finite ground plane will, of course, distort these patterns.) As seen in Fig. 5, the curves for variously sized bicones are quite similar with respect to the variations in the radiation pattern. At low frequencies when the antenna is electrically small, the pattern is similar to that of a short dipole [4] which is circular or oval in shape and in three dimensions has the appearance of a toroid (i.e., “doughnut”-shaped). The maximum radiation occurs at $\theta = \pi/2$. At the higher frequencies where the exit aperture is of the order of a wavelength or larger, the maximum radiation moves away from the horizontal direction and minor lobes appear. As a consequence, the horizontal directivity can decrease at higher frequencies, especially for the larger size bicones.

Directivity curves have also been computed. The directivity in the direction $\theta_m$, $D(\theta_m)$, is the ratio of the integral of the Poynting vector with the value of the electric field at $\theta = \theta_m$ to the actual value of the integral, or

$$D(\theta_m) = \frac{\int_{0}^{2\pi} \int_{0}^{\pi} E^*(\theta) E(\theta) \sin \theta \, d\theta \, d\Phi}{\int_{0}^{2\pi} \int_{0}^{\pi} E^*(\theta) E(\theta) \sin \theta \, d\theta \, d\Phi}.$$  \hspace{1cm} (7)

After the $\Phi$-integrations, (7) reduces to

$$D(\theta_m) = \frac{2(E(\theta_m))^2}{\int_{0}^{2\pi} \int_{0}^{\pi} E^*(\theta) E(\theta) \sin \theta \, d\theta \, d\Phi}.$$  \hspace{1cm} (8)

When the antenna is electrically short, $D(\theta_m) = 3/2$ which is the result for an infinitesimal dipole.

Horizontal directivity curves are shown in Fig. 6. Note that with the larger bicones there are frequencies at which the horizontal directivity is drastically reduced. On the other hand, a small bicone with $h = 3$ in., $a = 5$ in. (not shown) does not suffer from this effect and actually has values of $D(\pi/2)$ that increase with frequency.

V. CONCLUSION

A broadband omnidirectional antenna for use on a vehicle is described and analyzed. The associated far-field patterns including the effect of earth or sea are derived in [1].

**ACKNOWLEDGMENT**

The authors are pleased to acknowledge the graphics and computational assistance of Barbara H. Sandler and the editorial assistance of Margaret Owens.

**REFERENCES**


**A New Formulation for the Design of Chebyshev Arrays**

Ahmad Safaai-Jazi

Abstract—A new formulation for the design of Chebyshev arrays is presented which makes no direct use of Chebyshev polynomials. The array factor is expressed in terms of cosine/cosine-hyperbolic functions based on which all analysis and design steps are carried out. Zeros of the array factor are used to obtain a system of equations for excitation currents. Solving this system of equations, current magnitudes are determined in terms of the current of one element chosen as the independent variable. A general formulation for even or odd but otherwise arbitrary number of elements is presented which makes no direct use of Chebyshev polynomials. The process involves expressing the array factor as a polynomial whose coefficients are compared with a Chebyshev polynomial of the same order and their radiation pattern are of equal magnitude.

Chebyshev arrays have the unique property that all side lobes in their radiation pattern are of equal magnitude [1]. Traditionally, the analysis and design of Chebyshev arrays have been carried out with the help of Chebyshev polynomials [2]–[4]. The process involves expressing the array factor as a polynomial whose coefficients are in terms of element current magnitudes. This polynomial is then compared with a Chebyshev polynomial of the same order and coefficients of like power terms are equated. A linear system of equations then results which is solved for current magnitudes in terms of the current of one element chosen as the independent variable. This approach, while useful in illuminating the fundamental aspects of Chebyshev arrays, becomes quite involved when the number of elements is large.

In this article, a new approach to the design of Chebyshev arrays is presented which allows for a unified formulation to be readily obtained for an arbitrary number of elements. The key point in this approach is that no direct use is made of Chebyshev polynomials.

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Instead, the array factor is expressed in terms of cosine/cosine-
hyperbolic functions which serve as alternative representations to
Chebyshev polynomials \[5\]. Zeros of the array factor are easily
determined from the cosine function representation. Using these
zeros, a system of equations for element current magnitudes is then
readily obtained. The solution of this system of equations is presented
in a matrix form for even or odd but otherwise arbitrary number of
elements. Zeros of the array factor have been used before in a different
formulation of the Chebyshev array \[6\]. However, this formulation,
which uses a polynomial representation of the array factor, is not
general and becomes quite involved for large number of elements.

