Improved Statistical Method for System-Level ESD Tests

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A new method for system-level electrostatic discharge (ESD) tests is based on both upper and lower statistical limits for system failure probability in an ESD environment. The method is used to examine the ambiguous system reliability implications of the latest IEC 801-2 international ESD test specification. I give step-by-step procedures, including optimal methods to rectify unusual failures, for a reliability-based ESD test that fully meets IEC 801-2 requirements.

1. Introduction

In earlier approaches[1],[2] to system-level electrostatic discharge (ESD) test statistics, an upper bound P_h for the probability of field failure was estimated based on a model for the ESD environment. However, the same ESD test results also imply that the failure probability has a statistical lower bound, P_l. If no failures occur in the ESD test, then P_l equals zero, because there is no evidence that the system will ever fail. On the other hand, when there is even a single undesired response, P_l becomes non-zero, although it may be small.

While the system is probably at least as good as P_h, it is also probably at least as bad as P_l. No system can be considered reliable unless P_l is extremely low, because a non-zero P_l is essentially a guarantee of failure.

There is an easy way to make P_l equal zero: avoid doing any ESD tests. From this point of view, "no news is good news." This is particularly relevant for high voltage tests. Systems are commonly not required to pass at the highest level of IEC 801-2 (severity level 4: 8kV contact discharge, 15kV air discharge[3]). As manufacturers improve ESD hardness, systems will become increasingly robust at IEC level 4. To track this improvement, it is preferable to test at level 4 even if failures occur. By applying the statistical methodology given here, one can predict the impact of these failures on reliability.

If P_l equals zero because the system was not tested, P_h equals one for the same reason: since nothing is known about the ESD vulnerability of the system, its failure probability can be anywhere between zero and one. As more tests are done, P_h decreases from 1 (unless the system fails every test) while P_l increases from zero (unless the system passes every test). As a rule, P_h drops with increasing numbers of tests and is relatively insensitive to failures, while P_l increases rapidly with failures, and is the better indicator of unreliability. Thus, a test designed only around the upper limit P_h may allow too many failures.

Unusual failures sometimes occur during testing, failures that do not repeat with further testing. When this happens, a non-zero limit on P_l permits rectifying tests. If there are no additional failures during these tests, the system will eventually pass at the agreed value of P_l. Thus, even if the undesired responses reflect an actual failure mode of the system (rather than a testing error) the rectifying tests prove that it is rare enough.

This paper shows the statistical implications of the various test methods allowed under IEC 801-2. These tests are poorly specified, in that different systems may pass the same IEC severity level, and yet differ greatly in reliability. Using the methods developed here, I give step-by-step procedures for a statistically unambiguous test method, including optimal techniques for rectifying tests. The improved method does not require an unreasonable amount of testing, and fully satisfies the requirements of IEC 801-2.

2. Theory

2.1 System and Environment

To begin, let us review the statistical methods developed earlier[1]. The prevalence of ESD in a system's environment is characterized by the quantity E_{tot}, defined as the average number of ESD's per unit time, adding together the discharges at all voltage levels. The distribution of ESD's among voltages is given by the function $F(V)$, the fraction of the ESD's with voltage less than or equal to V. The voltage V here is the voltage setting of an ESD simulator that would give the same waveform as the measured environmental ESD.

The system is characterized by an intrinsic, voltage-dependent failure probability $p(V)$, which can never be known exactly without an infinite number of tests. Considering both the failure probability and the environment, we introduce the overall probability of an undesired response, defined by

$$P_{ur} = \int_0^{\infty} p(V) \frac{dF(V)}{dV} dV \qquad (1)$$

The quantity P_{ur} is the ratio of the total number of undesired responses to the total number of ESD's. (In the notation of the previous paper[1], I used the reciprocal quantity, $Z_{ur} = 1/P_{ur}$.) The average rate of undesired responses of the system in its environment is given by $R = E_{tot} P_{ur}$, while the mean time between undesired responses is $MTBUR = 1/R$. Like $p(V)$, P_{ur} cannot be known exactly with a finite amount of data. Our object will be to efficiently estimate P_{ur}.

