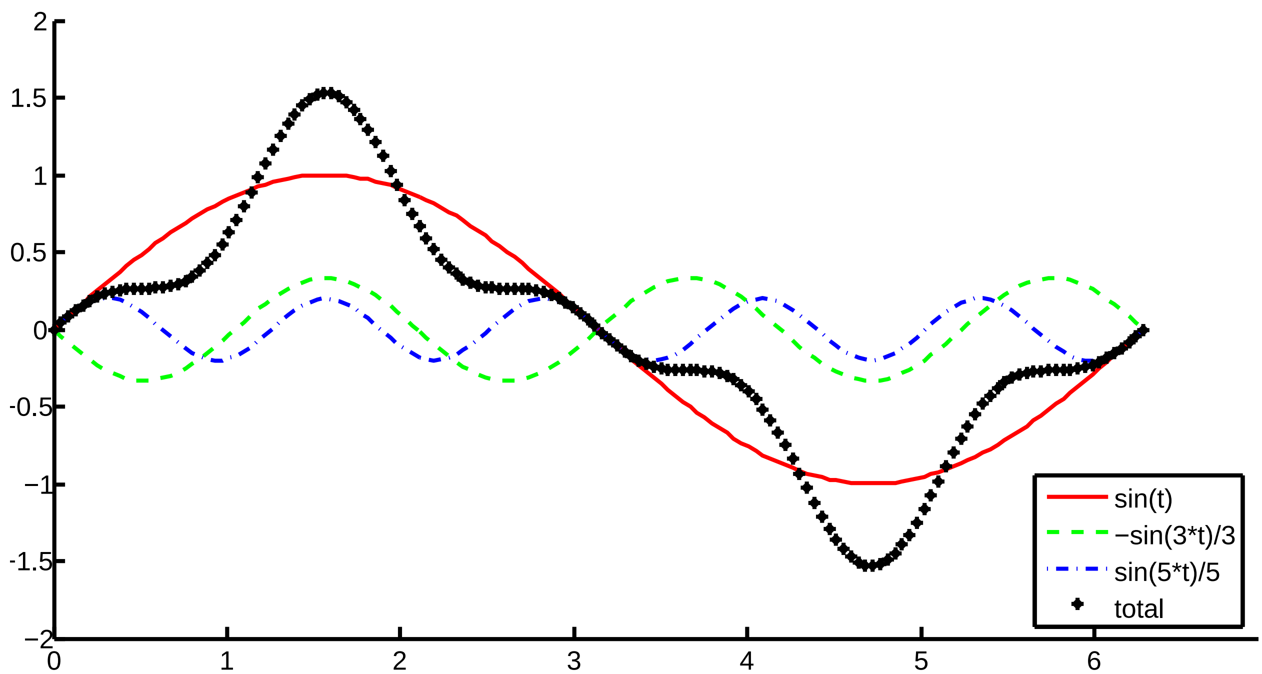


Figure 4.5: Test signal with constant phase shift $\theta = -90°$

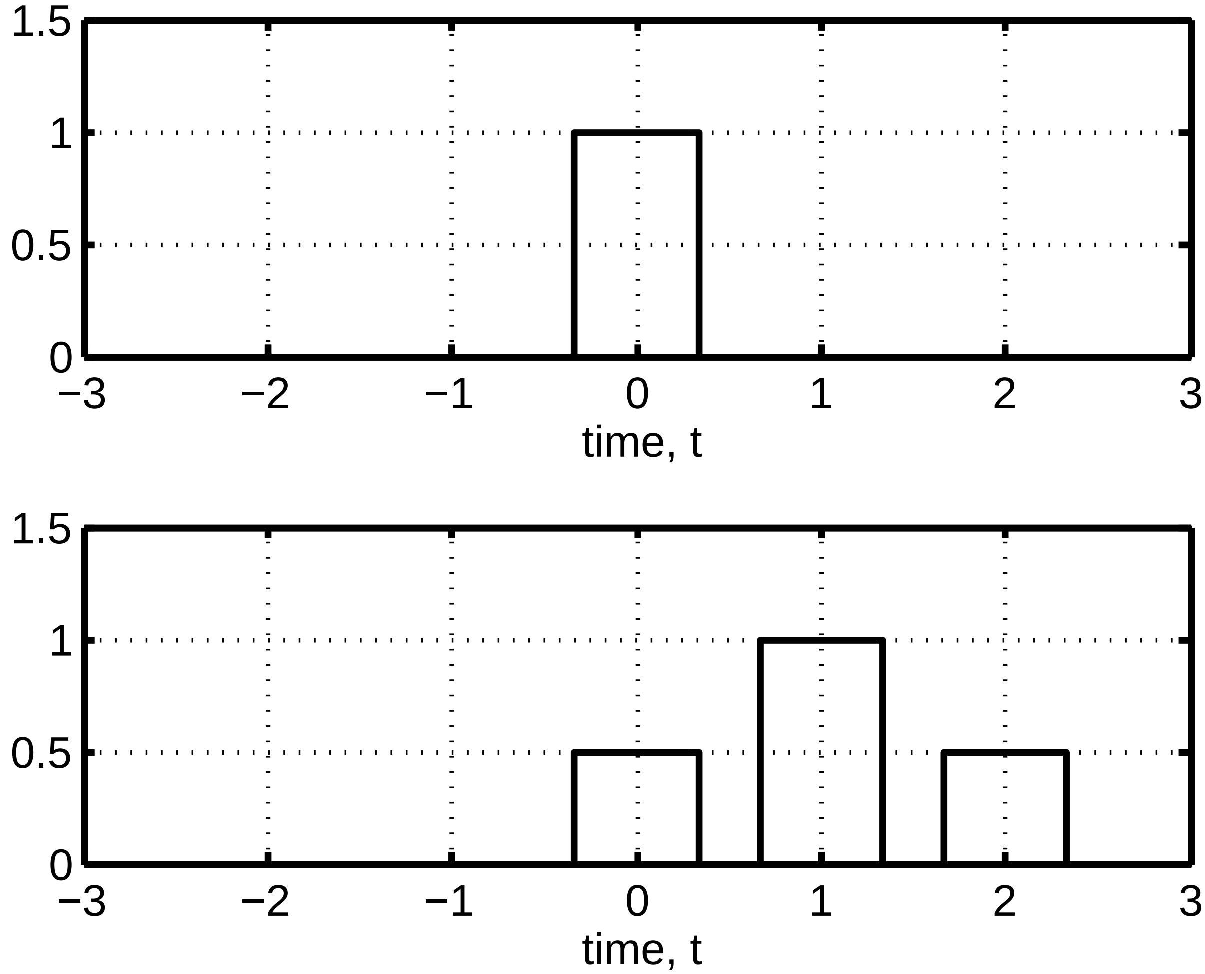


Figure 4.6: Multipath distortion, $y(t) = \alpha x(t) + x(t - T) + \alpha x(t - 2T)$, $x(t) = \Pi(t/\tau)$, $\alpha = 1/2$, $\tau = 2T/3$, $T = 1$

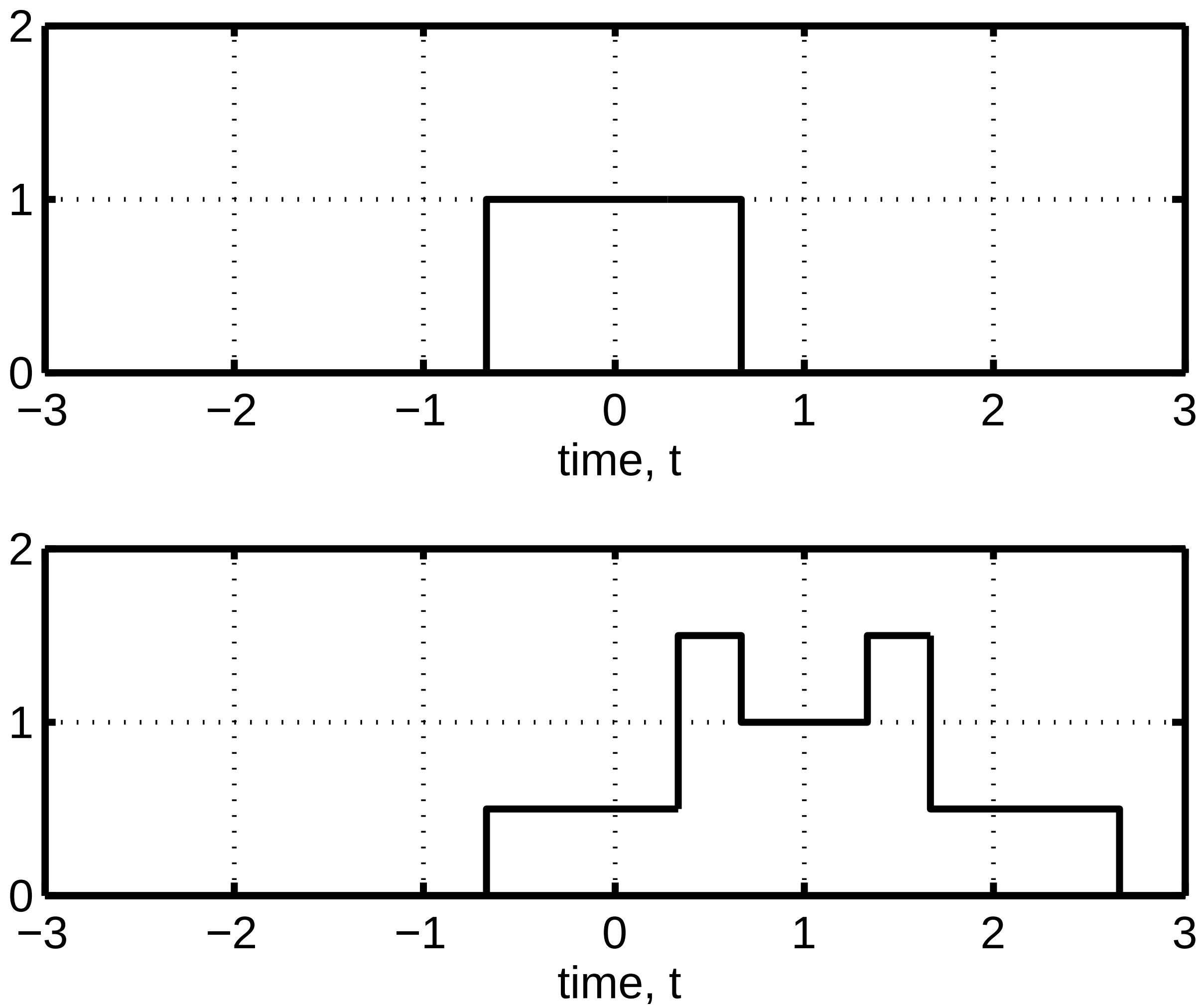


Figure 4.7: Multipath distortion, $y(t) = \alpha x(t) + x(t-T) + \alpha x(t-2T)$, $x(t) = \Pi(t/\tau)$, $\alpha = 1/2$, $\tau = 4T/3$, $T = 1$

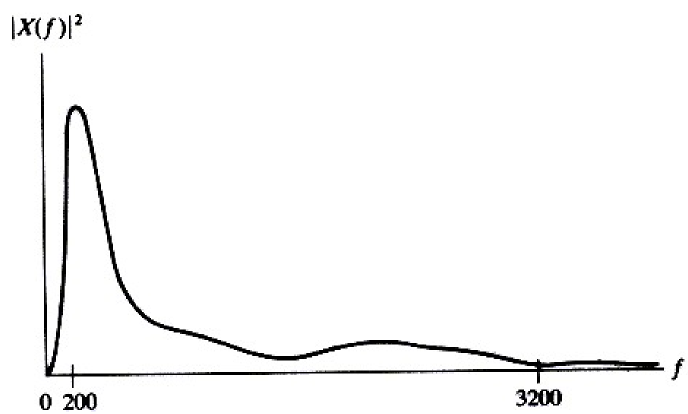


Figure 4.8: Energy spectral density of an average voice signal.

