Modeling of the Generalized Unified Power Flow Controller (GUPFC) in a Nonlinear Interior Point OPF

Xiao-Ping Zhang, *Member, IEEE*, Edmund Handschin, *Fellow, IEEE*, and Maojun "Mike" Yao, *Member, IEEE*

***Abstract*—With the progress of installing the latest generation of FACTS devices, namely, the Convertible Static Compensator (CSC) [1], several innovative operating concepts have been introduced to the historic development and application of FACTS. One of the novel concepts is the Generalized Unified Power Flow Controller (GUPFC) or multi-line UPFC, which can control bus voltage and power flows of more than one line or even a sub-network. The GUPFC should have stronger control capability than the UPFC. A mathematical model for the GUPFC consisting of one shunt converter and two or more series converters is developed and implemented in a nonlinear interior point OPF algorithm. Numerical results with various GUPFC devices based on the IEEE 30 bus system and IEEE 118 bus system demonstrate the feasibility as well as the effectiveness of the GUPFC model established and the OPF method proposed.**

***Index Terms*—AC transmission, FACTS, GUPFC, IPFC, optimal power flow, power flow, power flow controller, SSSC, STATCON, UPFC.**

I. Introduction

AN INNOVATIVE approach to utilization of FACTS controllers providing a multifunctional power flow management device was proposed in [1]. There are several possibilities of operating configurations by combing two or more converter blocks with flexibility. Among them, there are two novel operating configurations, namely the Interline Power Flow Controller (IPFC) and the Generalized Unified Power Flow Controller (GUPFC) [1], [2], which are significantly extended to control power flows of multi-lines or a sub-network rather than control power flow of single line by a Unified Power Flow Controller (UPFC) [3] or Static Synchronous Series Compensator (SSSC) [4].

In contrast to the practical applications of the GUPFC in power systems, very few publications have been focused on the mathematical modeling of this new FACTS controller in power system analysis. A fundamental frequency model of the GUPFC consisting of one shunt converter and two series converters for EMTP study was proposed quite recently in [5]. While modeling the GUPFC in power flow, optimal power flow (OPF) analysis has not been reported yet. Therefore, in this paper, a mathematical model of the GUPFC suitable for power flow and optimal power flow study is established. In the past three decades, techniques such as Newton method, sequential linear and quadratic programming method, PQ de-coupling method, etc. [6] have been used to solve optimal power flow problems. The optimal power flow problem with the GUPFC in this paper is solved by the newly developed Nonlinear Interior Point Methods [7]–[9]. Experience with application of interior point methods to power system optimization problems has been quite positive.

The rest of paper is organized as follows: Section II establishes the mathematical model of the GUPFC. Section III formulates the nonlinear interior point OPF algorithm. Analytical solutions are derived for the initialization of the GUPFC variables in the nonlinear interior point OPF algorithm. Numerical examples are given in Section IV, which demonstrate the computational performance of the proposed OPF with the GUPFC devices. General conclusions are given in Section V.

II. A Mathematical Model of the GUPFC

A. The Equivalent Circuit of the GUPFC

The GUPFC with combing three or more converters working together extends the concepts of voltage and power flow control beyond what is achievable with the known two-converter UPFC FACTS controller [1], [2]. The simplest GUPFC consists of three converters, one connected in shunt and the other two in series with two transmission lines in a substation [5]. It can control total five power system quantities such as a bus voltage and independent active and reactive power flows of two lines. Such a GUPFC, which is shown in Fig. 1, is used to show the basic operation principle for the sake of simplicity. However, the mathematical derivation is applicable to a GUPFC with an arbitrary number of series converters.

In the steady state operation, the main objective of the GUPFC is to control voltage and power flow. The equivalent circuit of the GUPFC consisting of one controllable shunt injected voltage source and two controllable series injected voltage sources is shown in Fig. 2. Real power can be exchanged among these shunt and series converters via the common DC link. The sum of the real power exchange should be zero if we neglect the losses of the converter circuits. For the GUPFC shown in Figs. 1 and 2, it has total 5 degrees of control freedom, that means it can control five power system quantities such as one bus voltage, and 4 active and reactive power flows of two

Manuscript received September 5, 2000.

X.-P. Zhang is with the School of Engineering, University of Warwick, Coventry, CV4 7AL, UK (e-mail: xpz@eng.warwick.ac.uk).

E. Handschin is with the Institute of Electric Power Systems, University of Dortmund, 44221 Dortmund, Germany (e-mail: handschin@ev.e-technik.uni-dortmund.de).

