Analysis and Design of a Repetitive Predictive-PID Controller for PWM Inverters

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***Abstract* — This paper presents a predictive PID controller including a feedforward control and a repetitive controller for UPS applications. This digital control scheme can minimize periodic distortions resulted from nonlinear cyclic loads. Design procedure and stability analysis of the control scheme are discussed. Simulation and experimental results for a single-phase PWM inverter (110 V_{RMS}, 1 kVA) controlled by a low cost microcontroller are presented to demonstrate the performance of the proposed control approach under periodic load disturbances.**

I. INTRODUCTION

Due to widespread use of the microcomputer-based systems in several applications, the demand of single-phase uninterruptible power systems to supply critical loads at low power (500 VA - 5 kVA) is increasing. On the other hand, with the fast development of the low cost microcontrollers, engineers and researchers have been using more sophisticated digital control techniques to improve the performance of the uninterruptible power supplies (UPS) [1]-[2]. As result, compact systems of low cost, good performance and great reliability have been obtained.

In this application, the purpose of a digital control scheme is to generate a PWM (Pulse Width Modulation) pattern to produce the sinusoidal output voltage of the UPS system with low total harmonic distortion (THD) even under both transient or load disturbances. Among the digital control techniques, the digital PID (proportional-integral-derivative) controller has been used in many industrial control systems. The main reason for this is its simplicity, along with the flexibility of a digital controller. The disadvantage of this controller is that the delay time caused by A/D conversion and computation time of the microprocessor reduces the maximum available pulse width. Hence, this limitation can result in output voltage waveform distortions. One method for overcoming this time delay was developed [3] and the resulting controller is called a PID predictor controller [3]-[4]. However, by using this controller in PWM inverters, the output voltage waveform may present a high THD specially when the load is nonlinear such as ac phase-controlled loads and rectifier loads.

Repetitive control theory [5] provides an alternative to minimize periodic errors occurred in a dynamic system. The repetitive control improves the accuracy of the steady state response of a control system when the reference input signal and disturbances are periodic, consisting of harmonic components of a common fundamental frequency. Several repetitive control schemes have been developed and applied to various industrial applications [6]-[7]. Besides, the repetitive control technique has been applied to minimize periodic errors which can appear in single-phase PWM inverters [8]-[10].

In this paper is proposed a predictive PID controller including a feedforward control and a repetitive controller for UPS applications. This digital control technique can minimize periodic distortions resulted from nonlinear cyclic loads, even in the presence of unmodeled dynamics. Moreover, due to error prediction, it is possible to implement this controller on a low cost microcontroller.

This paper is organized as follows. Section II describes the plant model. The structure, design procedure and stability analysis of the predictive PID-feedforward controller with a repetitive controller for a single-phase PWM inverter are presented in Section III. Section IV presents simulation results with nonlinear loads and Section V shows experimental results obtained for linear and nonlinear loads based on a microcontroller-controlled system.

II. PLANT MODEL

Fig. 1 shows the single-phase PWM inverter, where the full-bridge inverter, LC filter, and resistive load R are considered as the plant to be controlled. A triac connected in series with a resistive load or a full-bridge rectifier with RC filter can be used to evaluate the performance of the system with nonlinear loads. As a matter of fact, due to diversity of loads, it is not possible to formulate a general model to cover every kind of load. However, we can define a nominal load to derive its linear model and consider load variations and model uncertainties as a load disturbance.

The system transfer function of the Fig. 1 is given by

$$\frac{Y(s)}{U(s)}=G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2} \qquad (1)$$

where $\omega_n=1/\sqrt{LC}$, $\zeta=1/(2RC\omega_n)$, $Y(s)$ is the Laplace transform of the system output $y(t)=v_c(t)$ and $U(s)$ is the Laplace transform of the system input $u(t)=v_{in}(t)$.

In the following analysis the power switches are turned on and off once during each sampling interval T, such that $v_{in}(t)$ is a voltage pulse of magnitude V_B, 0 or $-V_B$ and width ΔT.