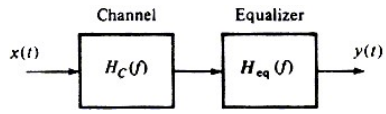


Figure 4.9: Channel equalisation with linear distortion.

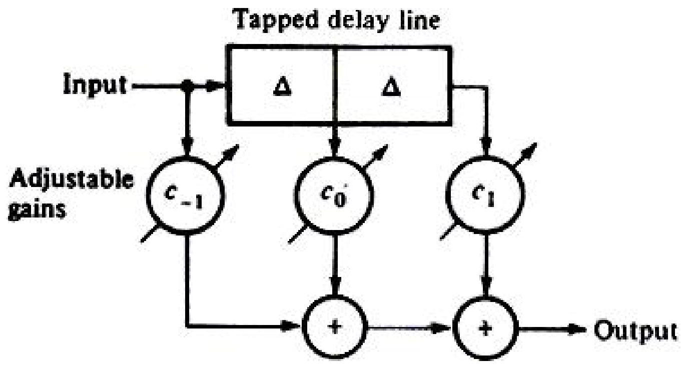


Figure 4.10: Tapped delay line equaliser

In general, with a $2M\Delta$ delay line with $2M+1$ taps

$$H_{eq}(f) = \left(\sum_{m=-M}^{M} c_m e^{-j\omega m\Delta} \right) e^{-j\omega M\Delta} \tag{4.8}$$

4.3 Nonlinear Distortion

$$y(t) = a_1 x(t) + a_2 x^2(t) + a_3 x^3(t) + \cdots \tag{4.9}$$

$$x(t) = \cos \omega_0 t \tag{4.10}$$

$$y(t) = \left(\frac{a_2}{2} + \frac{3a_4}{8} + \cdots \right) + \left(a_1 + \frac{3a_3}{4} + \cdots \right) \cos \omega_0 t + \left(\frac{a_2}{2} + \frac{a_4}{4} + \cdots \right) \cos 2\omega_0 t + \cdots$$

$$\text{Second-harmonic distortion} = \left| \frac{a_2/2 + a_4/4 + \cdots}{a_1 + 3a_3/4 + \cdots} \right| \times 100\% \tag{4.11}$$

If the input is a sum of two cosine waves, say $\cos \omega_1 t + \cos \omega_2 t$, the output will include all the harmonics of f_1 and f_2, plus crossproduct terms which yield $f_2 - f_1$, $f_2 + f_1$, $f_2 - 2f_1$, etc. These sums and difference frequencies are designated intermodulation distortion.

In the frequency domain $x_1(t)x_2(t)$ becomes $X_1(f) * X_2(f)$. These spectra overlap, producing one form of crosstalk.

power gain of an LTI system whose input and output signals have average power P_{in} and P_{out}, respectively is given by

$$g = P_{\text{out}}/P_{\text{in}} \tag{4.12}$$

Figure 4.11: Mesbah.

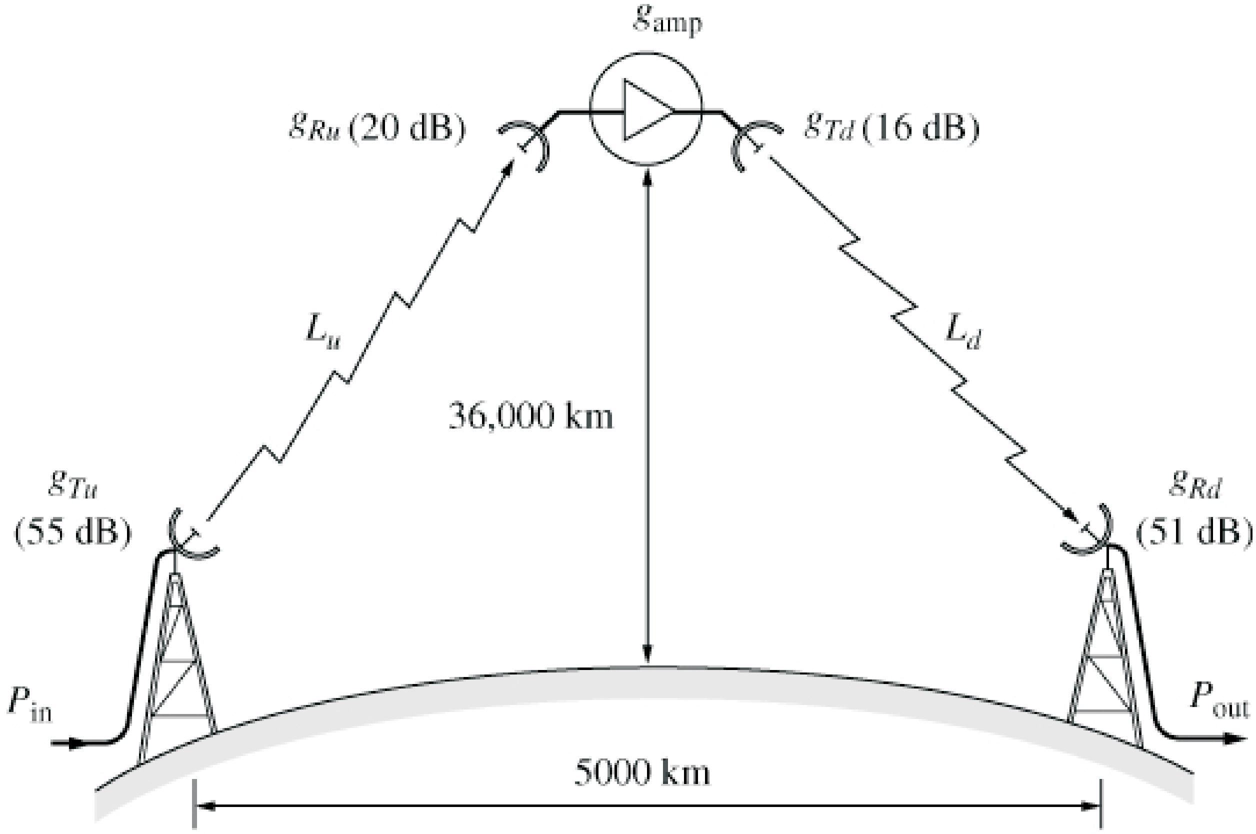


Figure 4.12: Satellite relay system.

$$g_{\rm dB} = 10\log_{10} g \tag{4.13}$$

$$P_{\rm dBW} = 10\log_{10}\frac{P}{1W} \tag{4.14}$$

$$P_{\rm dBm} = 10\log_{10}\frac{P}{1mW} \tag{4.15}$$

$$P_{\rm out} = g_{\rm dB} + P_{\rm in} \tag{4.16}$$

4.4 Transmission Loss and Repeaters

$$L = 1/g \tag{4.17}$$

$$P_{\rm out} = P_{\rm in} - L_{\rm dB} \tag{4.18}$$

In the case of transmission lines, coaxial and fiber-optic cables, and waveguides, the output power decreases exponentially with distance. We'll write this relation is the form

$$P_{\rm out} = 10^{-(\alpha l/10)} P_{\rm in} \tag{4.19}$$

where l is the path length between source and destination and α is the attenuation in dB per unit length.

$$L_{\rm dB} = \alpha l \tag{4.20}$$

4.5 Example

Suppose you transmit a signal on a 30 km length of cable having $\alpha = 3$ dB/km. Then $L_{\rm dB} = 3\times 30 = 90$ dB, and $P_{\rm out} = 10^{-9}P_{\rm in}$. Doubling the path length doubles the attenuation to 180 dB, so that $L = 10^{18}$ and $P_{\rm out} = 10^{-18}P_{\rm in}$. This loss is so great that you'd need an input power of one megawatt to get an output power of one picowatt!

Large attenuation certainly calls for amplification to boost the output signal.

$$P_{\rm out} = \frac{g_2 g_4}{L_1 L_3} P_{\rm in} \tag{4.21}$$

4.6 Radio transmission free space loss

$$L = \left(\frac{4\pi l}{\lambda}\right)^2 \tag{4.22}$$

4.7 Problem

Consider a transmission channel with $H_C(f) = (1 + 2\alpha\cos\omega T)e^{-j\omega T}$, which has amplitude ripples. (a) Show that $y(t) = \alpha x(t) + x(t-T) + \alpha x(t-2T)$, so the output includes a leading and trailing echo. (b) Let $x(t) = \prod(t/\tau)$ and $\alpha = 1/2$. Sketch $y(t)$ for $\tau = 2T/3$ and $4T/3$. Design a tapped-delay line equalizer with $\alpha = 0.4$.

4.8 Problem

Consider a transmission channel with $H_C(f) = \exp[-j(\omega T - \alpha \sin \omega T)]$, which has phase ripples. Assume $|\alpha| \ll \pi/2$ and use a series expansion to show that the output includes a leading and trailing echo. Design a tapped-delay line equalizer with $\alpha = 0.4$.

4.9 Problem

Suppose $x(t) = A\cos\omega_0 t$ is applied to a nonlinear system with $y(t) = 2x(t) - 3x^3(t)$. Write $y(t)$ as a sum of cosines. Then evaluate the second-harmonic and third-harmonic distortion when $A = 1$ and $A = 2$. Repeat for $y(t) = 5x(t) - 2x^2(t) + 4x^3(t)$.

4.10 Problem

A 400 km repeater system consists of m identical cable sections with $\alpha = 0.4$ dB/km and m identical amplifiers with 30 dB maximum gain. Find the required number of sections and the gain per amplifier so that $P_{\text{out}} = 50$ mW when $P_{\text{in}} = 2W$.

4.11 Problem

A 3000 km repeater system consists of m identical fiber-optic cable sections with $\alpha = 0.5$ dB/km and m identical amplifiers. Find the required number of sections and the gain per amplifier so that $P_{\text{out}} = P_{\text{in}} = 5$ mW and the input power to each amplifier is at least $67\,\mu$W. Repeat your calculations with $\alpha = 2.5$ dB/km.

Chapter 5

Double Sideband-Suppressed Carrier Modulation

Most of the common signalling techniques consist of modulating an analog or digital baseband signal onto a carrier. All can be represented by the generic bandpass form

$$s(t) = g(t)\cos(2\pi f_c t) \tag{5.1}$$

or

$$s(t) = \Re\{g(t)\exp(j2\pi f_c t)\} \tag{5.2}$$

where f_c is the carrier frequency. The function $g(t)$ is referred to as the envelope of the modulated signal. The desired type of modulating signal is obtained by selecting the appropriate modulation function $g[m(t)]$ where $m(t)$ is the analog or digital baseband message signal. We always assume that $m(t)$ is real but the function $g(t)$ is complex or real. Either the frequency, phase or amplitude of the carrier is varied in proportion to the baseband signal $m(t)$.