Figure 1 shows the expected appearance of $p(V)$, dF/dV and their product, the distribution of failures. Because measured ESD's are much more prevalent at low than at high voltages, even a small system failure probability permits a high field failures rate, shown by the dashed curve. The method developed here is designed to alert the user to such situations.

2.2 Statistical Basics

We now examine the statistical procedure for estimating the intrinsic failure probability p for a single voltage. Consider an ESD test in which there are n failures out of N tests, where for each

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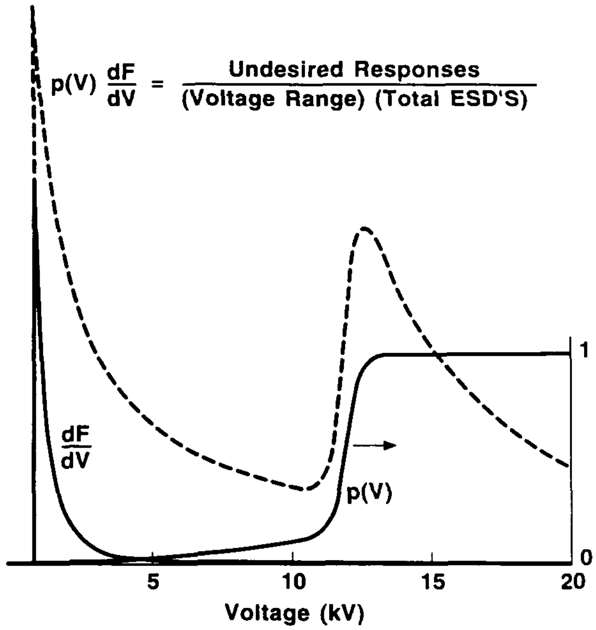

Figure 1. Typical $p(V)$, dF/dV and Failure Distribution Curves

test the system has the same independent failure probability p. The random variable n obeys the binomial distribution with parameter p, the unknown probability of failure. One can show[4] that statistical confidence limits on p can be derived from the cumulative binomial distribution as follows:

$$\sum_{k=n+1}^{N} \frac{N!}{k!(N-k)!} p_h^k (1-p_h)^{N-k} = C \qquad (2)$$

$$\sum_{k=0}^{n-1} \frac{N!}{k!(N-k)!} p_l^k (1-p_l)^{N-k} = C \qquad (3)$$

In these equations, p_h is the upper limit, p_l is the lower limit, and C is the one-sided statistical confidence. Given n, N and C, these equations are solved for p_l and p_h. This calculation may be done directly with a software package such as Mathematica™[5], from tables of the cumulative binomial distribution, or by exploiting the relations between the binomial distribution and the F distribution[6] or the Incomplete Beta Function[7]. Several special cases are easily computed. When $n = 0$ (no failures), p_l equals 0 and $p_h = 1-(1-C)^{1/N}$. When $n = N$ (every test fails), $p_l = (1-C)^{1/N}$ and p_h equals 1. The quantities p_l and p_h have the following properties[6]:

1. The probability is C that $0 \le p \le p_h$.
2. The probability is C that $p_l \le p \le 1$.
3. The probability is $2C-1$ that $p_l \le p \le p_h$.

Another way of thinking about these equations is that our estimate p_h [p_l] is so high [low], that if p were really that high [low], the system would have a high probability (C) of failing [passing] the test more times than it actually did.