M. Yao is with Alstom ESCA Corporation, Bellevue, WA 98004 USA (e-mail: yao@ieee.org).

Publisher Item Identifier S 0885-8950(01)06057-6.

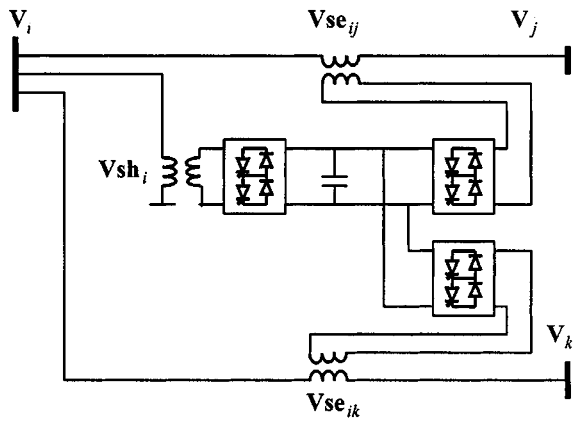

Fig. 1. Operational principle of the GUPFC with three converters.

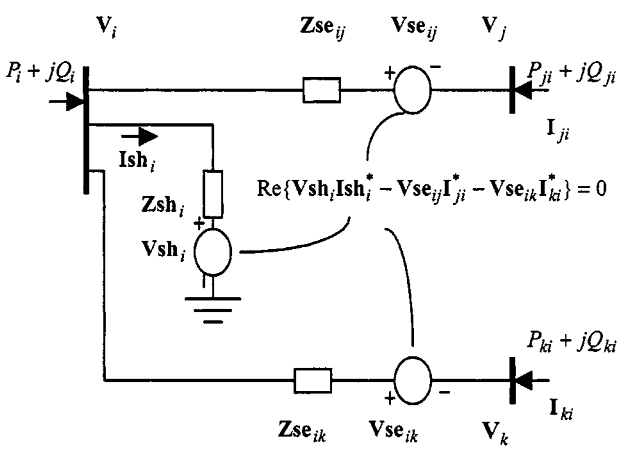

Fig. 2. The equivalent circuit of the GUPFC.

lines. It can be seen that with more series converters included within the GUPFC, more degrees of control freedom can be introduced and hence more control objectives can be achieved.

$\mathbf{Zsh}_i\mathbf{Zse}_{in}$ $(n = j, k, \ldots)$ in Fig. 2 are shunt and series transformer impedances. The controllable injected voltage sources shown in Fig. 2 are defined as,

$$\mathbf{Vsh}_i = Vsh_i\angle\theta sh_i \tag{1.1}$$

$$\mathbf{Vse}_{in} = Vse_{in}\angle\theta se_{in} \tag{1.2}$$

where $n = j, k, \ldots$.

B. Power Flow Equations of the GUPFC

According to the equivalent circuit of the GUPFC shown in Fig. 2, the power flow equations can be derived:

$$\begin{aligned} P_i =& V_i^2 g_{ii} - V_i Vsh_i (gsh_i \cos(\theta_i - \theta sh_i) \\ &+ bsh_i \sin(\theta_i - \theta sh_i)) \\ &- \sum_n V_i V_n (g_{in}\cos\theta_{in} + b_{in}\sin\theta_{in}) \\ &- \sum_n V_i Vse_{in}(g_{in}\cos(\theta_i - \theta se_{in}) \\ &+ b_{in}\sin(\theta_i - \theta se_{in})) \end{aligned} \tag{2.1}$$

$$\begin{aligned} Q_i =& -V_i^2 b_{ii} - V_i V_{sh}(g_{sh}\sin(\theta_i - \theta_{sh}) - b_{sh}\cos(\theta_i - \theta_{sh})) \\ &- \sum_n V_i V_n (g_{in}\sin\theta_{in} - b_{in}\cos\theta_{in}) \\ &- \sum_n V_i Vse_{in}(g_{in}\sin(\theta_i - \theta se_{in}) \\ &- b_{in}\cos(\theta_i - \theta se_{in})) \end{aligned} \tag{2.2}$$

$$\begin{aligned} P_{ni} =& V_n^2 g_{nn} - V_i V_n (g_{in}\cos(\theta_n - \theta_i) \\ &+ b_{in}\sin(\theta_n - \theta_i)) \\ &+ V_n Vse_{in}(g_{in}\cos(\theta_n - \theta se_{in}) \\ &+ b_{in}\sin(\theta_n - \theta se_{in})) \end{aligned} \tag{3.1}$$