In a design problem with unknown number of elements but
specified side lobe level and beamwidth, the common practice has
been to determine the minimum number of elements required to meet
the design specifications through a trial and error procedure. Here,
an equation is derived from which the minimum number of elements
is obtained in a single step without resorting to an iterative process,
therefore a considerably more efficient design approach.

Two cases corresponding to even and odd number of elements
are considered. It is assumed that among side lobe level, half-power
beamwidth, and number of elements, two quantities are specified
and the third quantity as well as excitation currents (relative to the
beamwidth, and number of elements, two quantities are specified
for optimum spacing between the elements are given and numerical
results for example design cases are also presented.

II. ARRAY FACTOR

Let us consider a linear array of \( N \) equally spaced elements
without uniform excitations. Assuming a symmetrical distribution
for magnitude of currents and a constant inter-element phase shift
\( \alpha \), the array factor for even and odd number of elements can be
expressed as

\[
I(\psi) = 2 \sum_{n=1}^{N} I_n \cos \left( \frac{N-1}{2} \psi \right), \quad N = 2n, \quad \text{(1a)}
\]

and

\[
I(\psi) = I_0 + 2 \sum_{n=1}^{N} I_n \cos(m\psi), \quad N = 2n + 1, \quad \text{(1b)}
\]

where \( \psi = \beta d \cos \theta + \alpha, \beta = 2\pi f \lambda, \) \(\lambda = \) the wavelength, \( d \) is
the element spacing, \( \theta \) is the angle measured from the line of array,
\( I_n \) is the magnitude of current for the \( n \)th element on either side of
the array midpoint, and \( I_0 \) denotes the current of the center element
when \( N \) is odd.

Taking advantage of the fact that Chebyshev polynomials satisfy
the relationships \( T_n(\cos \psi) = \cos(n\psi) \) for \( T_n < 1 \) and \( T_n(\cos \psi) = \cos(h/n\psi) \) for \( T_n \geq 1 \), the array factor in (1a) or
(1b) is written as

\[
|I(\psi)| = |I(\psi)| = \begin{cases} C \cos \left( \frac{(N-1)}{2} \psi \right), & |I(\psi)| \leq C, \quad \text{(2a)} \\ C \cos \left( \frac{(N-1)}{2} \psi \right), & |I(\psi)| \geq C, \quad \text{(2b)} \end{cases}
\]

where \( C \) is a positive constant coefficient and the relationship between \( \psi \) and \( \psi \) is given by

\[
\gamma \cos \left( \frac{\psi}{2} \right) = \begin{cases} \cos \left( \frac{\psi}{2} \right), & |I(\psi)| \leq C, \quad \text{(3a)} \\ \cos \left( \frac{\psi}{2} \right), & |I(\psi)| \geq C, \quad \text{(3b)} \end{cases}
\]

In (3a) and (3b), \( \gamma \) is a real coefficient which can be determined when
design specifications are known. It is evident from (1a) and (1b) that
\( I(\psi) \) assumes its maximum value at \( \psi = 0 \). This corresponds to
\( \psi = \psi_0 = 2 \cos h^{-1}(\gamma) \). Let us, for the time being, assume that
the number of elements \( N \) and beam maximum-to-side lobe level ratio
\( R \) (side lobe level in dB = \( 20 \log R \)) are specified. Since all side
lobes are of the same magnitude \( C \) as implied by equation (2a), the
ratio \( R \) satisfies the relation

\[
R = \cos h \left( \frac{(N-1)}{2} \psi_0 \right) = \cos h((N-1) \cos h^{-1}(\gamma)). \quad \text{(4)}
\]

Solving (4) for \( \gamma \), yields

\[
\gamma = \cos h \left( \frac{1}{N-1} \ln(R + \sqrt{R^2-1}) \right). \quad \text{(5)}
\]

Calculation of \( \gamma \) when \( N \) and half-power beamwidth \( \text{HPBW} \) or \( R 
and \( \text{HPBW} \) are given will be discussed later. The coefficient \( C \) if
the array factor is required to be normalized to unity is determined
as \( C = 1/R \).