If p_h and p_l are small, while N is large, equations 2 and 3 are greatly simplified by the Poisson approximation:

$$\sum_{k=n+1}^{\infty} \frac{T_h^k}{k!} e^{-T_h} = C \qquad (4)$$

$$\sum_{k=0}^{n-1} \frac{T_l^k}{k!} e^{-T_l} = C \qquad (5)$$

where $T_l = Np_l$ and $T_h = Np_h$. These equations may also be solved either numerically, with tables, or by using the connections between the Poisson distribution and the Chi-Square distribution[6] or the Incomplete Gamma Function[7]. For the special case $n = 0$, $T_l = 0$ and $T_h = -\ln(1-C)$. The special case $n = N$ cannot be handled by the Poisson approximation, which breaks down for high failure rates. The advantage of using the Poisson approximation is that for a given confidence level, T_l and T_h depend only on the number of failures n, which makes them easy to tabulate. The following table gives these quantities for a single-sided confidence level $C = 95\%$, or a two-sided confidence $2C-1 = 90\%$.

Statistical Functions for the Poisson Approximation

n	T_l	T_h	n	T_l	T_h
0	0	2.996	11	6.169	18.21
1	.0513	4.744	12	6.924	19.44
2	.3554	6.296	13	7.690	20.67
3	.8177	7.754	14	8.464	21.89
4	1.366	9.154	15	9.246	23.10
5	1.970	10.51	16	10.04	24.30
6	2.613	11.84	17	10.83	25.50
7	3.285	13.15	18	11.63	26.69
8	3.981	14.43	19	12.44	27.88
9	4.695	15.71	20	13.25	29.06
10	5.425	16.96			

Table 1

In the Poisson approximation, the ratio of either p_h or p_l to the measured failure rate, n/N, depends only on the number of failures. These ratios are plotted in figure 2, where one observes that the upper and lower confidence limits converge to the measured rate as the number of measurements increases. This figure illustrates the danger of using failure rate as a performance criterion. For a given failure rate, the confidence interval narrows as the number of tests increases. In particular, the lower limit increases. Thus, the more tests one does, the higher the guaranteed failure rate. By contrast, the method described here sets a limit on the lower failure rate bound. While for a small number of tests, it is possible for a few exceptional failures to occur, with more tests, it is expected that the measured failure rate will be small, as it converges to the actual system failure probability.

In table 1, the ratios of successive values for the lower probability limit, $T_l(n)/T_l(n-1)$ are greater than the ratios of the upper limit, $T_h(n)/T_h(n-1)$. Figure 3 shows these ratios for the first few failures. The difference in the ratios is greatest for small numbers of failures, and decreases with increasing failures. This