$$\begin{aligned} Q_{ni} =& -V_n^2 b_{nn} - V_i V_n (g_{in}\sin(\theta_n - \theta_i) \\ &- b_{ij}\cos(\theta_n - \theta_i)) \\ &+ V_n Vse_{in}(g_{in}\sin(\theta_n - \theta se_{in}) \\ &- b_{in}\cos(\theta_n - \theta se_{in})) \end{aligned} \tag{3.2}$$

$$gsh_i + jbsh_i = 1/\mathbf{Zsh}_i \tag{4.1}$$

$$g_{in} + jb_{in} = 1/\mathbf{Zse}_{in} \tag{4.2}$$

$$g_{nn} + jb_{nn} = 1/\mathbf{Zse}_{in} \tag{4.3}$$

$$g_{ii} = gsh_i + \sum_n g_{in} \tag{4.4}$$

$$b_{ii} = bsh_i + \sum_n b_{in} \tag{4.5}$$

where $n = j, k, \ldots$.

C. Operating Constraints of the GUPFC

According to the operating principle of the GUPFC, the operating constraint representing active power exchange between converters via the common DC link is:

$$PE = \mathrm{Re}\left(Vsh_i Ish_i^* - \sum_n Vse_{in} I_{ni}^*\right) = 0$$

or

$$\begin{aligned} PE =& Vsh_i^2 gsh_i \\ &- V_i Vsh_i (gsh_i\cos(\theta_i - \theta sh_i) - bsh_i \sin(\theta_i - \theta sh_i)) \\ &+ \sum_n (Vse_{in}^2 g_{in} - V_i Vse_{in}(g_{in}\cos(\theta_i - \theta se_{in}) \\ &- b_{in}\sin(\theta_i - \theta se_{in}))) \\ &+ \sum_n V_n Vse_{in}(g_{in}\cos(\theta_n - \theta se_{in}) \\ &- b_{in}\sin(\theta_n - \theta se_{in})) = 0 \\ & n = j, k, \ldots . \end{aligned} \tag{5}$$

The equivalent controllable injected voltage source bound constraints:

$$Vsh_i^{\min} \le Vsh_i \le Vsh_i^{\max} \tag{6.1}$$

$$\theta sh_i^{\min} \le \theta sh_i \le \theta sh_i^{\max} \tag{6.2}$$

$$Vse_{in}^{\min} \leq Vse_{in} \leq Vse_{in}^{\max} \tag{6.3}$$

$$\theta se_{in}^{\min} \leq \theta se_{in} \leq \theta se_{in}^{\max} \tag{6.4}$$

where $n = j, k, \ldots$.

D. Control Constraints of the GUPFC

The GUPFC shown in Figs. 1 and 2 can control the voltage at bus i and active and reactive power flows of line i–j and line i–k. Suppose the sending ends of two lines are connected with bus j and k, respectively. Therefore the active and reactive power flows of the two lines at the sending ends are $-P_{ni}, -Q_{ni}$ $(n = j, k)$. The voltage constraint of the GUPFC is,

$$V_i - V_i^{Spec} = 0$$

where V_i^{Spec} is the specified bus voltage.

In the implementation, the above equality constraint is replaced by the following inequality constraints,

$$V_i^{Spec} - \varepsilon \leq V_i \leq V_i^{Spec} + \varepsilon \tag{7}$$

where ε is a specified very small value.

The power flow control constraints of the GUPFC are,

$$P_{ni} - P_{ni}^{Spec} = 0 \tag{8.1}$$

$$Q_{ni} - Q_{ni}^{Spec} = 0 \tag{8.2}$$

where $n = j, k, \ldots$. $P_{ni}^{Spec}, Q_{ni}^{Spec}$ are specified active and reactive power flows, respectively.

III. Nonlinear Interior Point OPF

A. Formulation of the Nonlinear Interior Point OPF

Mathematically, as an example, an OPF for minimization of the total operating cost can be formulated as follows,

$$\text{Objective: Min } f(x) = \sum_{i}^{Ng} (\alpha_i * Pg_i^2 + \beta_i * Pg_i + \gamma_i). \tag{9.1}$$

Subject to the following constraints:

$$g(x) = 0 \tag{9.2}$$

$$h^{\min} \leq h(x) \leq h^{\max} \tag{9.3}$$

where

- x $= [\theta sh, Vsh, \theta se, Vse, V, \theta, T, Pg, Qg]^T$
- $\alpha_i, \beta_i, \gamma_i$ —coefficients of quadratic production cost functions of generator.
- $g(x)$ —equality constraints including bus power flow equations, operating and control constraints of the GUPFC given by (5), (7), (8).
- $h(x)$ —inequality-constraints including line flow constraints, simple inequality constraints of variables such as voltage-magnitudes, generator active power, generator reactive power, transformer tap ratio and bound constraints of the GUPFC variables in (6).
- θsh —angle of shunt voltage source of the GUPFC
- Vsh —magnitude of shunt voltage source of the GUPFC
- θse —angle of series voltage source of the GUPFC
- Vse —magnitude of series voltage source of the GUPFC
- θ —bus angle
- V —bus voltage magnitude
- T —tap ratio vector of transformer
- Pg —bus active generation
- Qg —bus reactive generation.