III. CALCULATION OF EXCITATION CURRENTS

Zeros of the array factor are used to establish the system of
equations governing current magnitudes. These zeros are solutions of
\( \cos \left( (N-1) \psi / 2 \right) = 0 \), which yields \( \psi = \left( (m \pm 1) \pi / (N-1) \right) \) with
\( m \) being an integer. Due to the symmetry of the array factor about
\( \psi = \pi \), it suffices to consider only the zeros in the range \( 0 < \psi < \pi \). It can be shown that such zeros are obtained from

\[
\psi_m = \frac{(2m-1)\pi}{N-1}, \quad m = 1, 2, \ldots, M. \quad \text{(6)}
\]

where \( M = (N-2)/2 \) for \( N \) even and \( M = (N-1)/2 \) for \( N \) odd. Using (3a), the zeros in terms of \( \psi \) are given by

\[
\psi_m = 2 \cos^{-1} \left[ \frac{1}{1 \cos \left( \frac{(2m-1)\pi}{N-1} \right)} \right], \quad m = 1, 2, \ldots, M. \quad \text{(7)}
\]

On the other hand, \( \psi_m \) are zeros of \( f(\psi) \) in (1a) or (1b); that is,

\[
f(\psi_m) = 0, \quad m = 1, 2, \ldots, M. \quad \text{(8)}
\]

(8) is, in fact, a system of \( M \) equations and \( M + 1 \) unknowns.
The unknowns are current magnitudes \( I_m \), one of which is chosen
as an independent variable. For even number of elements, \( I_{M+1} \) is
chosen as the independent variable. The remaining \( M \) currents are
then obtained from the following matrix equation,

\[
\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_{M+1} \end{bmatrix} = -I_{M+1} [A]^{-1} \begin{bmatrix} \cos \left( \frac{M}{2} \psi_1 \right) \\ \cos \left( \frac{M}{2} \psi_2 \right) \\ \vdots \\ \cos \left( \frac{M}{2} \psi_M \right) \end{bmatrix}, \quad \text{(9)}
\]

where

\[
[A] = \begin{bmatrix} \cos \left( \frac{\psi_1}{2} \right) \cos \left( \frac{\psi_2}{2} \right) \cdots \cos \left( \frac{\psi_M}{2} \right) \\ \cos \left( \frac{\psi_1}{2} \right) \cos \left( \frac{\psi_2}{2} \right) \cdots \cos \left( \frac{\psi_M}{2} \right) \\ \vdots \\ \cos \left( \frac{\psi_1}{2} \right) \cos \left( \frac{\psi_2}{2} \right) \cdots \cos \left( \frac{\psi_M}{2} \right) \end{bmatrix}. \quad \text{(10)}
\]

For odd number of elements, \( I_M \) is chosen as the independent variable.
The remaining \( M \) currents are then obtained from

\[
\begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_M \end{bmatrix} = -I_M [B]^{-1} \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \end{bmatrix}, \quad \text{(11)}
\]

where

\[
[B] = \begin{bmatrix} \cos \psi_1 \cos \left( 2\psi_1 \right) \cdots \cos \left( M\psi_1 \right) \\ \cos \psi_2 \cos \left( 2\psi_2 \right) \cdots \cos \left( M\psi_2 \right) \\ \vdots \\ \cos \psi_M \cos \left( 2\psi_M \right) \cdots \cos \left( M\psi_M \right) \end{bmatrix}. \quad \text{(12)}
\]
IV. RELATIONSHIP AMONG BEAMWIDTH, SIDE LOBE LEVEL AND NUMBER OF ELEMENTS

Let us assume that \( \theta = \theta_h \) corresponds to a half-power point on the main beam of the array pattern. For broadside array (\( \alpha = 0 \)) there exist two half-power points, on the left and right sides of \( \theta = \pi/2 \).

Considering \( \theta_h < \pi/2 \), the half-power beamwidth for broadside arrays is HPBW = \( \pi - 2\theta_h \). For endfire arrays (\( \alpha = \pm \beta d \)), there exists only one solution for \( \theta_h \) (there \( d \leq d_{\text{opt}} \) is assumed, see Eq. (20)), and HPBW = \( \theta_h \) for \( \alpha = -\beta d \). Without loss of generality, endfire arrays with \( \alpha = -\beta d \) are considered in the following analysis. For endfire arrays with \( d = \lambda/2 \), \( \theta_h = \pi/2 \). Let \( \psi = \psi_h \) at \( \theta = \theta_h \), then \( \psi_h = \beta d \cos \theta_h \) for broadside case, and \( \psi_h = 3\beta d \cos \theta_h - 1 \) for endfire case. From (3b)

\[
\psi_h = 2 \cos^{-1} \left( \gamma \cos \left( \frac{\psi_h}{2} \right) \right), \quad (13)
\]

Recalling that \( \psi = \psi_h \), the array factor assumes its maximum value, from (2b), (4), (5), and (13) it follows that

\[
\cos \left( \frac{\psi_h}{2} \right) \cos h \left[ \frac{1}{(N-1)} \cos h^{-1} \left( \frac{\sqrt{2}}{R} \right) \right] - \cos h \left[ \frac{1}{(N-1)} \cos h^{-1} \left( \frac{\sqrt{2}}{R} \right) \right] = 0. \quad (14)
\]

Equation (14) is a relationship among half-power beamwidth, side lobe level, and the number of elements. Thus, if two of these three quantities are specified the third one can be determined from (14).