5.1 Modulation

Modulation is a process that causes a shift in the range of frequencies in a signal.

5.2 Baseband

The term baseband is used to designate the band of frequencies of the signal delivered by the source or the input transducer. In telephony, the baseband is 0 to 3.5 kHz. In television, the baseband is 0 to 4.3 MHz. Baseband communication does not use modulation.

5.3 Example

Long-distance PCM over optical fibers.

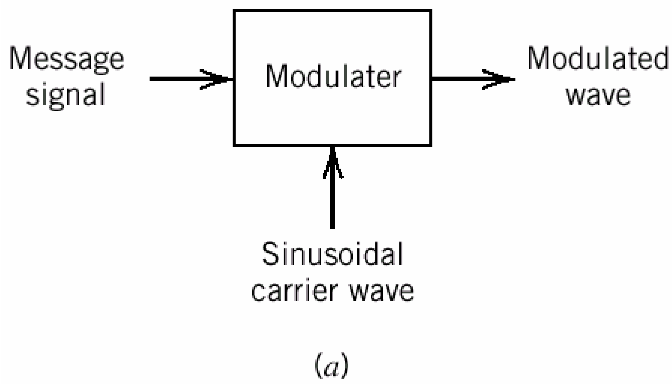


Figure 5.1: Transmitter.

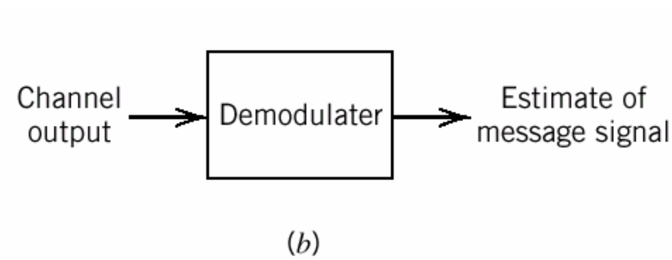


Figure 5.2: receiver.

5.4 Carrier Communication

Carrier Communication uses modulation. Long-haul communication over a radio link is an example of carrier communication.

In carrier communication, one of the basic parameters (amplitude, frequency, or phase) of a sinusoidal carrier of high frequency ω_c is varied in proportion to $m(t)$. This results in AM, FM, or PM, respectively.

FM and PM belong to the class of modulation known as angle modulation.

The carrier amplitude A is made directly proportional to the modulating signal $m(t)$. The DSB signal is

$$m(t)\cos\omega_c t \leftrightarrow \frac{1}{2}[M(\omega+\omega_c)+M(\omega-\omega_c)] \tag{5.3}$$

That is, the DSB modulation translates or shifts the frequency spectrum to the left and the right by ω_c.

If the bandwidth of $m(t)$ is B Hz, then the bandwidth of the modulated signal is $2B$ Hz.

5.5 Lower sideband

Lower sideband (LSB) is a portion of DSB spectrum which lies below ω_c.

5.6 Upper sideband

Upper sideband (USB) is a portion of DSB spectrum which lies above ω_c.

5.7 DSB-SC

DSB signal does not contain a discrete component of the carrier frequency ω_c. For this reason DSB is called DSB-SC.

5.8 Demodulation

Demodulation, or detection, is the process of recovering the signal from the modulated signal.

5.9 Synchronous

In synchronous or coherent detection the local oscillator has the same frequency and phase as the carrier used for modulation.

5.10 Modulation

Modulation can be obtained by multiplier modulators, nonlinear modulators, and switching modulators.

5.10.1 Multiplier Modulator

Here modulation is achieved directly by multiplying $m(t)$ by $\cos \omega_c t$ using an analog multiplier whose output is proportional to the product of two input signal. Another way to multiply two signals is through logarithmic amplifiers. Here the basic components are a logarithmic and an antilogarithmic amplifier with outputs proportional to the log and antilog of their inputs, respectively. Using two logarithmic amplifiers, we generate and add the logarithms of the two signals to be multiplied. The sum is then applied to an antilogarithmic amplifier to obtain the desired product. That is,

$$\begin{aligned} AB &= \text{antilog}[\log(AB)] \\ &= \text{antilog}[\log A + \log B] \end{aligned} \tag{5.4}$$

It is rather difficult to maintain linearity in this kind of amplifier, and they tend to be rather expensive.

5.10.2 Nonlinear Modulator

Modulation can be achieved by using nonlinear devices, such as a semiconductor diode or a transistor.

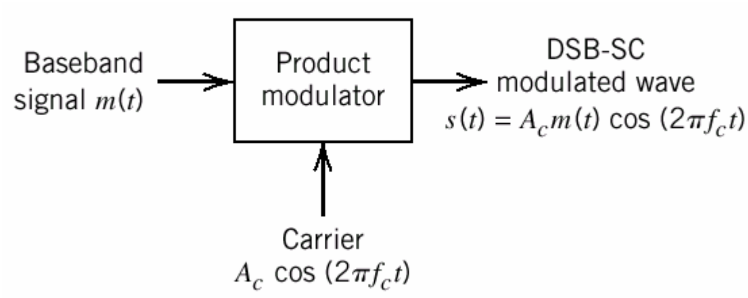


Figure 5.3: Block diagram of product modulator.

5.10.3 Switching Modulator

The multiplication operation required for modulation can be replaced by a simpler switching operation in which a modulated signal can be obtained by multiplying $m(t)$ not only by a pure sinusoidal but by any periodic signal $w(t)$ of the fundamental radian frequency ω_c. For example with square pulse train

$$w(t) = \frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right) \tag{5.5}$$

or with a square waveform

$$w(t) = \frac{4}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right) \tag{5.6}$$

5.11 Frequency Mixing

Frequency mixing or frequency conversion is used to change the carrier frequency of a modulated signal $m(t)\cos\omega_c t$ from ω_c to some other frequency ω_I. This can be done by multiplying $m(t)\cos\omega_c t$ by $2\cos\omega_{\text{mix}} t$, where $\omega_{\text{mix}} = \omega_c + \omega_I$ for up-conversion or $\omega_{\text{mix}} = \omega_c - \omega_I$ for down-conversion and then bandpass-filtering the product.

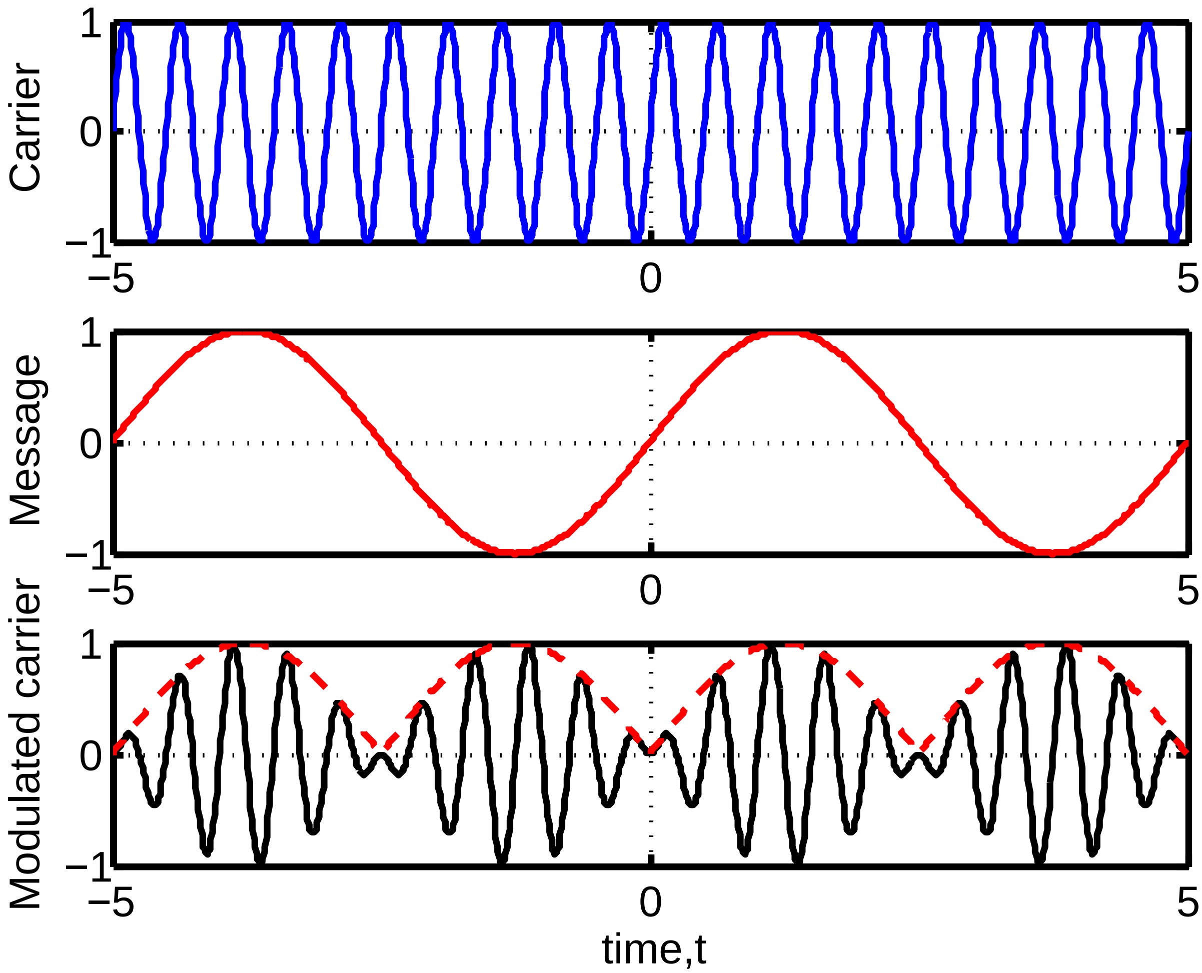


Figure 5.4: DSB modulation.

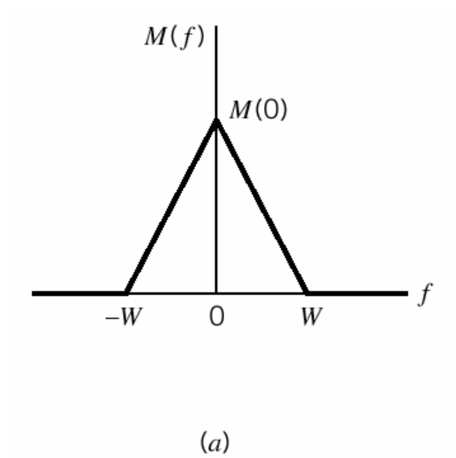


Figure 5.5: Spectrum of baseband signal.

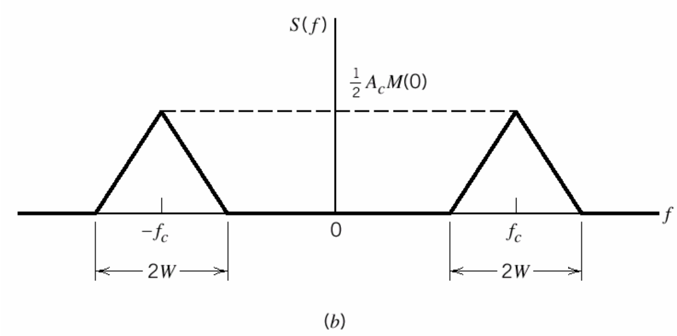


Figure 5.6: Spectrum of DSB-SC modulated wave.