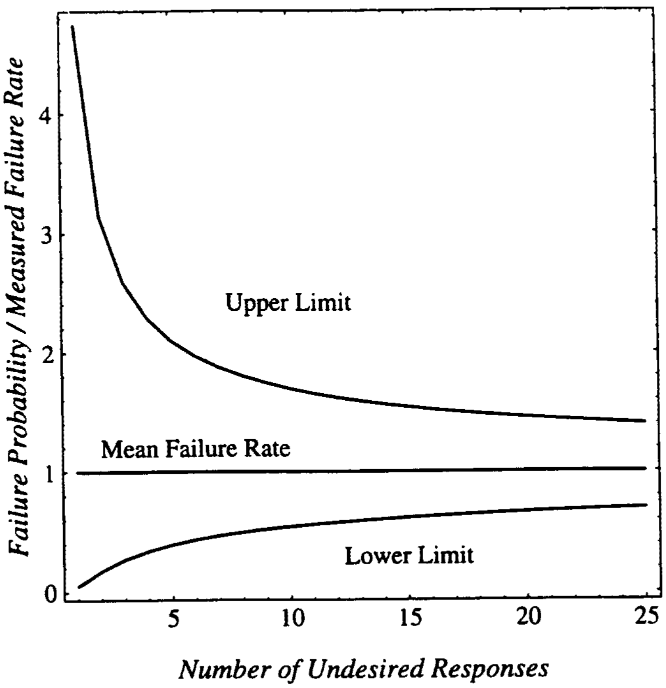

Figure 2. Ratios of Failure Probability Limits to Measured Failure Rates

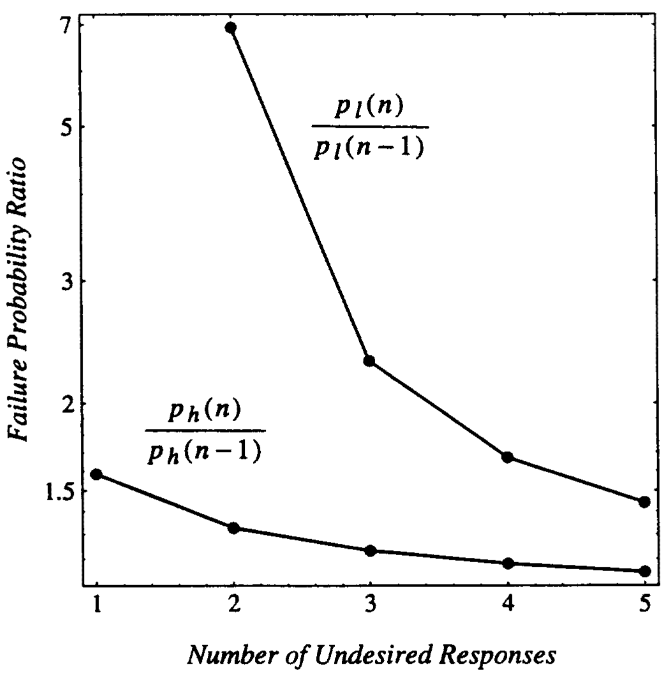

Figure 3. Ratios of Successive Failure Probability Limits

implies that the lower limit p_l is much more sensitive to small numbers of failures than the upper limit p_h. In section 4 I will explain what this means for the proposed test method.

2.3 Multiple Voltage Levels

Now we can extend these methods to evaluate systems tested at multiple voltage levels. I define two estimates for P_{ur}, a lower bound P_l and an upper bound P_h,

$$P_l = \sum_{i=1}^{L} f_i p_{li} \ , \tag{6}$$

$$P_h = \sum_{i=1}^{L} f_i p_{hi} + f_{\max} \ . \tag{7}$$

In these formulas, the integral in equation 1 has been replaced by a sum over the L test voltage levels. The quantities f_i are the fractions of the environmental ESD associated with each test level. If we allow tests at one voltage to represent all ESD's between that voltage and the next lower test voltage, then

$$f_i = F(V_i) - F(V_{i-1}) \tag{8}$$

for $i = 2,3,\ldots,L$, while $f_1 = F(V_1)$. In equation 7, $f_{\max}$ represents the fraction of ESD at voltages higher than the highest test voltage, or

$$f_{\max} = 1 - F(V_L) \ . \tag{9}$$

Equation 7 includes the $f_{\max}$ term because one pessimistically fears that the system might fail for every ESD higher than the highest test voltage, while in equation 6, one optimistically hopes that since no testing was done at those voltages, the system might always pass.

The probability bounds p_{hi} and p_{li} for each level may be evaluated exactly with equations 2 and 3 or approximately with equations 4 and 5 for small failure probabilities. In the Poisson approximation, we have

$$p_{li} = \frac{T_l(n_i)}{N_i} \tag{10}$$

$$p_{hi} = \frac{T_h(n_i)}{N_i} \tag{11}$$

If there are no failures, all the p_{li} equal zero, and hence P_l equals zero. Nevertheless, P_h remains nonzero even after many tests with no failures, because you cannot prove perfection. When starting the test, it is thus reasonable to ask how many tests would be needed to achieve a desired P_h if there are no failures. As shown in my earlier paper[1], using the Poisson approximation and assuming no failures at any voltage level ($n_i = 0$), the method of Lagrange undetermined multipliers may be used to calculate the optimum number of tests at each voltage level needed to verify the desired value of P_h. To summarize, we have

$$N_i = \frac{T_h(0)\sqrt{f_i}}{P_h - f_{\max}} \left[\sum_{k=1}^{L} \sqrt{f_k} \right] \ . \tag{12}$$

Equation 7 implies that the best possible value of P_h is always larger than $f_{\max}$, the fraction of ESD above the highest test level. A possible pitfall in designing the test is to set the upper bound P_h too close to $f_{\max}$. As is clear from equation 12, this error can result in a test plan requiring vastly more tests than are practical or necessary[8]. Lower values of P_h are achieved more appropriately by increasing the highest test voltage, thereby reducing $f_{\max}$.