The above discussions and results are applicable to both even and odd number of elements.

V. OPTIMUM SPACING

The spacing \( d \) may be optimized such that for a specified side lobe level and given number of elements the smallest possible beamwidth is achieved. This amounts to accommodating as many side lobes as possible in the radiation pattern of the array. In doing so, the visible range in the \( \psi \) domain, corresponding to the range \( 0 \leq \psi \leq \pi \), is so determined to include all side lobes on both sides of the main beam in broadside arrays, and on one side of it in endfire arrays, as well as a portion of the grating lobe which will produce a side lobe with a magnitude equal to the specified side lobe level. The cutoff point on the grating lobe should satisfy the relation

\[
\cos h \left[ \frac{(N-1)\psi}{2} \right] = \cos \left[ \frac{(N-1)\psi}{2} \right], \quad (18)
\]

which is possible only if \( \psi = 0 \). The value of \( \psi \) corresponding to \( \psi = 0 \) can be obtained from (3a). It should, however, be noted that in (3a), \( 0 \leq \psi \leq \pi \) is presumed. Thus, the solution \( \psi = 2 \cos^{-1} (1/\gamma) \) has to be discarded, because this corresponds to a point on the main beam at which \( \psi = 0 \). The solution of \( \psi \) corresponding to grating lobe lies in the range \( \pi < \psi < 2\pi \). However, in this range \( \cos \psi < 0 \) and (3a) should be modified as \( -\gamma \cos (\psi/2) = \cos (\psi/2) \). Putting \( \psi = 0 \) in this equation, \( \psi = 2\pi - 2 \cos^{-1} (1/\gamma) \) is obtained.

For broadside arrays, \( \psi \) should correspond to \( \theta = \theta_h \), that is \( \psi = \beta d_{\text{opt}} \), resulting in

\[
d_{\text{opt}} = \lambda \left[ 1 - \cos^{-1} \left( \frac{1}{\gamma} \right) \right], \quad \text{broadside.} \quad (19)
\]

For endfire arrays with the main beam in the \( \theta = \pi \) direction, \( \psi \) should correspond to \( \theta = 0 \), that is \( \psi = 2\beta d_{\text{opt}} \), resulting in

\[
d_{\text{opt}} = \frac{\lambda}{2} \left[ 1 - \cos^{-1} \left( \frac{1}{\gamma} \right) \right]. \quad \text{endfire.} \quad (20)
\]

VI. NUMERICAL RESULTS

For given number of elements \( N \) and side lobe level SLL, current magnitudes are calculated by first finding \( \psi_{h0} \) from (7) and then solving (9) if \( N \) is even or (11) if \( N \) is odd. Currents are normalized such that the current of edge elements is unity; i.e., \( I_0 = 1 \). Currents of the remaining elements of the array: namely, \( I_1, I_2, \ldots, I_{N-1} \), when \( N \) is even (\( = 2n \)) and \( I_1, I_2, \ldots, I_{n-1} \) when \( N \) is odd (\( = 2n + 1 \)) are summarized in Table I for \( N = 3, 4, \ldots, 10 \) and SLL.
A. Endfire arrays are presented in Tables I and II. For each value of \( N \), the first and second rows correspond to \( d = \lambda/2 \) and \( d = d_{\text{opt}} \), respectively.

Table I

<table>
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<tr>
<th>( N )</th>
<th>SLL (dB)</th>
<th>-10</th>
<th>-20</th>
<th>-30</th>
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<tbody>
<tr>
<td>3</td>
<td>36.45</td>
<td>40.38</td>
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<td>30.08</td>
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<td>7</td>
<td>19.61</td>
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<td>8</td>
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<td>15.11</td>
<td>18.33</td>
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<td>9</td>
<td>15.84</td>
<td>19.46</td>
<td>22.06</td>
<td>23.82</td>
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<td>10</td>
<td>8.91</td>
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<td>14.42</td>
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Table II

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In these two tables, there are two entries for each SLL-S combination. The first entry is the half-power beamwidth corresponding to element spacing \( d = \lambda/2 \) while the second entry is that for \( d = d_{\text{opt}} \) as specified in (19) and (20). As an example, for a broadside array with \( N = 9 \) and SLL = -30 dB, the half-power beamwidth is 14.55° if \( d = \lambda/2 \) and 8.63° if \( d = d_{\text{opt}} = 0.8419 \lambda \). The optimum spacings for the broadside case are given in Table IV.