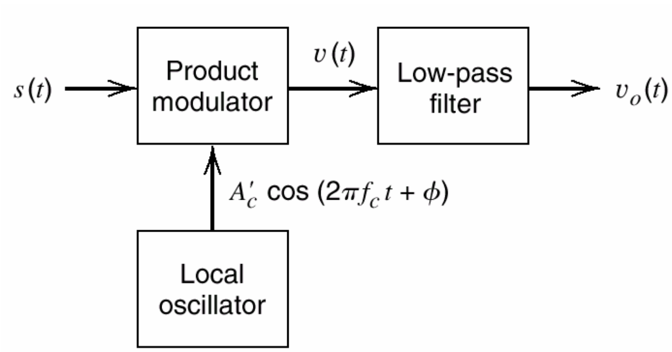


Figure 5.7: Coherent detector for demodulating DSB-SC modulated wave.

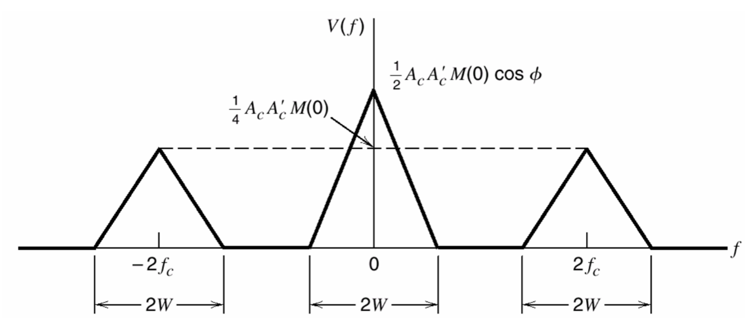


Figure 5.8: Spectrum of a product modulator output with a DSB-SC modulated wave as input.

5.12 Problem

Consider a multiplex system in which four input signals $m_1(t)$, $m_2(t)$, $m_3(t)$, and $m_4(t)$ are respectively multiplied by the carrier waves

$$[\cos(2\pi f_a t) + \cos(2\pi f_b t)] \tag{5.7}$$

$$[\cos(2\pi f_a t + \alpha_1) + \cos(2\pi f_b t + \beta_1)] \tag{5.8}$$

$$[\cos(2\pi f_a t + \alpha_2) + \cos(2\pi f_b t + \beta_2)] \tag{5.9}$$

$$[\cos(2\pi f_a t + \alpha_3) + \cos(2\pi f_b t + \beta_3)] \tag{5.10}$$

and the resulting DSB-SC signals are summed and then transmitted over a common channel. In the receiver, demodulation is achieved by multiplying the sum of the DSB-SC signals by the four carrier waves separately and then using filtering to remove the unwanted components.

(a) Determine the conditions that the phase angles α_1, α_2, α_3 and β_1, β_2, β_3 must satisfy in order that the output of the kth demodulator is $m_k(t)$, where $k = 1, 2, 3, 4$.

(b) Determine the minimum separation of carrier frequencies f_a and f_b in relation to the bandwidth of the input signals so as to ensure a satisfactory operation of the system.

Chapter 6

Ordinary Amplitude Modulation

6.1 Time Domain

$$\begin{aligned}\varphi_{\text{AM}}(t) &= A\cos\omega_c t + m(t)\cos\omega_c t \\ &= [A + m(t)]\cos\omega_c t\end{aligned} \tag{6.1}$$

6.2 Spectrum

$$\varphi_{\text{AM}}(t) \leftrightarrow \frac{1}{2}[M(\omega+\omega_c) + M(\omega-\omega_c)] + \pi A[\delta(\omega+\omega_c) + \delta(\omega-\omega_c)] \tag{6.2}$$

That is, the AM spectrum is simply a translated version of the modulated signal (both positive and negative with power equally distributed between the two) plus delta functions at the carrier line spectral component.

Larger power at the transmitter, which makes it rather expensive. Less expensive receivers. The envelope of AM has the information about the message $m(t)$ only if the AM signal $[A + m(t)]\cos\omega_c t$ satisfies the condition $A + m(t) \geq 0$ for all t. This means that

$$A \geq m_p \tag{6.3}$$

where m_p is the peak amplitude (positive or negative) of $m(t)$.

6.3 Modulation Index

$$\mu = \frac{m_p}{A} \tag{6.4}$$

Because $A \geq m_p$ and because there is no upper bound on A, it follows that

$$\mu \leq 1 \tag{6.5}$$

as the required condition for the viability of demodulation of AM by an envelope detector.

6.4 Overmodulation

Overmodulation results if $\mu > 1$ or $A < m_p$.

6.5 Power Efficiency

$$\eta = \frac{\text{useful power}}{\text{total power}} = \frac{P_s}{P_c + P_s} \tag{6.6}$$

where P_c the carrier power, and P_s is the power in sidebands.

6.5.1 Example

For tone modulation $m(t) = \mu A \cos \omega_m t$, $P_s = \frac{1}{2}\frac{(\mu A)^2}{2}$, and $P_c = \frac{A^2}{2}$. Hence

$$\eta = \frac{\mu^2}{2 + \mu^2} 100\% \tag{6.7}$$

With maximum modulation index, that is $\mu = 1$, $\eta = 33.3\%$, which means that only 33.3% of the power is used to transmit the sidebands.

6.6 Positive Modulation

The percentage of positive modulation on an AM signal is

$$(A_{\max} - A)/(A) \times 100 = \max[\mu m(t)] \times 100 \tag{6.8}$$

6.7 Negative Modulation

The percentage of negative modulation is

$$(A - A_{\min})/(A) \times 100 = \min[\mu m(t)] \times 100 \tag{6.9}$$

6.8 Overall Modulation Percentage

The overall modulation percentage is

$$(A_{\max} - A_{\min})/(2A) \times 100 \tag{6.10}$$

or

$$(\max[\mu m(t)] - \min[\mu m(t)])/(2) \times 100 \tag{6.11}$$

So, the positive modulation refers to the maximum amount the amplitude of the modulated carrier is increased over the maximum amplitude of the unmodulated carrier. And, the negative modulation refers to the maximum amount the amplitude of the modulated carrier is decreased in comparison with the unmodulated carrier. Note that for any message signal which is symmetric (at least in terms of maximum and minimum values) about the x axis, the percentage of positive and negative modulation (and therefore the overall modulation) will always be equal. The percentage of modulation can be greater than 100%, in which case A_{min} has a negative value

6.9 Generation of AM Signal

$$v_{bb'}(t) = [c\cos\omega_c t + m(t)]w(t) \tag{6.12}$$

where

$$w(t) = \left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right] \tag{6.13}$$

Hence

$$v_{bb'}(t) = \underbrace{\frac{c}{2}\cos\omega_c t + \frac{2}{\pi}m(t)\cos\omega_c +}_{\text{AM}} \underbrace{\text{other terms}}_{\text{suppressed by bandpass filter}} \tag{6.14}$$

6.10 Demodulation of AM Signals

6.10.1 Rectifier Detector

$$v_R = \{[A + m(t)]\cos\omega_c t\}w(t) \tag{6.15}$$

where

$$w(t) = \left[\frac{1}{2} + \frac{2}{\pi}\left(\cos\omega_c t - \frac{1}{3}\cos 3\omega_c t + \frac{1}{5}\cos 5\omega_c t - \cdots\right)\right] \tag{6.16}$$

Hence

$$v_R = \frac{1}{\pi}[A + m(t)] + \text{other terms of higher frequencies} \tag{6.17}$$

6.10.2 Envelope Detector

In an envelope detector, the output of the detector follows the envelope of the modulated signal. The time constant RC must be large compared to $1/\omega_c$ but should be small compared to $1/2\pi B$, where ω_c and B and radian frequency of carrier and bandwidth of the message to be recovered.

Compared with an AM signal, a DSBSC signal has infinite percentage modulation because there is no carrier line component.

6.11 Modulation Efficiency

The modulation efficiency of a DSBSC signal is 100% since no power is wasted in a discrete carrier.

However, a product detector is always required for a DSBSC signal (more expensive than an envelope detector).

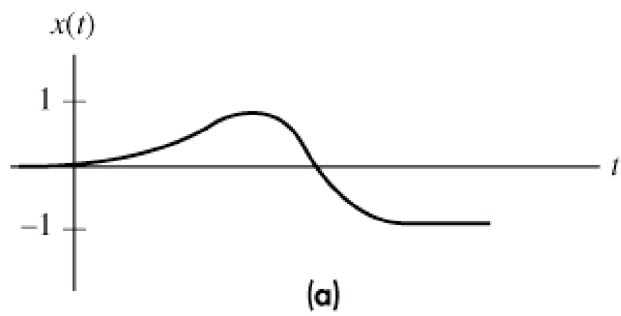


Figure 6.1: Message.

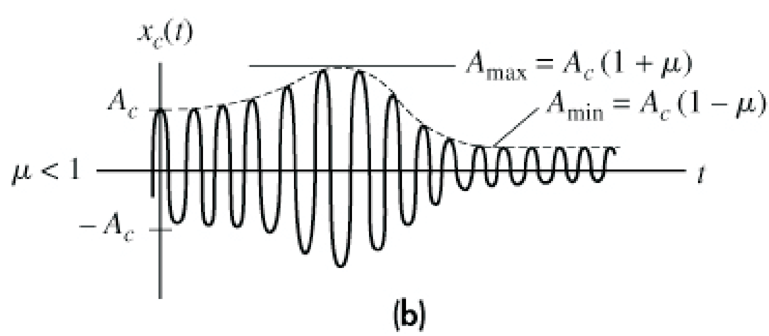


Figure 6.2: AM signal with $\mu < 1$.