2.4 Rectifying Inspections

Equation 6 and 10 imply that if there are any failures, P_l will be nonzero. Because P_l represents the best expected reliability of the system, its value must be very small. However, when a system malfunctions during test, the failure may be an unusual one that will not repeat during further tests or in normal operation. To verify this possibility, one may conduct a rectifying inspection. Using the same method used to derive equation 12, one may show that the optimum total number of tests needed to verify a given value of P_l is

$$N_i = \frac{\sqrt{f_i T_l(n_i)}}{P_l} \left[\sum_{k=1}^{L} \sqrt{f_k T_l(n_k)} \right] . \qquad (13)$$

If the calculated number is less than the number of tests already done (or zero), no additional tests are needed at that level. However, if there are failures at a level, the value of N_i calculated by equation 13 will normally be larger than the value calculated by equation 12, because P_l will normally be chosen much smaller than P_h. Thus, additional tests will be needed. If the system passes these tests without additional failures, then a calculation with equation 6 will show that the desired value of P_l has been met or exceeded.

3. Testing to IEC 801-2

IEC 801-2 requires testing at "severity levels" numbered 1 through 4, which are defined as: 2, 4, 6 and 8 kV for contact discharges or 2, 4, 8 and 15kV for air discharges. The minimum test satisfying IEC 801-2 thus has four test voltage levels ($L = 4$). Following Simonic[9], I will model the ESD environment with a power-law distribution. I further assume that the relevant parameter for the distribution is the *air-discharge* voltage, and that the Severity Levels for air and contact discharges correlate. The distribution function $F(V)$ will be taken to be zero below a voltage $V_0 = 1kV$, and for $V > V_0$ equal to

$$F(V) = 1 - (V/V_0)^y . \qquad (14)$$

The exponent $y = -1.86$ represents a worst-case environment[1] with few ESD controls, where high-voltage ESD's are much more common than in controlled environments. Using the air-discharge voltages for the four test levels V_i, equations 8 and 9 imply the following numbers for the ESD environmental parameters:

Model Environmental ESD Factors for IEC 801-2 Voltage Levels

Level i	1	2	3	4	f_{max}
f_i	.72	.20	.055	.014	.0065

Table 2

The IEC method specifies a minimum of 10 discharges per test point per test voltage and polarity. Using this minimum number for each test point implies that N_i equals 10 times the number of test points. This contrasts with the method suggested here, defined by equation 12, that calls for more tests at the lower voltages. Figure 4 compares the efficiency of these two approaches. For a given value of P_h, the figure shows that the IEC method requires about 40% more tests.

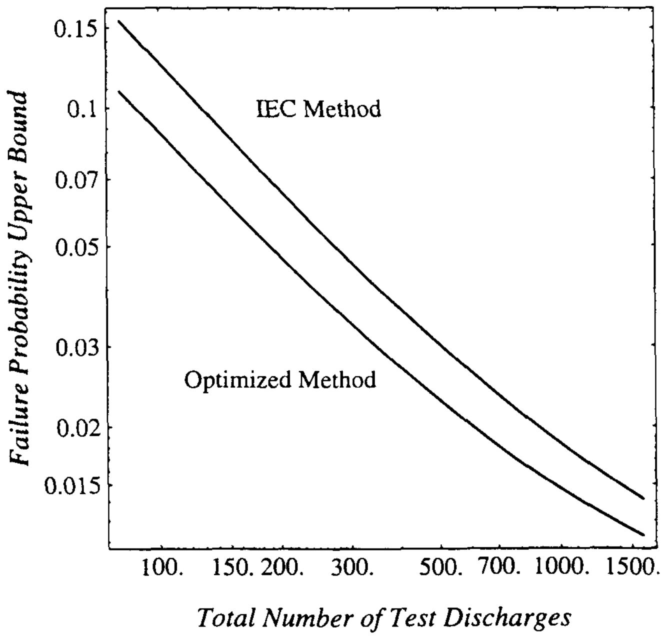

Figure 4. Failure Probability Upper Bounds Versus ESD Tests (No Measured Failures)