The optimum spacings for the endfire case \( \alpha = \pm 5d \) are one-half of those for the broadside array. The results of the tables in Table I are available elsewhere [7]. They have been produced here primarily for the purpose of comparison and checking the correctness of the proposed method.

A. Design Example

An endfire array with side lobe level of -20 dB and half-power beamwidth of 22.5° is to be designed. Determine the minimum number of elements, the excitation currents, and the optimum spacing.

Step 1 From \( R = 10^{-5} \text{SLL}/20 \), \( R = 10 \) is calculated. Then, with \( \theta_0 = 22.5^\circ \) and combining (5), (14), and (20), we have an equation in which only \( N \) is unknown. Solving this equation numerically, \( N = N_0 = 13.596 \) is obtained. However, since \( N \) must be an integer and in order to have HPBW \( \leq 22.5^\circ \), according to (17) \( N = 14 \) is considered.

Step 2 For \( N = 14 \) and \( \epsilon = 10 \) from (5) and (7) \( \gamma \) and \( \epsilon_{\text{opt}} \) are determined, respectively. Then, assuming \( I_1 = 1 \), equation (9) is solved with the following results: 

\( I_1 = 1.2684, I_2 = 1.2190, I_3 = 1.1245, I_4 = 0.9931, I_5 = 0.8359, \) and \( I_6 = 0.6655 \). Finally, from (20) the optimum spacing \( d_{\text{opt}} = 0.4636 \lambda \) is obtained.

To ensure that the design meets the specified requirement, the half-power beamwidth is examined. From (15b) for \( N = 14, R = 10 \), and \( d = 0.4636 \lambda \), we obtain HPBW \( \leq 22.10^\circ \) which is slightly less than maximum required beamwidth of 22.5°. \( N = 14 \) should be the minimum number of elements. This is indeed the case, because for \( N = 13 \) and \( R = 10 \) the smallest achievable beamwidth is calculated to be 23.06° which exceeds the maximum beamwidth of 22.5°. A plot of the radiation pattern for this endfire array is shown in Figure 1.

VII. CONCLUSION

A unified formulation for Chebyshev arrays with arbitrary number of elements has been presented. This formulation is sufficiently general, lends itself to simple computer-aided design algorithms, and allows for quick analysis of Chebyshev arrays. Design parameters can be obtained without the need for a trial and error process. An equation in terms of the number of elements, side lobe level, and...
half-power beamwidth has been derived which facilitates the design process considerably.

ACKNOWLEDGMENT

The author would like to thank Y. Kim for performing the computations.

REFERENCES


Comments on “An Improved Pulse-Basis Conjugate Gradient FFT Method for the Thin Conducting Plate Problem”

A. Peter M. Zwamborn

In the above cited paper, Tran and McCowen present pulse-basis expansion functions and Dirac $\delta$ testing functions within a conjugate gradient FFT (CGFFT) formulation. In the conclusions, they state: “Although these spurious effects do not appear in the CGFFT formulation using the rooftop basis function proposed by Zwamborn and van den Berg [1], there are more computational costs associated with their method than with ours.” At this point, I strongly disagree with this statement.

Close examination of (3) reveals that the method proposed by Tran and McCowen uses four scalar convolutions to be evaluated for each operator computation. The weak form of the conjugate gradient FFT (WCGFFT) method introduced by Zwamborn and van den Berg [1] needs only two scalar convolutions. Since these convolutions consume a dominant part of the computation of the operator, it is straightforwardly observed that the WCGFFT method needs significantly less CPU time per iteration. Further, we remark that Tran and McCowen have left out the numerical convergence rate of the WCGFFT method while comparing their method with other CGFFT methods. Therefore, we present in Fig. 1 the numerical convergence rates of all various schemes. This figure is composed by copying Fig. 2 of the above paper, but now including the WCGFFT results from [1, Fig. 1]. From the present figure, it is shown that the WCGFFT method converges within far fewer iterations than Tran and McCowen’s CGFFT method. As mentioned in [1] in the last paragraph of Section IV, a considerable decrease of computation time is achieved by using the discrete cyclic convolution (see also [2]). For example, in the case where we have discretized the plate into 31 subdomains, the CPU time per WCGFFT iteration on a VAX6400 minicomputer amounts to 2.5 s instead of 10 s for the spectral representation that is presented in [1].

Further, Tran and McCowen claim in the conclusions that: “The extraneous ripples which are formed in the interior of the induced surface densities in our method do not significantly affect the overall quality of the current distributions... Yet, there is little compromise in accuracy, particularly in the far field...” To discuss this statement in more detail, we consider the results of the circular disk as...