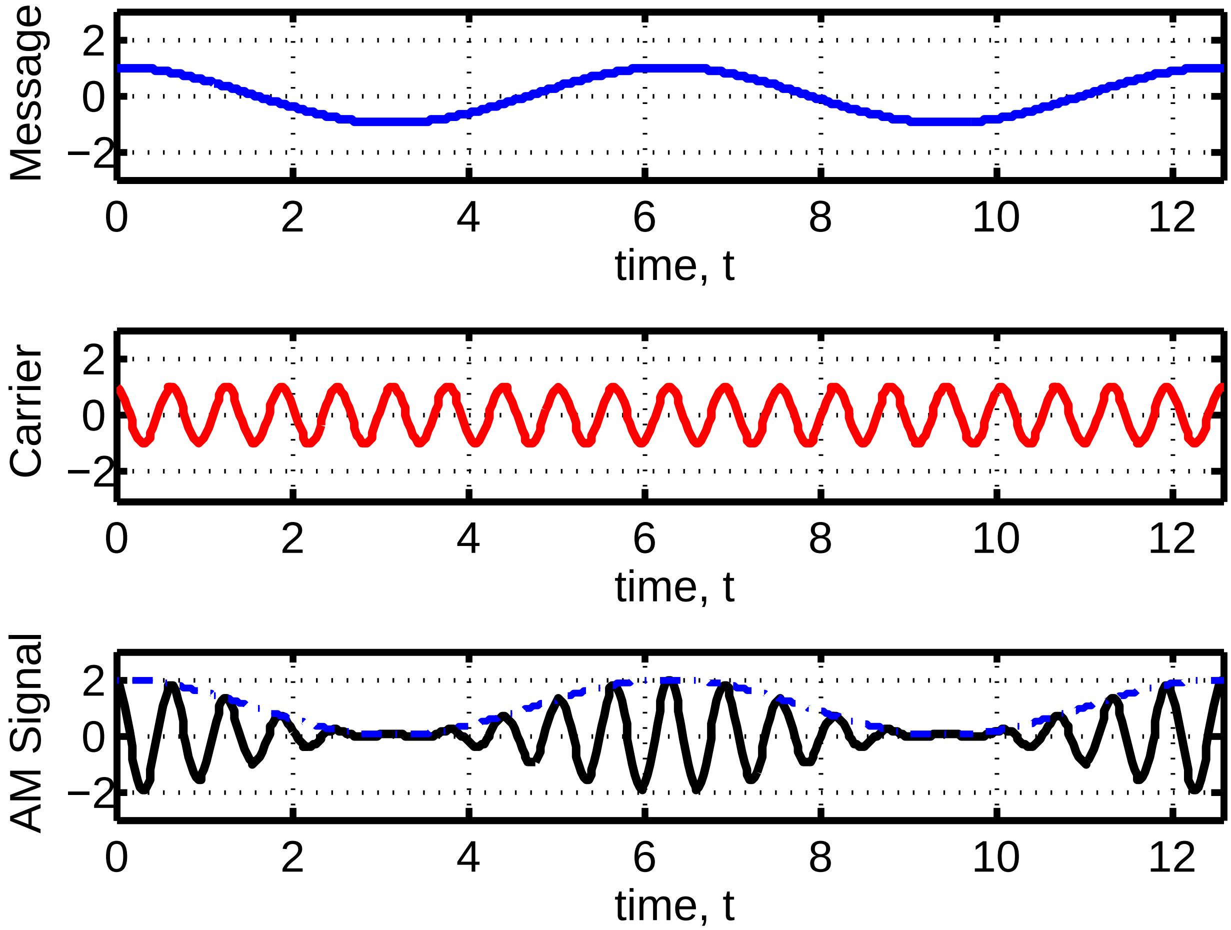
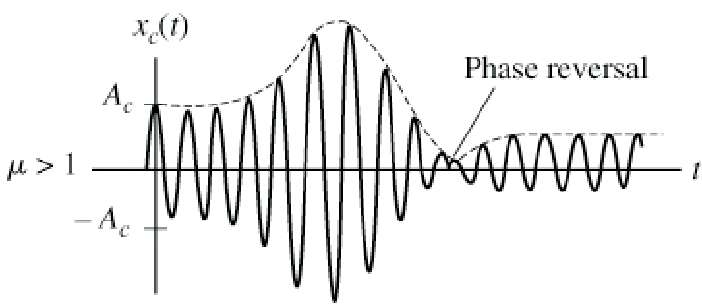


Figure 6.3: AM signal with $\mu = 1$ and $\omega_m = 2.5$.



(c)

Figure 6.4: AM signal with $\mu > 1$.

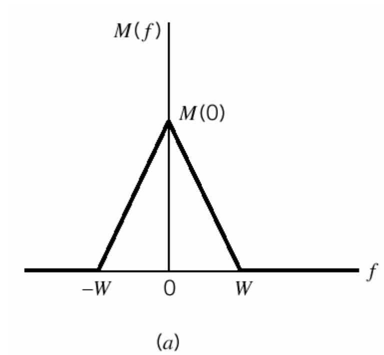


Figure 6.5: Spectrum of baseband signal.

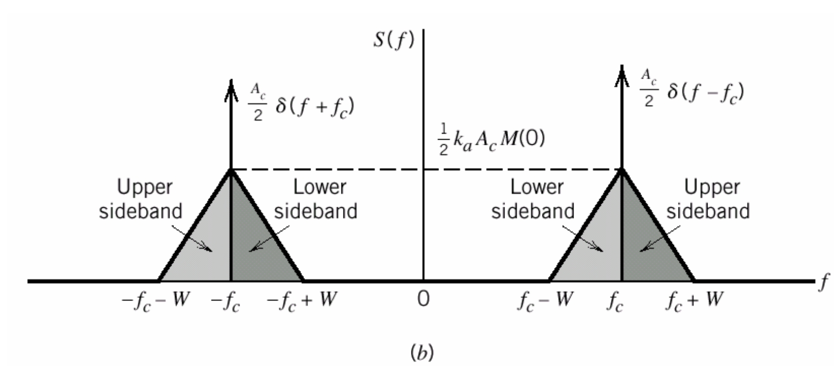


Figure 6.6: Spectrum of AM signal.

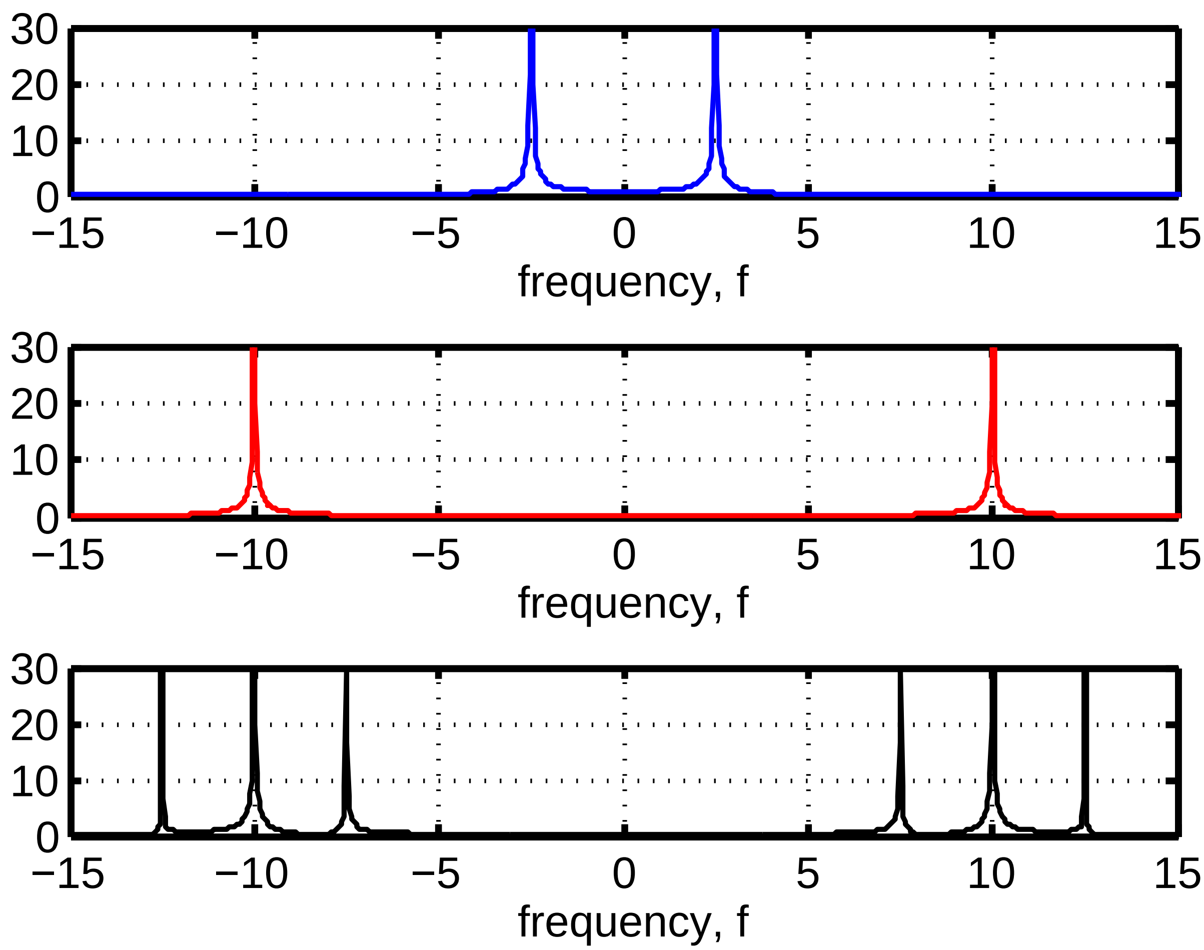


Figure 6.7: Spectrum of AM signal with $\mu = 1$ and $\omega_m = 2.5$.

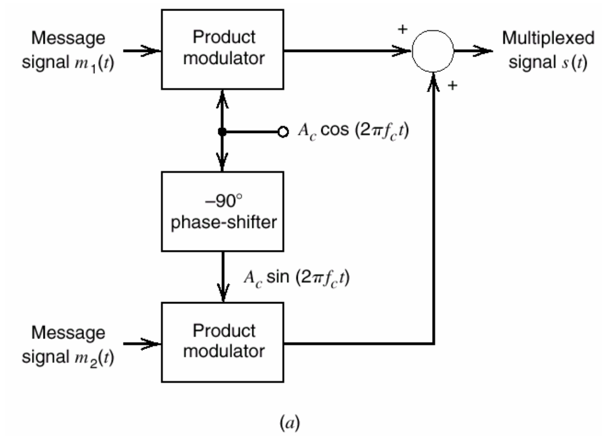


Figure 6.8: Transmitter.

6.12 Quadrature Amplitude Modulation (QAM)

- AKA quadrature multiplexing
- The DSB signals occupy twice the bandwidth required for the baseband. This disadvantage can be overcome by transmitting two DSB signals using carriers of the same frequency but in phase quadrature.
- Modulation
$$\varphi_{\text{QAM}}(t) = m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t \tag{6.18}$$
- Demodulation
$$2\varphi_{\text{QAM}}(t)\cos\omega_c t = 2[m_1(t)\cos\omega_c t + m_2(t)\sin\omega_c t]\cos\omega_c t \tag{6.19}$$

  or
$$2\varphi_{\text{QAM}}(t)\cos\omega_c t = m_1(t) + \underbrace{m_1(t)\cos 2\omega_c t + m_2(t)\sin 2\omega_c t}_{\text{suppressed by lowpass filtering}} \tag{6.20}$$
- Phase error results in crosstalk
$$2\varphi_{\text{QAM}}(t)\cos(\omega_c t + \theta) \tag{6.21}$$

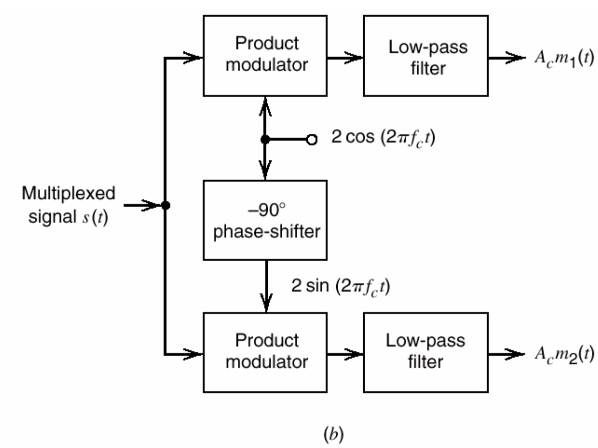


Figure 6.9: Receiver.

Chapter 7

Single Sideband Modulation

The DSB spectrum has two sidebands; the upper sideband (USB) and the lower sideband (LSB), both containing the complete information of the baseband signal. A scheme in which only one sideband is transmitted is known as sideband (SSB) transmission, which requires only one-half the bandwidth of the DSB signal.