Figure 4 also illustrates another, more basic point. The IEC 801-2 method does not specify how many test points to use. Therefore, the statistical significance of the test is completely unspecified. At 40 discharges per test point, figure 4 covers the range of 2 to 40 test points. The upper bound of verified system reliability varies there by a factor of 10, yet all such tests pass the same set of simulator voltages. To quantify system reliability, both the number of test points and the number of discharges per test point must be specified.

4. Proposed Statistical Test Method

To explore the system's vulnerability, the test discharges should be distributed among a number of test points that may be vulnerable in the judgement of the tester. Thus, the failure probability for the system as a whole is based in part on the likelihood of touching a particular part of the equipment, as was explicitly modeled by Pratt and Davis[10].

Equations 10 and 11 with table 1 may be used to carry out the calculations of this method with acceptable accuracy. However, if the system fails more than one third of the tests, or more than 20 times, p_{li} and p_{hi} should either be determined exactly from the binomial distribution or overestimated using the expressions for $n = N$ in section 1.

The quantity P_h is relatively insensitive to small numbers of failures, but it can only be made small by doing many tests. Thus, setting a limit on P_h is an effective means of ensuring that an adequate number of tests are done. For the IEC test voltages, I suggest a test criterion of $P_h = .025$ for the system as a whole. This goal can be met by passing a total of 480 discharges of each polarity distributed among the test voltages. To reach $P_h = .012$ requires tripling the discharge numbers (see figure 4), because P_h is getting too close to f_{max}. The 480 discharges may be distributed among 6 test points with each test point stressed as follows for each polarity:

Discharges Per Test Point

Severity Level	Test Discharges
1	40
2	20
3	10
4	10
Total:	80

Table 3

This method thus satisfies the IEC requirement of 10 discharges per test point, and is close to the optimum of 444 discharges implied by equation 12. If one wishes to test with exactly 10 discharges at each voltage level, the $P_h = .025$ objective can be met with 17 test points, a total of 680 discharges at each polarity.

The sensitivity of P_l to small numbers of failures (see figure 3), makes it a useful indicator of system reliability. A limit on P_l may be imposed both at each individual test point and for the test as a whole. If an individual test point fails the P_l criterion, it means trouble at that location. However, another possibility is that many test points will meet the criterion marginally. Then one should be concerned that there is an overall problem with the system.

The harm resulting from system malfunction depends on the type of system and on how it is being used. Depending on the system user's needs, one can differentiate between the severity of allowable failure by classifying systems according to measured P_l. Although this could be done in many ways, one reasonable method is as follows:

Relation Between Failure Severity and P_l

Failure Severity:	Maximum P_l
Critical	0
Severe	.00001
Moderate	.0001
Minor	.001

Table 4

4.1 How Many Failures Are Allowed?

If P_l is required to equal zero, no failures are permitted. However, for any of the other categories, sufficiently rare failures may be tolerated. To understand how many errors are permissible, note that for failures at only one level, equations 6 and 10 imply that $N_i = f_i T_l(n_i)/P_l$. This equation was used to generate the following table, based on the "Minor" failure category:

Required Tests Versus Allowed Malfunctions

Lower Bound on N_i for Failures at a Single Level
Minor Malfunctions (P_l = 0.001)

	i = 1	2	3	4
n_i = 1	36.93	10.26	2.975	0.769
2	255.89	71.08	20.613	5.331
3	588.74	163.54	47.427	12.266
4	983.52	273.20	79.228	20.490
5	1418.40	394.00	114.260	29.550
6	1881.36	522.60	151.554	39.195
7	2365.20	657.00	190.530	49.275
8	2866.32	796.20	230.898	59.715
9	3380.40	939.00	272.310	70.425

Table 5

For "Moderate" or "Severe" failures, multiply the table entries by 10 or 100 respectively. When all failures occur at a single severity level, the system passes if the number of test discharges is greater than the tabulated number. The table may be used either for individual test points or the test as a whole.