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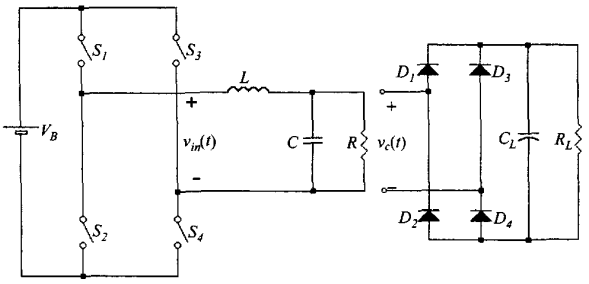

Fig. 1 – PWM inverter system.

From (1), a discrete transfer function can be obtained using a zero-order hold with an appropriate sampling time T [11],

$$G(z)=\frac{b_1 z+b_2}{z^2+a_1 z+a_2} \tag{2}$$

III. Predictive PID-Feedforward Controller with Repetitive Controller

A. Structure of the Controller

The classic PID controller, shown in Fig. 2, has been used in many industrial control systems. The main reasons are its simple structure, which can be easily understood and implemented in the practice, and its excellent flexibility made possible by the adjustment of the coefficients K_P, K_I e K_D.

The equation for the controller in Fig. 2 is

$$u_{PID}(t)=K_P e(t)+K_I\int e(t)dt+K_D\frac{de(t)}{dt} \tag{3}$$

For a microcontroller implementation, (3) becomes

$$u_{PID}(k)=K_P e(k)+K_I T\sum_{n=1}^{k} e(n)+\frac{K_D}{T}[e(k)-e(k-1)] \tag{4}$$

According to (4), the control variable $u_{PID}(k)$ depends upon the present value of the error signal $e(k)$ as well as its previous values. If the required computation time to implement $u_{PID}(k)$ is not small, benefits may be reached in predicting the value of $e(k)$ at time $t = (k-1)T$, and then computing $u_{PID}(k)$ from (4). However, errors are introduced into the system since the predicted value of $e(k)$ is used rather than the true one [4].

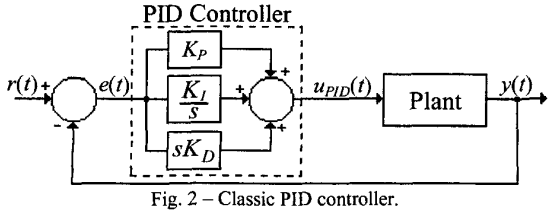

Fig. 2 – Classic PID controller.

Aylor, Ramey and Cook [3] used the simple linear prediction algorithm, given by:

$$e(k)=e(k-1)+[e(k-1)-e(k-2)] \tag{5}$$

where the error signal is predicted according to their last two measured values.

Using (5) to predict $e(k)$ in (4), the predictive PID controller equation becomes

$$u_{PID}(k)=[2K_P+2K_I T+K_D/T]e(k-1) -[K_P+K_I T+K_D/T]e(k-2)+[K_I T]\sum_{n=1}^{k-1} e(n) \tag{6}$$

To minimize the tracking error for linear and nonlinear cyclic loads, a feedforward control and a repetitive controller are added to predictive PID controller, as shown in Fig. 3.

Then, the control law $u_P(k)$ becomes:

$$u_P(k)=u_{PID}(k)+u_{RP}(k)+r(k) \tag{7}$$

In a similar way as presented in [9], the repetitive control law can be written as:

$$u_{RP}(k)=c_1 e(k+N-n)+c_2\sum_{i=1}^{\infty} e(k+N-i.n) \tag{8}$$

where $e(k)$ is the tracking error, c_1 and c_2 are the gains of the repetitive controller, N is the time advance step size and n is the number of samples in a period of reference voltage.

Due to inclusion of the repetitive controller in the control law, it is possible to eliminate the integral action from the PID controller, therefore reducing the computational effort for the controller implementation without degrade the performance of the system. By removing the integration from the predictive PID controller, (6) becomes,

$$u_{PID}(k)=K_1 e(k-1)+K_2 e(k-2) \tag{9}$$

where

$$K_1=2K_P+K_D/T \qquad K_2=-K_P-K_D/T \tag{10}$$

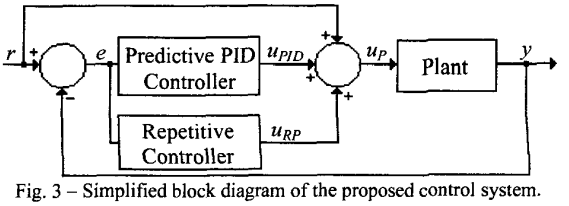

Fig. 3 – Simplified block diagram of the proposed control system.