7.1 Time-domain representation

$$M(\omega) = M_+(\omega) + M_-(\omega) \tag{7.1}$$

$$m(t) = m_+(t) + m_-(t) \tag{7.2}$$

$|M_+(\omega)|$ and $|M_-(\omega)|$ are not even functions of ω. Therefore, $m_+(t)$ and $m_-(t)$ cannot be real; they are complex.

$$m_+(t) = \frac{1}{2}[m(t) + jm_h(t)] \tag{7.3}$$

and

$$m_-(t) = \frac{1}{2}[m(t) - jm_h(t)] \tag{7.4}$$

To determine $m_h(t)$, we note that

$$\begin{aligned} M_+(\omega) &= M(\omega)u(\omega) \\ &= \frac{1}{2}M(\omega)[1 + \mathrm{sgn}(\omega)] \\ &= \frac{1}{2}M(\omega) + \frac{1}{2}M(\omega)\mathrm{sgn}(\omega) \end{aligned} \tag{7.5}$$

It can be seen that

$$jm_h(t) \leftrightarrow M(\omega)\mathrm{sgn}(\omega) \tag{7.6}$$

Hence

$$M_h(\omega) = -jM(\omega)\mathrm{sgn}(\omega) \tag{7.7}$$

From the table of Fourier transform pairs

$$1/\pi t \leftrightarrow -j\mathrm{sgn}(\omega) \tag{7.8}$$

Therefore

$$m_h(t) = m(t) * 1/\pi t \tag{7.9}$$

or

$$m_h(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{m(\alpha)}{t-\alpha} d\alpha \tag{7.10}$$

which is known as the Hilbert transform of $m(t)$. The Hilbert transformer has the following frequency response

$$H(\omega) = -j\mathrm{sgn}(\omega) \tag{7.11}$$

It follows that $|H(\omega)| = 1$ and $\theta_h(\omega) = -\pi/2$ for $\omega > 0$ and $\pi/2$ for $\omega < 0$. Thus, if we delay the phase of every component of $m(t)$ by $\pi/2$ (without changing its amplitude), the resulting signal is $m_h(t)$, the Hilbert transform of $m(t)$.

7.2 Example

For the simple case of a tone modulation, that is, when the modulating signal is a sinusoid $m(t) = \cos \omega_m t$, the Hilbert transform is

$$m_h(t) = \cos\left(\omega_m t - \frac{\pi}{2}\right) = \sin \omega_m t \tag{7.12}$$

7.3 SSB-USB spectrum

$$\Phi_{\text{SSB-USB}}(\omega) = M_+(\omega - \omega_c) + M_-(\omega + \omega_c) \tag{7.13}$$

7.4 SSB-USB signal

$$\phi_{\text{SSB-USB}}(t) = m_+(t)e^{j\omega_c t} + m_-(t)e^{-j\omega_c t} \tag{7.14}$$

Substituting for $m_+(t)$ and $m_-(t)$

$$\phi_{\text{SSB}}(t) = m(t)\cos \omega_c t \mp m_h(t) \sin \omega_c t \tag{7.15}$$

where the minus sign applies to USB and the plus signa applies to LSB.

7.5 Generation of SSB signals

7.5.1 Selective-Filtering Method

A DSB-SC signal is passed through a bandpass filter to eliminate the undesired sideband. This method is used in speech processing.

7.5.2 Phase-Shift Method

$$\phi_{\text{SSB}}(t) = m(t)\cos \omega_c t \mp m_h(t) \sin \omega_c t \tag{7.16}$$

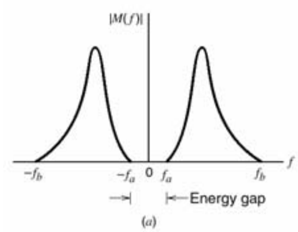


Figure 7.1: (a) Spectrum of a message signal m(t) with an energy gap of width $2fa$ centered on the origin.

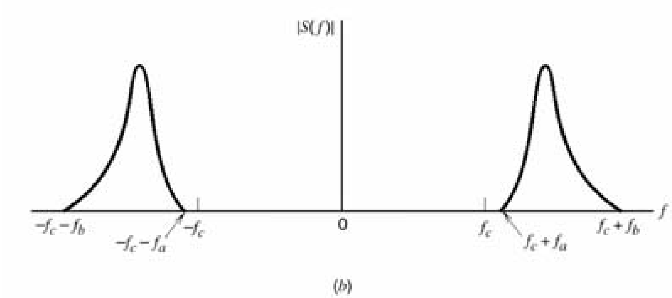


Figure 7.2: (b) Spectrum of corresponding SSB signal containing the upper sideband.

7.6 Demodulation of SSB signals

7.6.1 Synchronous Method

$\phi_{\text{SSB}}(t)$ is multiplied by $\cos\omega_c t$ and is then passed through a low-pass filter.

7.6.2 Envelope Detection Method

$$\phi_{\text{SSB+C}} = A\cos\omega_c t + [m(t)\cos\omega_c t + m_h(t)\sin\omega_c t] \tag{7.17}$$

where the envelope $E(t)$ is given by

$$\begin{aligned} E(t) &= \{[A+m(t)]^2 + m_h^2(t)]\}^{1/2} \\ &= A\left[1 + \frac{2m(t)}{A} + \frac{m^2(t)}{A^2} + \frac{m_h^2(t)}{A^2}\right]^{1/2} \end{aligned} \tag{7.18}$$

With $A \gg |m(t)|$ and $A \gg |m_h(t)|$

$$E(t) \simeq A\left[1 + \frac{2m(t)}{A}\right]^{1/2} \tag{7.19}$$

Using binomial expansion and discarding higher order terms

$$\begin{aligned} E(t) &\simeq A\left[1 + \frac{m(t)}{A}\right] \\ &= A + m(t) \end{aligned} \tag{7.20}$$

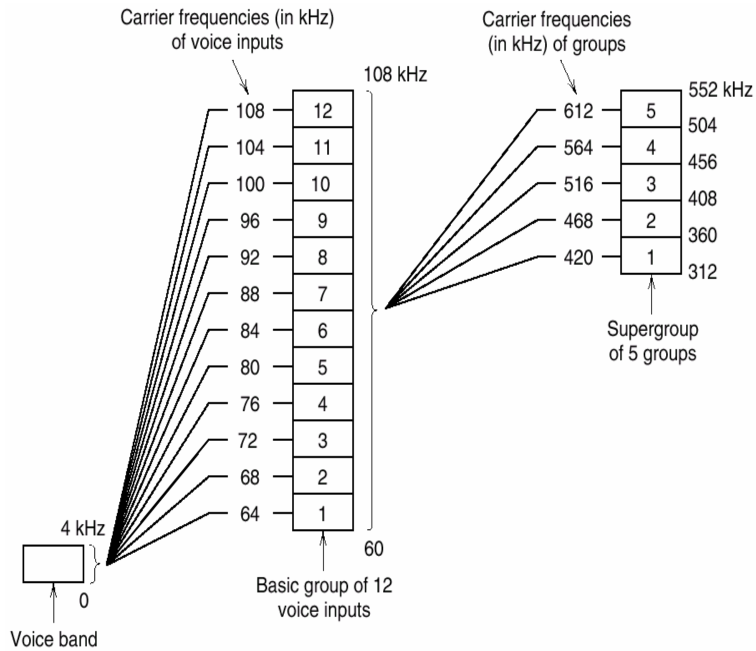


Figure 7.3: Modulation steps in an FDM system.

7.7 Problem

Consider the modulated waveform

$$s(t) = A_c \cos(2\pi f_c t) + m(t)\cos(2\pi f_c t) - \hat{m}(t)\sin(2\pi f_c t) \tag{7.21}$$

which represents a carrier plus an SSB signal, with $m(t)$ denoting the message and $\hat{m}(t)$ its Hilbert transform. Determine the conditions for which an ideal envelope detector, with $s(t)$ as input, would produce a good approximation to the message signal $m(t)$.

7.8 CCITT standard

- group: 12 channels
- supergroup: 5 groups
- mastergroup: 10 supergroups

Chapter 8

Vestigial Sideband Modulation

AKA asymmetric sideband system is a compromise between DSB and SSB.

VSB inherits the advantages of DSB and SSB but avoids their disadvantages at a small cost.

VSB signals are relatively easy to generate, and, at the same time, their bandwidth is only (typically 25%) greater than that of SSB signal.

In VBS, instead of rejecting one sideband completely (as in SSB), a gradual cutoff of one sideband, is accepted.

8.1 Demodulation Methods

1. Synchronous detection.
2. Envelope (or rectifier) detector.

8.2 VSB Spectrum

If the vestigial shaping filter that produces VSB from DSB is $H_i(\omega)$, then the resulting VSB signal spectrum is

$$\Phi_{\text{VSB}}(\omega) = [M(\omega + \omega_c) + M(\omega - \omega_c)]H_i(\omega) \tag{8.1}$$

This shaping filter allows the transmission of one sideband, but suppresses the other sideband, not completely, but gradually.

The product $e(t)$ of multiplying the incoming VSB signal $\phi_{VSB}(t)$ by $2\cos\omega_c t$ is

$$e(t) = 2\phi_{\text{VSB}}(t)\cos\omega_c t \leftrightarrow [\Phi_{\text{VSB}}(\omega + \omega_c) + \Phi_{\text{VSB}}(\omega - \omega_c)] \tag{8.2}$$

The signal $e(t)$ is further passed through a low-pass equalizer filter of transfer function $H_0(\omega)$ whose output is required to be $m(t)$. Hence,

$$M(\omega) = [\Phi_{\text{VSB}}(\omega + \omega_c) + \Phi_{\text{VSB}}(\omega - \omega_c)]H_0(\omega) \tag{8.3}$$

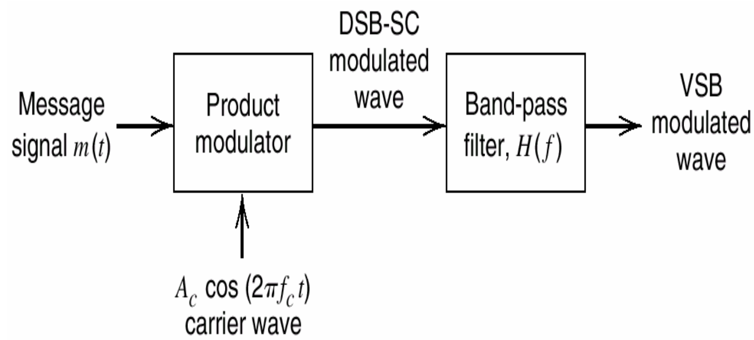


Figure 8.1: Filtering scheme for the generation of VSB modulated wave.

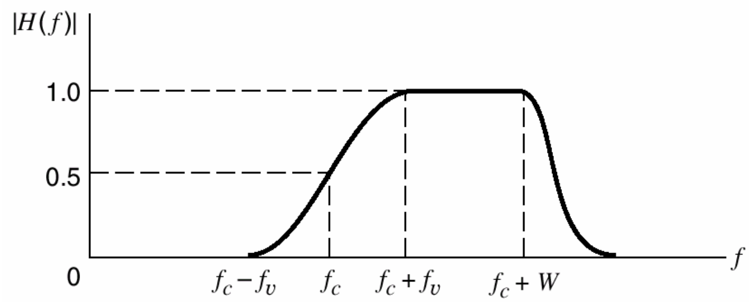


Figure 8.2: Magnitude response of VSB filter; only the positive-frequency portion is shown.