The nonlinear OPF problem given in (9) can be solved by the Nonlinear Interior Point Methods [7]–[9], which include three important achievements in optimization. Those achievements are Fiacco and McCormick's barrier method for optimization with inequalities, Lagrange's method for optimization with equalities and Newton's method for solving nonlinear equations.

By applying Fiacco and McCormick's barrier method, we transform the OPF problem (9) into the following equivalent OPF problem,

$$\text{Objective: Min } \left\{ f(x) - \mu \sum_{i}^{Nh} \ln(sl_i) - \mu \sum_{i}^{Nh} \ln(su_i) \right\} \tag{10.1}$$

Subject to the following constraints:

$$g(x) = 0 \tag{10.2}$$

$$h(x) - sl - h^{\min} = 0 \tag{10.3}$$

$$h(x) + su - h^{\max} = 0 \tag{10.4}$$

where $\mu > 0$.

The Lagrangian function for equalities optimization for problem (10) is

$$\begin{aligned} L = f(x) &- \mu \sum \ln(sl) - \mu \sum \ln(su) - \lambda^T g(x) \\ &- \pi l^T \left(h(x) - sl - h^{\min}\right) - \pi u^T (h(x) + su - h^{\max}) \end{aligned} \tag{11}$$

where $\lambda, \pi l, \pi u$ are Lagrangian multipliers for constraints (10.2), (10.3), (10.4), respectively.

The Karush–Kuhn–Tucker (KKT) first order conditions for the Lagrangian function of (11) are,

$$\nabla_x L_\mu = \nabla f(x) - \nabla g(x)^T \lambda - \nabla h(x)^T \pi l - \nabla h^T \pi u = 0 \tag{12.1}$$

$$\nabla_\lambda L_\mu = -g(x) = 0 \tag{12.2}$$

$$\nabla_{\pi l} L_\mu = -\left(h(x) - sl - h^{\min}\right) = 0 \tag{12.3}$$

$$\nabla_{\pi u} L_\mu = -(h(x) + su - h^{\max}) = 0 \tag{12.4}$$

$$\nabla_{sl} L_\mu = \mu e + Sl * \Pi l = 0 \tag{12.5}$$

$$\nabla_{su} L_\mu = \mu e - Su * \Pi u = 0 \tag{12.6}$$

where $Sl = \text{diag}(sl_j), Su = \text{diag}(su_j), \Pi l = \text{diag}(\pi l_j), \Pi u = \text{diag}(\pi u_j)$.

The Newton equation for the nonlinear interior point OPF algorithm derived above may be expressed as the following compact form,

$$\begin{bmatrix} -\Pi l^{-1}Sl & 0 & -\nabla h & 0 \\ 0 & \Pi u^{-1}Su & -\nabla h & 0 \\ -\nabla h^T & -\nabla h^T & H & -J^T \\ 0 & 0 & -J & 0 \end{bmatrix} \begin{bmatrix} \Delta\pi l \\ \Delta\pi u \\ \Delta x \\ \Delta\lambda \end{bmatrix} = \begin{bmatrix} -\nabla_{\pi l}L_{\mu+}\Pi l^{-1}\nabla_{Sl}L_\mu \\ -\nabla_{\pi u}L_{\mu+}\Pi u^{-1}\nabla_{Su}L_\mu \\ -\nabla_x L_\mu \\ g(x) \end{bmatrix} \quad (13.1)$$

$$\Delta sl = \Pi l^{-1}(-\nabla_{sl}L_\mu - Sl\Delta\pi l) \quad (13.2)$$

$$\Delta su = \Pi u^{-1}(\nabla_{su}L_\mu - Su\Delta\pi u) \quad (13.3)$$

where

$$H(x, \lambda, \pi l, \pi u) = \nabla^2 f(x) - \lambda\nabla^2 g(x) - (\pi l + \pi u)\nabla^2 h(x),$$