For example, a system is allowed one Minor error at any one severity level, because the number of tests at every level exceed the corresponding entries in the first line of table 5. As another example, if a system has one Severe failure at severity level 4, the table entry, multiplied by 100, implies that there must be a total of 77 discharges for the system to pass. Thus, after one failure in an initial 10 discharges, the system would need to pass a rectifying test of an additional 67 successive discharges.

As an example of a system that passes at every test point, but fails the overall test, consider a system that has one Minor malfunction at level 3 for all 6 test points. Each individual test point passes, because the number of level 3 discharges (10) at each point exceeds the number in the table for one failure (2.975). When the test as a whole is considered, there are a total of 6 failures out of 60 discharges at level 3. Thus the system as a whole fails, because table 5 tolerates $n_i = 6$ only if there are at least 152 tests. This behavior is another application of figure 2, where the lower failure bound increases with the absolute number of failures, even though the measured failure rate stays the same.

Note that when there are failures at more than one level, the required number of discharges is greater than the sum of the corresponding entries from this table. The general procedure for evaluating the test using this table is as follows. For each level where there are failures, divide the appropriate table entry by N_i. Compute the sum of all such quotients. The EUT satisfies the P_l criterion if the sum is less than 1 (see equations 6 and 10). For example, if the EUT has 2 minor errors at level 4 and 1 at level 3, it passes, because $(5.331+10) + (2.975+10) = .8306 < 1$.

5. Conclusion

I have described a method of analysis for ESD tests that relates the performance of a system under test to its reliability in use. Based on the results of any ESD test, one can compute upper and lower bounds on expected failure rates for a model field environment.

The new ESD test method described here specifies test performance with statistical parameters instead of with the more conventional, but less informative ESD threshold voltage. The method is under consideration for a proposed revision to the ESD test section in EIA/TIA-571-1991, a specification for telephone terminal equipment of the Telecommunications Industry Association (TIA)[11].

These results extend earlier work that was limited to upper failure bounds[1]. In the present method, the upper bound P_h serves primarily to ensure that enough tests are done, while the lower bound P_l guards against excessive failures. The limit on P_l may be set by the user to ensure the desired level of reliability. Although P_l and P_h do not form a confidence interval as do the single-level quantities p_l and p_h (see section 2.2), they define performance in the following sense. If the system's probability of an undesired response P_{ur} in the model environment is actually larger [smaller] than P_h [P_l], the probability is *at least C* that the results of the test would have been worse [better] at some level than they actually were.

Because most people doing system-level ESD tests do not have the resources to measure the abundance of ESD in their environment, I have proposed a test method that uses a model environment. However, the same method can also be used if environmental information is available. The environmental model does not need any specific distribution function as I used here (equation 14). One only needs to know the fractions f_i of ESD associated with each test level, and the fraction f_{max} above the highest test level (table 2). If measured data on the overall ESD rate (E_{tot}) is available, the user can also calculate statistical limits for the system's mean time between undesired responses (MTBUR, section 2.1). On the other hand, an advantage in using a model environment rather than the user's real environment is that it allows comparison of systems whose ultimate use environment is not known.

ESD tests at multiple voltages are needed to look for "windows of vulnerability." But there is a trade-off between the number of test voltage levels and the number of discharges needed to achieve a desired statistical significance[1]. I have suggested a method that uses the IEC 801-2 test voltages, a choice that leads to a reasonable number of test discharges, and more importantly, allows one to meet the IEC requirements. With the European Community's forthcoming mandatory ESD standards, manufacturers are increasingly interested in complying with the baseline IEC 801-2 test requirements[12].

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