By substitution of $\Phi_{\text{VSB}}(\omega)$ in this equation and eliminating the spectra $\pm 2\omega_c$ by a low-pass filter $H_0(\omega)$, we obtain

$$H_0(\omega) = \frac{1}{H_i(\omega + \omega_c) + H_i(\omega - \omega_c)} \qquad |\omega| \leq 2\pi B \tag{8.4}$$

8.3 Use of VSB in Broadcast Television

The baseband video signal of television occupies an enormous bandwidth of 4.5 MHz.

The vestigial shaping filter $H_i(\omega)$ cuts off the lower sideband of the DSB spectrum of a television signal gradually at 0.75 MHz to 1.25 MHz below the carrier frequency f_c. The resulting VSB spectrum bandwidth is 6 MHz.

8.4 Linearity of Amplitude Modulation

Principle of superposition applies in all amplitude modulation schemes (AM, DSB, SSB, VSB).

8.5 Problem

The single tone modulating signal $m(t) = A_m \cos(2\pi f_m t)$ is used to generate the VSB signal

$$s(t) = \frac{1}{2} a A_m A_c \cos[2\pi(f_c + f_m)t] + \frac{1}{2} A_m A_c (1 - a) \cos[2\pi(f_c - f_m)t] \tag{8.5}$$

where a is a constant, less than unity, representating the attenuation of the upper side frequency.

(a) Find the quadrature component of the VSB signal $s(t)$.

(b) The VSB signal, plus the carrier $A_c \cos(2\pi f_c t)$, is passed through an envelope detector. Determine the distortion produced by the quadrature component.

(c) What is the value of constant a for which this distortion reaches its worst possible condition?

Chapter 9

Angle (Exponential) Modulation

9.1 Introduction

Generalized sinusoidal signal $\phi(t)$

$$\phi(t) = A\cos\theta(t) \tag{9.1}$$

where $\theta(t)$ is the generalized angle and is a function of t.

$$\phi(t) = A\cos(\omega_c t + \theta_0) \tag{9.2}$$

Instantaneous frequency and phase

$$\omega_i(t) = \frac{d\theta}{dt} \tag{9.3}$$

$$\theta(t) = \int_{-\infty}^{t} \omega_i(\alpha)\,d\alpha$$

9.2 Example

The instantaneous frequency in hertz of $\cos 200\pi t\cos(5\sin 2\pi t) + \sin 200\pi t\sin(5\sin 2\pi t)$ is found by noticing that the phase is $\theta(t) = 200\pi t - 5\sin 2\pi t$. Hence, $\omega_i = 200\pi - 10\pi\cos 2\pi t$.

9.3 Angle modulation possibilities

phase modulation (PM) and frequency modulation (FM). In PM, the angle $\theta(t)$ is varied linearly with $m(t)$:

$$\theta(t) = \omega_c t + \theta_0 + k_p m(t) \tag{9.4}$$

where k_p is a constant and ω_c is the carrier frequency. Assuming $\theta_0 = 0$, without loss of generality

$$\theta(t) = \omega_c t + k_p m(t) \tag{9.5}$$

9.4 PM signal

$$\phi_{\mathrm{PM}}(t) = A\cos[\omega_c t + k_p m(t)] \tag{9.6}$$

Instantaneous frequency $\omega_i(t)$ in PM

$$\omega_i(t) = \frac{d\theta}{dt} = \omega_c + k_p \dot{m}(t) \tag{9.7}$$

Hence, in PM, the instantaneous frequency ω_i varies linearly with the derivative of the modulating signal. In FM the instantaneous frequency ω_i is varied linearly with the modulating signal.

$$\omega_i(t) = \omega_c + k_f m(t) \tag{9.8}$$

where k_f is a constant. In FM

$$\begin{aligned}\theta(t) &= \int_{-\infty}^{t} [\omega_c + k_f m(\alpha)]\, d\alpha \\ &= \omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\end{aligned} \tag{9.9}$$

FM signal

$$\phi_{\mathrm{FM}}(t) = A\cos\left[\omega_c t + k_f \int_{-\infty}^{t} m(\alpha)\, d\alpha\right] \tag{9.10}$$

The generalized angle-modulated carrier ϕ_{EM} can be expressed as

$$\begin{aligned}\phi_{\mathrm{EM}}(t) &= A\cos[\omega_c t + \psi(t)] \\ &= A\cos\left[\omega_c t + \int_{-\infty}^{t} m(\alpha) h(t-\alpha)\, d\alpha\right]\end{aligned} \tag{9.11}$$

PM results if $h(t) = k_p \delta(t)$, and FM results if $h(t) = k_f u(t)$ Power of an angle-modulated wave is $A^2/2$.

9.5 Narrow-band FM

$$\phi_{\mathrm{FM}}(t) \simeq A[\cos\omega_c t - k_f a(t) \sin\omega_c t] \tag{9.12}$$

9.6 Narrow-band PM

$$\phi_{\mathrm{PM}}(t) \simeq A[\cos\omega_c t - k_p m(t) \sin\omega_c t] \tag{9.13}$$

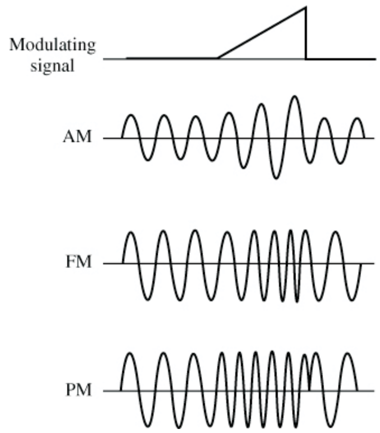


Figure 9.1: Illustrative AM, FM, and PM waveforms.

9.7 Carson's rule for FM

$$B_{\mathrm{FM}} = 2B(\beta + 1) \tag{9.14}$$

where B is the message BW, and the deviation ratio β is

$$\beta = \frac{\Delta f}{B} \tag{9.15}$$

where the frequency deviation Δf is

$$\Delta f = \frac{k_f m_p}{2\pi} \tag{9.16}$$

where k_f and m_p are the FM modulator sensitivity and the peak of the message, respectively.

9.8 Carson's rule for PM

$$B_{\mathrm{PM}} = 2(\Delta f + B) \tag{9.17}$$

where the frequency deviation Δf is

$$\Delta f = \frac{k_p m'_p}{2\pi} \tag{9.18}$$

where k_p and m'_p are the PM modulator sensitivity and the peak of the derivative of the message, respectively.

9.9 Example

For

$$x_c(t) = 10\cos[(10^8)\pi t + 5\sin 2\pi(10^3)t] \tag{9.19}$$

the phase $\theta(t) = \omega_c t + \phi(t)$. Hence, $\phi(t) = 5\sin 2\pi(10^3)t$ and $\phi'(t) = 5(2\pi)(10^3)\cos 2\pi(10^3)t$. Therefore, $|\phi(t)|_{\max} = 5$ rad, and $\Delta\omega = |\phi'(t)|_{\max} = 5(2\pi)(10^3)$ rad/s or $\Delta f = 5$ kHz.

9.10 Example

Given the angle-modulated signal $x_c(t) = 10\cos(2\pi 10^8 t + 200\cos 2\pi 10^3 t)$, the instantaneous frequency is $\omega_i = 2\pi(10^8) - 4\pi(10^5)\sin 2\pi(10^3)t$. So $\Delta\omega = 4\pi(10^5)$, $\omega_m = 2\pi(10^3)$, and $\beta = \dfrac{\Delta\omega}{\omega_m} = 200$. The BW is $W_B = 2(\beta+1)\omega_m = 8.04\pi(10^5)$ rad/s

9.11 Immunity to Nonlinearities

For example, consider a second-order nonlinear device whose input $x(t)$ and output $y(t)$ are related by

$$y(t) = a_1 x(t) + a_2 x^2(t) \tag{9.20}$$

If

$$x(t) = \cos[\omega_c t + \psi(t)] \tag{9.21}$$

then

$$y(t) = \frac{a_2}{2} + a_1\cos[\omega_c t + \psi(t)] + \frac{a_2}{2}\cos[2\omega_c t + 2\psi(t)] \tag{9.22}$$

A similar nonlinearity in AM not only causes unwanted modulation with carrier frequencies $n\omega_c$ but also causes distortion of the desired signal. For instance, if a DSB-SC signal $m(t)\cos\omega_c t$ passes through a nonlinearity $y(t) = ax(t) + bx^3(t)$, the output is

$$y(t) = \left[am(t) + \frac{3b}{4}m^3(t)\right]\cos\omega_c t + \frac{b}{4}m^3(t)\cos 3\omega_c t \tag{9.23}$$

Passing this signal through a bandpass filter yields $[am(t) + (3b/4)m^3(t)]\cos\omega_c t$. The distortion component $(3b/4)m^3(t)$ is present along with the desired signal $am(t)$.

In telephone systems, several channels are multiplexed using SSB signals. The multiplexed signal is frequency modulated and transmitted over a microwave radio relay system with many links in tandem.

9.12 Generation of FM Waves by Method of Armstrong

In this method the NBFM is converted to WBFM by using frequency multipliers. Thus, if we want a 12-fold increase in the frequency deviation, we can use a 12th-order nonlinear device or two second-order and one third-order device in cascade.

9.13 Demodulation of FM

If we apply $\varphi_{\mathrm{FM}}(t)$ to an ideal differentiator, the output is

$$\dot{\varphi}_{\mathrm{FM}}(t) = A[\omega_c + k_f m(t)]\sin\left[\omega_c t + k_f\int_{-\infty}^{t} m(\alpha)d(\alpha)\right] \tag{9.24}$$

The signal $\dot{\varphi}_{\mathrm{FM}}(t)$ is both amplitude and frequency modulated, the envelope being $A[\omega_c + k_f m(t)]$. The message can be obtained by envelope detection of $\dot{\varphi}_{\mathrm{FM}}(t)$.

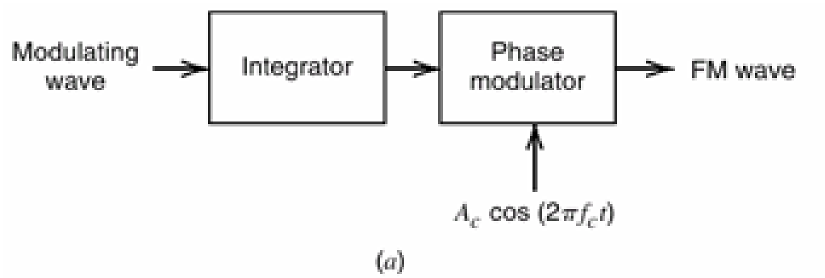


Figure 9.2: (a) Scheme for generating an FM wave by using a phase modulator.

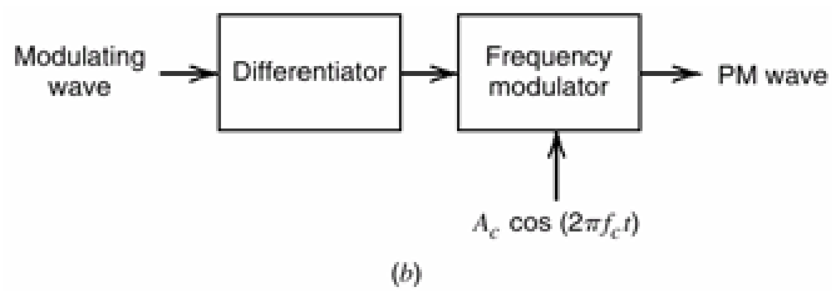


Figure 9.3: (b) Scheme for generating a PM wave by using a frequency modulator.