$$J(x) = \frac{\partial g(x)}{\partial x}.$$

By solving the Newton equation of (13), $\Delta\pi l$, $\Delta\pi u$, Δx, $\Delta\lambda$, Δsl, Δsu can be obtained. Then the Newton solution can be updated as follows,

$$sl = sl + \sigma\alpha_p\Delta sl \quad (14.1)$$

$$su = su + \sigma\alpha_p\Delta su \quad (14.2)$$

$$x = x + \sigma\alpha_p\Delta x \quad (14.3)$$

$$\pi l = \pi l + \sigma\alpha_d\Delta\pi l \quad (14.4)$$

$$\pi u = \pi u + \sigma\alpha_d\Delta\pi u \quad (14.5)$$

$$\lambda = \lambda + \sigma\alpha_d\Delta\lambda \quad (14.6)$$

where $\sigma = 0.995 \sim 0.999\,95$. α_p, α_d are primal and dual step length respectively. They can be determined by

$$\alpha_p = \min\left\{\min\left(\frac{sl}{-\Delta sl}\right), \min\left(\frac{su}{-\Delta su}\right), 1.0\right\} \quad (15.1)$$

$$\alpha_d = \min\left\{\min\left(\frac{-\pi l}{\Delta\pi l}\right), \min\left(\frac{\pi u}{-\Delta\pi u}\right), 1.0\right\} \quad (15.2)$$

for those $\Delta sl < 0$, $\Delta su < 0$, $\Delta\pi l > 0$, $\Delta\pi u < 0$.

The complementary gap for the nonlinear interior point OPF is,

$$Cgap = su^T\pi u - sl^T\pi l. \quad (16)$$

The barrier parameter can be determined by,

$$\mu = \frac{\beta * Cgap}{2 * m} \quad (17)$$

where $\beta = 0.01 \sim 0.2$, m is the number of inequality constraints in (9.3).

B. Solution Procedure of the Nonlinear Interior Point OPF

In the practical solving of the equation (13.1), the variables $\Delta\pi l$, $\Delta\pi u$ are explicitly eliminated from the above equation at first. Then the remaining part, which includes variables such as Δx, $\Delta\lambda$has the similar sparse structure as that of system matrix in Newton's method [10]. The remaining system equation can be arranged as a light border and heavy border. The light border consists of variables such as the GUPFC injected voltage variables, transformer tap ratios, generator active and reactive power injections, etc. The heavy border has $4 * 4$ block structure with coupled formulation, which has the same sparse structure as that of bus admittance matrix. The remaining Newton equation can be solved by first eliminating the light border as the same manner used in the Newton OPF. Then block sparse matrix techniques can be employed to solve the resulting heavy border equation. This is the key idea not only for the implementation of the Newton method based OPF [10] but also for the implementation of the nonlinear primal–dual interior point OPF.

The solution procedure for the nonlinear interior point OPF is summarized as the following:

Step 0) Set iteration count $K = 0$, $\mu = \mu 0$, and initialize the OPF solution
Step 1) If KKT conditions are satisfied and complementary gap is less than a tolerance, output results. Otherwise go to step 2)
Step 2) Form and solve Newton equation in (13.1), then (13.2) and (13.3)
Step 3) Update Newton solution by equation (14)
Step 4) Compute complementary gap by (16)
Step 5) Determine barrier parameter by (17)
Step 6) $K = K + 1$, go to step 1).

C. Initialization of the Nonlinear Interior Point OPF

In the implementation of the nonlinear interior point OPF, the initial state vector x excluding variables of the GUPFC can be chosen as a flat start or as the middle point between the upper and lower bounds. Initial values for dual variables are discussed in [6].

Good starting points for the GUPFC are helpful to improve the convergence of the OPF algorithm. Suppose bus voltage V_i, V_n, θ_i, θ_n $(n = j, k, \ldots)$ are set to flat start values, obviously $\theta_i = \theta_n = 0$, then we can find the following analytical solutions in (18) for the initial values of Vse_{in}, θse_{in} by solving two simultaneous equations (8.1) and (8.2).

$$Vse_{in} = \frac{1}{V_n}\sqrt{\frac{a1}{(g_{in}^2 + b_{in}^2)}} \quad (18.1)$$

$$\theta se_{in} = \tan^{-1}\left(\frac{P_{ni}^{Spec} - V_n^2 g_{nn} + V_iV_ng_{in}}{Q_{ni}^{Spec} + V_n^2 b_{nn} - V_iV_nb_{in}}\right) - \tan^{-1}\left(\frac{g_{in}}{-b_{in}}\right) \quad (18.2)$$

$$a1 = (P_{ni}^{Spec} - V_n^2 g_{nn} + V_iV_ng_{in})^2 + (Q_{ni}^{Spec} + V_n^2 b_{nn} - V_iV_nb_{in})^2 \quad (18.3)$$

where $n = j, k, \ldots$

With $V_i, V_n, \theta_i, \theta_n, Vse_{in}, \theta se_{in}$ $(n = j, k, \ldots)$ known, further suppose,

$$Vsh_i = \frac{Vsh_i^{\max} + Vsh_i^{\min}}{2} \quad \text{or} \quad Vsh_i = 1.0 \text{ p.u.} \tag{19.1}$$

then we can find the following analytical solution for the initial value of θsh_i by solving equation (5),

$$\theta sh_i = -\sin^{-1}\left(\frac{a2}{V_i Vsh_i \sqrt{(gsh_i^2 + bsh_i^2)}}\right) + \tan^{-1}\left(\frac{gsh_i}{-bsh_i}\right) \tag{19.2}$$

and define,

$$\begin{aligned} a2 =& Vsh_i^2 gsh_i \\ &+ \sum_n (Vse_{in}^2 g_{in} - V_i Vse_{in}(g_{in}\cos(\theta_i - \theta se_{in}) \\ &\quad - b_{in}\sin(\theta_i - \theta se_{in}))) \\ &+ \sum_n V_n Vse_{in}(g_{in}\cos(\theta_n - \theta se_{in}) \\ &\quad - b_{in}\sin(\theta_n - \theta se_{in})) \end{aligned} \tag{19.3}$$

where $n = j, k, \ldots$.

IV. Numerical Examples

A. Test Systems

Test cases in this paper are carried out on the IEEE 30 bus system and IEEE 118 bus system. The IEEE 30 bus system has 6 generators, 4 OLTC transformers and 37 transmission lines. The IEEE 118 bus system has 18 controllable active power generation, 54 controllable reactive power generation, 9 OLTC transformers, 177 transmission lines. For all cases in this paper, the convergence tolerances are 5.0e-4 for complementary gap and 1.0 e-4 (0.01 MW/Mvar) for maximal absolute bus power mismatch, respectively.

B. The IEEE 30 Bus System Results

In order to show the power flow control capability of the nonlinear interior point OPF algorithm proposed, four cases based on the IEEE 30 bus system are carried out. In the discussion thereafter, the control settings of active and reactive power flow are referred to $-P_{ni}^{Spec}$, $-Q_{ni}^{Spec}$, which are at the sending end of a line. Active power flow and reactive power flow at the sending end of a line are referred to $-P_{ni}$, $-Q_{ni}$ ($n = j$ or k) since the sending end of a line is connected to bus n ($n = j$ or k).

- Case 1) This is a base case without GUPFC.
- Case 2) This is similar to the case 1 except that there is a GUPFC installed for control of voltage of bus 12 and active and reactive power flow of line 12–15 and line 12–16. The control setting of the bus voltage is 1.0 p.u. The control settings for active and reactive power flow of line 12–15 and line 12–16 are 25 MW $+j$ 5 Mvar and 10 MW $+j$ 2 Mvar, respectively. The active power flow settings are about 150% of their corresponding base case active power flow.
- Case 3) This is similar to the case 2 except that second GUPFC is further installed for control of voltage at bus 10 and control of active and reactive power flow of line 10–21 and line 10–22. The control setting of voltage at bus 10 is 1.0 p.u. The control settings of active and reactive power flow are 10 MW $+j$ 6 Mvar and 12 MW $+j$ 4 Mvar for line 10–21 and line 10–22, respectively. The active power flow setting of line 10–21 is about 60% of the corresponding base case active power flow. While the active power flow setting of line 10–22 is about 160% of the base case active power flow.
- Case 4) This is similar to the case 3 except that third GUPFC is further installed for control of voltage at bus 6 and control of active and reactive power flow of line 6–2 and line 6–8. The control setting of voltage at bus 6 is 1.0 p.u. The control settings of active and reactive power flow are -60 MW $+j$ 4 Mvar, and 10 MW $-j$ 4 Mvar for line 6–2 and line 6–8, respectively.

The test results of the above cases are summarized in Table I.