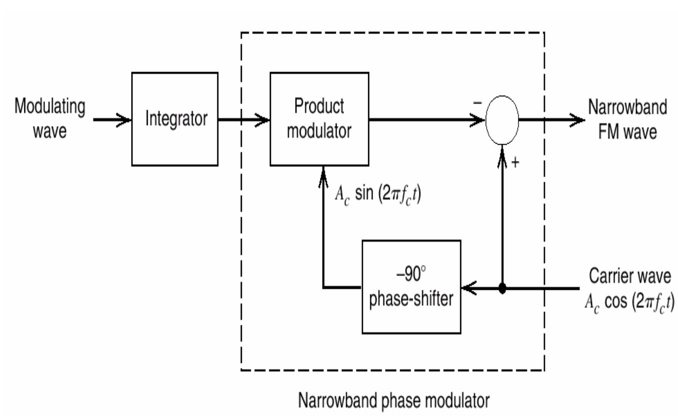


Figure 9.4: Narrowband FM.

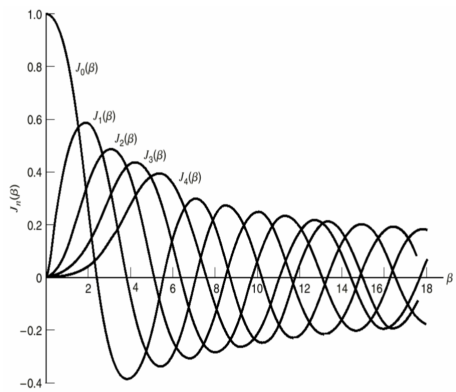


Figure 9.5: Plots of Bessel functions of the first kind for varying order.

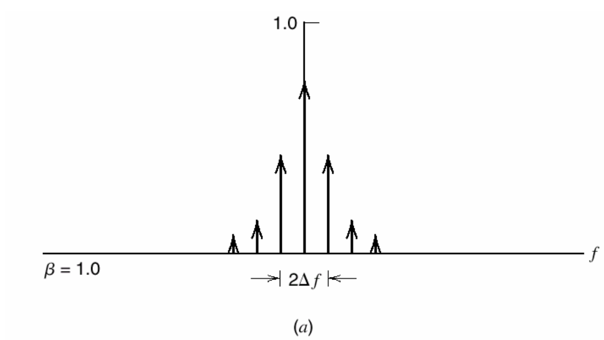


Figure 9.6: Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of fixed frequency and varying amplitude. Only the spectra for positive frequencies are shown.

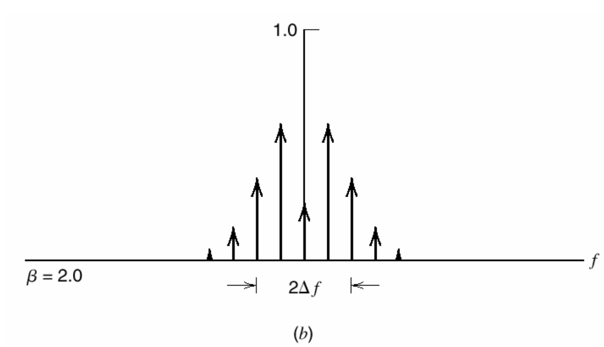


Figure 9.7: Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of fixed frequency and varying amplitude. Only the spectra for positive frequencies are shown.

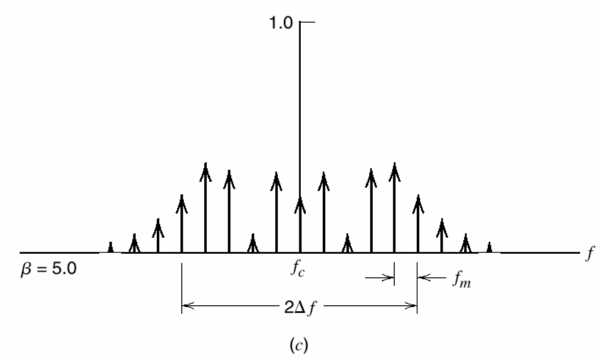


Figure 9.8: Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of fixed frequency and varying amplitude. Only the spectra for positive frequencies are shown.

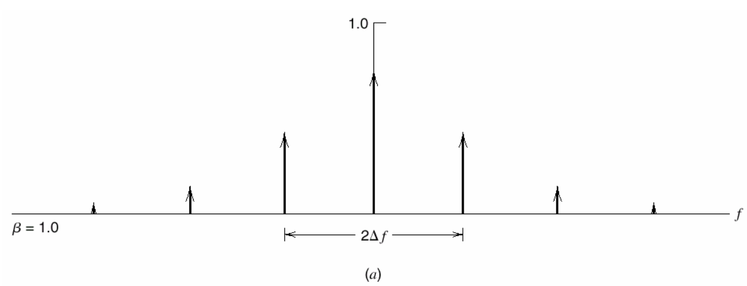


Figure 9.9: Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of varying frequency and fixed amplitude. Only the spectra for positive frequencies are shown.

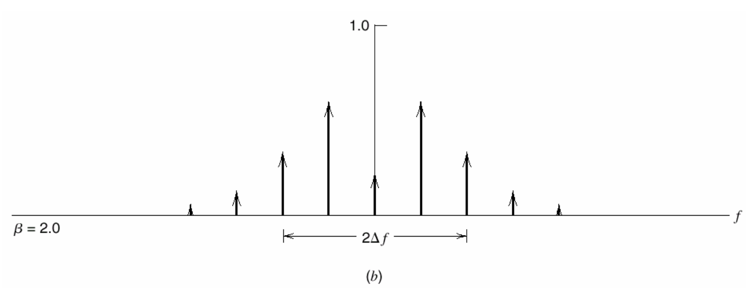


Figure 9.10: Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of varying frequency and fixed amplitude. Only the spectra for positive frequencies are shown.

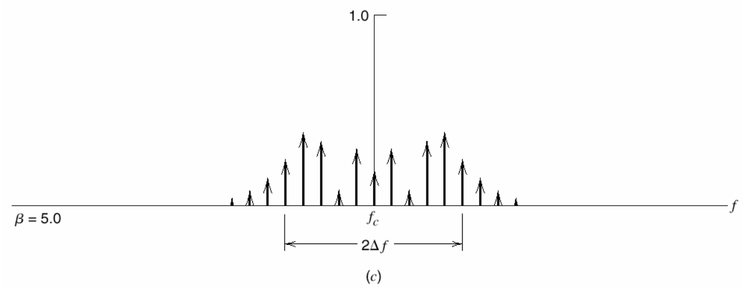


Figure 9.11: Discrete amplitude spectra of an FM signal, normalized with respect to the carrier amplitude, for the case of sinusoidal modulation of varying frequency and fixed amplitude. Only the spectra for positive frequencies are shown.

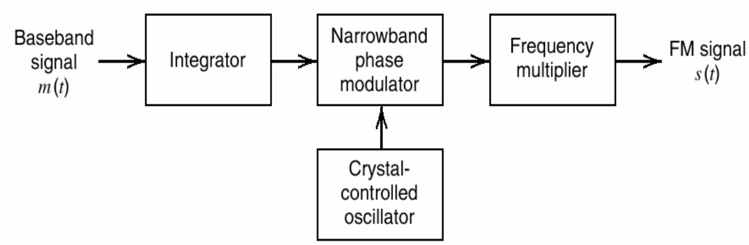


Figure 9.12: Block diagram of the indirect method of generating a wideband FM signal.

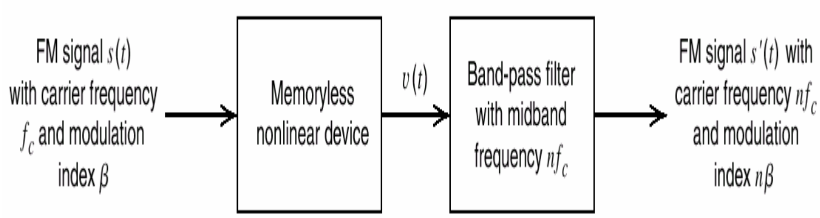


Figure 9.13: Block diagram of frequency multiplier.

9.14 Problem

Consider a narrow-band FM signal approximately defined by

$$s(t) \simeq A_c \cos(2\pi f_c t) - \beta A_c \sin(2\pi f_c t) \sin(2\pi f_m t) \tag{9.25}$$

(a) Determine the envelope of this modulated signal. What is the ratio of the maximum to the minimum value of this envelope? Plot this ratio versus β, assuming that β is restricted to the interval $0 \leq \beta \leq 0.3$.

(b) Determine the average power of the narrow-band FM signal, expressed as a percentage of the average power of the unmodulated carrier wave. Plot this result versus β, assuming that β is restricted to the interval $0 \leq \beta \leq 0.3$.

(c) By expanding the angle $\theta_i(t)$ of the narrow-band FM signal $s(t)$ in the form of a power series, and restricting the modulation index β to a maximum value of 0.3 radians, show that

$$\theta_i(t) \simeq 2\pi f_c t + \beta \sin(2\pi f_m t) - \frac{\beta^3}{3} \sin^3(2\pi f_m t) \tag{9.26}$$

What is the value of the harmonic distortion for $\beta = 0.3$?

9.15 Problem

The sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t) \tag{9.27}$$

is applied to a phase modulator with phase sensitivity k_p. The unmodulated carrier wave have frequency f_c and amplitude A_c. Determine the spectrum of the resulting phase-modulated signal, assuming that the maximum phase deviation $\beta_p = k_p A_m$ does not exceed 0.3 radians.

9.16 Problem

Suppose that the phase-modulated signal of the previous problem has an arbitrary value for the maximum phase deviation β_p. This modulated signal is applied to an ideal band-pass filter with mid-band frequency f_c and a passband externding from $f_c - 1.5f_m$ to $f_c + 1.5f_m$. Determine the envelope, phase, and instantaneous frequency of the modulated signal at the filter output as functions of time.

9.17 Problem

An FM signal with modulation index $\beta = 1$ is transmitted through and ideal band-pass filter with mid-band frequency f_c and bandwidth $5f_m$, where f_c is the carrier frequency and f_m is the frequency of the sinusoidal modulating wave. Determine the amplitude spectrum of the filter output.

9.18 Problem

Consider a wide-band PM signal produced by a sinusoidal modulating wave $A_m \cos(2\pi f_m t)$, using a modulator with a phase sensitivity equal to k_p radian per volt.

(a) Show that if the maximum phase deviation of the PM signal is large compared with one radian, the bandwidth of the PM signal varies linearly with the modulation frequency f_m.

(b) Compare this characteristic of a wide-band PM signal with that of wide-band FM signal.

9.19 Problem

An FM signal with a frequency deviation of 10 kHz at a modulation frequency of 5 kHz is applied to two frequency multipliers connected in cascade. The first multiplier doubles the frequency and the second multiplier triples the frequency. Determine the frequency deviation and the modulation index of the FM signal obtained at the second multiplier output. What is the frequency separation of the adjacent side frequencies of this FM signal?