TABLE I
TEST RESULTS OF THE IEEE 30 BUS SYSTEM

	Case 1	Case 2	Case 3	Case 4
The Number of GUPFC	-	1	2	3
The Total Number of Control Objectives by GUPFC	-	5	10	15
The Total Number of Controllable Active and Reactive Power Flow by GUPFC	-	2 P Flow 2Q Flow	4 P Flow 4Q Flow	6P Flow 6Q Flow
The Total Number of Controllable Bus Voltage by GUPFC	-	1	2	3
The Number of Iterations	12	13	13	14

C. The IEEE 118 Bus System Results

Case studies are further carried out on the IEEE 118 bus system. Four cases are presented as follows,

- Case 5) This is a base case IEEE 118 bus system without GUPFC.
- Case 6) This is similar to the case 5 except that there is a GUPFC installed for control of voltage at bus 45 and active and reactive power flow of line 45–44 and line 45–46. The control setting of the bus voltage is 1.0 p.u. The control settings for active and reactive power flow of line 45–44 and line 45–46 are 40 MW $+j$ 7 Mvar and -50 MW $-j$ 7 Mvar, respectively.
- Case 7) This is similar to the Case 6 except that there is second GUPFC further installed for control voltage of bus 94 and power flow of line 94–95, line 94–93, line 94–100. The voltage control objective is 1.0 p.u. The control settings for active and reactive power flow of line 94–95, line 94–93, line 94–100 are 50 MW $+j$ 5 Mvar, -50 MW $-j$ 20 Mvar and -35 MW $-j$ 10 Mvar, respectively.

TABLE II
TEST RESULTS OF THE IEEE 118 BUS SYSTEM

	Case 5	Case 6	Case 7	Case 8
The Number of GUPFC	-	1	2	3
The Total Number of Control Objectives by GUPFC	-	5	12	17
The Total Number of Controllable Active and Reactive Power Flow by GUPFC	-	2 P Flow 2Q Flow	5 P Flow 5Q Flow	7P Flow 7Q Flow
The Total Number of Controllable Bus Voltage by GUPFC	-	1	2	3
The Number of Iterations	13	13	13	13

Case 8) This is similar to the Case 7 except that there is third GUPFC further installed for control voltage of bus 12 and power flow of line 12–3 and line 12–11. The control settings for active and reactive power flow of line 12–3, line 12–11 are 15 MW $+j$ 4 Mvar and − 40 MW $+j$ 15 Mvar, respectively.

Test results based on the cases 5–8 are summarized in Table II. In these cases, active power flow settings are over 125% of their corresponding base case active power flows.

D. Discussion of the Results

From these results on the IEEE 30 and 118 bus systems, it can be seen:

1) Numerical results demonstrate the feasibility as well as the effectiveness of the GUPFC model established and the OPF method proposed.
2) The OPF with GUPFC can find a solution in reasonable iterations. The number of iterations for an OPF solution with the GUPFC devices is comparable with that of a base case OPF solution, which can be found in Tables I and II. It should be pointed out here that initialization of the GUPFC variables based on the analytical solutions derived in this paper is very helpful to improve the convergence characteristics of the OPF.
3) The GUPFC is a quite flexible and powerful FACTS controller. It can control bus voltage and active and reactive power flows of several lines simultaneously. It may be installed in some central substations to manage power flows of multi-lines or a group of lines and provide voltage support as well.
4) By using the GUPFC devices, the transfer capability of transmission lines can be increased significantly. Further more, by using the multi-line management capability of the GUPFC, active power flows on lines can not only be increased, but also be decreased with respect to operating and market transaction requirements in an open access environment. In the cases 3) and 4) above, such scenarios are simulated. Therefore, the GUPFC can be used to increase transfer capability and relieve congestion as well in power systems.
5) The OPF with global coordinating capability is a very useful tool to minimize (or maximize) an objective while satisfying power flow constraints, thermal constraints, as well as the operating and control constraints of the GUPFC devices.
6) The flexibility of the GUPFC with controlling bus voltage and multi-line active and reactive power flows offers a great potential in solving many of the problems facing the electric utilities in a competitive environment. OPF with the GUPFC FACTS devices would be a very useful tool to operate and manage the electric power systems in efficient ways.

V. CONCLUSION

A mathematical model of the Generalized Unified Power Flow Controller (GUPFC), which is suitable for power flow and optimal power flow study, is established. The model with one shunt converter and two or more series converters is implemented in a nonlinear interior point OPF. Further more analytical solutions for the initial values of the GUPFC are derived. With these starting points, the OPF can converge rapidly. The number of iterations for an OPF solution with the GUPFC devices is comparable with that of a base case OPF solution without the GUPFC devices.

Numerical results based on the IEEE 30 bus system and 118 bus system with various GUPFC devices demonstrate the feasibility as well as the effectiveness of the GUPFC model established and the OPF method proposed. It is obvious that the implementation principles of the GUPFC proposed can also be used in modeling other members of the Convertible Static Compensator (CSC) family in power flow and OPF analysis.

Clearly, the GUPFC can construct a multi-terminal (at least 3-terminals: bus i, j, k shown in Figs. 1 and 2) sub-network, which can control active and reactive power flows for a group of lines and selected bus voltage within a substation to their specified objectives. This has significantly extended the voltage and power flow control capability that was achieved by the independent STATCON or SSSC or UPFC.

The strong control capability of the GUPFC with controlling bus voltage and multi-line power flows or even a sub-network power flows offers a great potential in solving many of the problems facing the electric utilities in a competitive environment. OPF with the GUPFC devices would be a very useful tool to operate, plan and manage power systems efficiently.

REFERENCES

[1] B. Fardanesh, M. Henderson, B. Shperling, S. Zelingher, L. Gyugyi, C. Schauder, B. Lam, J. Moundford, R. Adapa, and A. Edris, "Convertible static compensator: Application to the New York transmission system," in *CIGRE 14-103*, Paris, France, Sept. 1998.

[2] L. Gyugyi, K. K. Sen, and C. D. Schauder, "The interline power flow controller: A new approach to power flow management in transmission systems," *IEEE Trans. Power Delivery*, vol. 14, no. 3, pp. 1115–1123, July 1999.

[3] L. Gyugyi, C. D. Shauder, S. L. Williams, T. R. Rietman, D. R. Torgerson, and A. Edris, "The unified power flow controller: A new approach to power transmission control," *IEEE Trans. Power Delivery*, vol. 10, no. 2, pp. 1085–1093, Apr. 1995.

[4] L. Gyugyi, C. D. Shauder, and K. K. Sen, "Static synchronous series compensator: A solid-state approach to the series compensation of transmission lines," *IEEE Trans. Power Delivery*, vol. 12, no. 1, pp. 406–413, Jan. 1997.

[5] B. Fardanesh, B. Shperling, E. Uzunovic, and S. Zelingher, "Multi-converter FACTS devices: The generalized unified power flow controller (GUPFC)," in *IEEE 2000 PES Summer Meeting*, Seattle, USA, July 2000.

[6] M. Huneanult and F. D. Galiana, "A survey of the optimal power flow literature," *IEEE Trans. Power Systems*, vol. 6, no. 2, pp. 762–770, May 1991.

[7] S. Granville, "Optimal reactive power dispatch through interior point methods," *IEEE Trans. Power Systems*, vol. 9, no. 1, pp. 136–146, 1994.

[8] Y.-C. Wu, A. Debs, and R. E. Marsten, "A direct nonlinear predictor–corrector primal–dual interior point algorithm for optimal power flows," *IEEE Trans. Power Systems*, vol. 9, no. 2, pp. 876–883, 1994.

[9] A. S. El-Bakry, R. A. Tapia, T. Tsuchiya, and Y. Zhang, "On the formulation and theory of the Newton interior-point method for nonlinear programming," *Journal of Optimization Theory and Applications*, vol. 89, no. 3, pp. 507–541, 1996.

[10] D. I. Sun, B. Ashley, B. Brewer, A. Hughes, and W. F. Tinney, "Optimal power flow by Newton approach," *IEEE Trans. PAS*, vol. 103, no. 10, pp. 2864–2880, 1984.

Xiao-Ping Zhang is a Lecturer at the University of Warwick, England. He received the B.Eng., M.Sc. and Ph.D. degrees in electrical engineering from Southeast University in 1988, 1990, 1993, respectively. He worked at NARI, Ministry of Electric Power, China on EMS/DMS advanced application software research and development between 1993–1998. He was an Alexander-von-Humboldt Research Fellow with the University of Dortmund, Germany.

Edmund Handschin received his diploma in electrical engineering in 1965 from the Swiss Federal Institute of Technology, Zürich, Switzerland and the Ph.D. degree in 1968 from the Imperial College London, United Kingdom. From 1969 till 1974, he was a staff member of the Brown Boveri Research Center in Baden, Switzerland. Since 1974, he has been Professor and Chair of Electrical Energy Systems at the University of Dortmund. He is a Fellow of IEEE.

Maojun "Mike" Yao received the B.S.E.E., M.S.E.E. degrees from the Department of Electrical Engineering, Shanghai Jiao Tong University in 1987, 1990, respectively. He received the Ph.D. degree in electrical engineering from the University of Texas at Arlington in 2000. He was employed by Central and South West Service, Inc. as an EMS engineer. He is currently a power systems engineer employed by Alstom ESCA Corporation.