B. Controller Design

Fig. 4 shows the block diagram of the feedback system, using the predictive PID-feedforward controller. The closed-loop transfer function of the system shown in Fig. 4 is given by

$$G_{PID}(z)=\frac{b_1z^3+(b_1K_1+b_2)z^2+(b_1K_2+b_2K_1)z+b_2K_2}{z^4+a_1z^3+(a_2+b_1K_1)z^2+(b_1K_2+b_2K_1)z+b_2K_2} \quad (11)$$

From (11), it is observed that the characteristic equation of the feedback system is a fourth-order polynomial. However, for this case, it was verified that the closed-loop system has two poles that are dominant. Then, the characteristic equation can be represented by

$$P_c(z)=(z-p_1)(z-p_2)(z-p_D)(z-\overline{p}_D) \quad (12)$$

where p_D and $\overline{p}_D$ are dominant poles of the closed-loop system in the z domain and, p_1 and p_2 are the other two poles of the feedback system. The location of the dominant poles of the closed-loop system in the s domain is given by:

$$S_{1,2}=-\zeta_c\omega_c\pm j\omega_c\sqrt{1-\zeta_c^2} \quad (13)$$

where ω_c and ζ_c are, respectively, the natural frequency and damping ratio of the desired closed-loop system. Consequently, the location of the dominant poles in the discrete domain (z domain) is given by:

$$\begin{aligned} p_D &= e^{S_1T} \\ \overline{p}_D &= e^{S_2T} \end{aligned} \quad (14)$$

Therefore, by using the predictive PID-feedforward controller, which has two parameters (K_1 and K_2), it is possible to place the two dominant poles of the closed-loop system, such that the feedback system satisfies the desired specifications.

For example, we choose a damping ratio $\zeta_c = 0.4$ and a natural frequency (ω_c) 10% greater than the natural frequency of the plant. Then, the parameters of predictive PID controller, by using the denominator of (11) and (12)-(14), are given by

$$K_1=0.1033$$
$$K_2=-0.2523$$

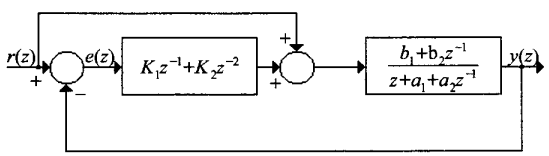

Fig. 4 – Block diagram of the closed-loop system using the predictive PID-feedforward controller.

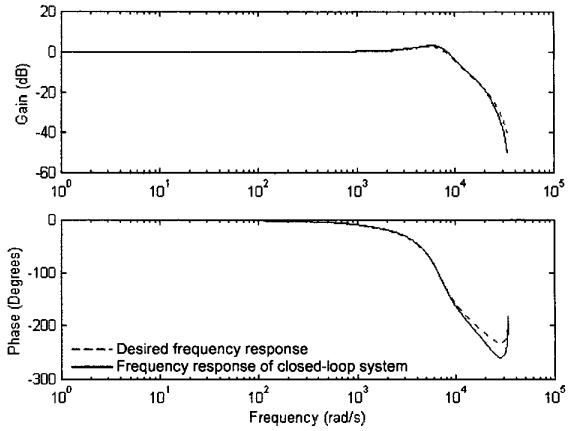

Fig. 5 – Frequency response of the system.

In Fig. 5 is shown the small influence of the poles p_1 and p_2 in the frequency response of the closed-loop system, verifying that the frequency response of the closed-loop system is similar to the desired frequency response.

The repetitive controller gains used in all simulations and experiments are $c_1 = 0.02$ and $c_2 = 0.20$. With these gains, the feedback system has a good steady-state response for any resistive load and a fast convergence of the output error for nonlinear cyclic loads.

C. Stability Analysis

The closed-loop system with the predictive PID-feedforward controller is stable if all roots of the denominator of (11) lie within the unit circle.

By simulation, various values of K_1 and K_2 were found for different values of load resistance R, which caused one or more roots of the denominator of (11) to lie on the unit circle (the stability boundary) with all other roots inside the circle. In this way, the family of curves shown in Fig. 6 was generated, where each curve represents the limiting values of K_1 and K_2 for a selected load resistance R.

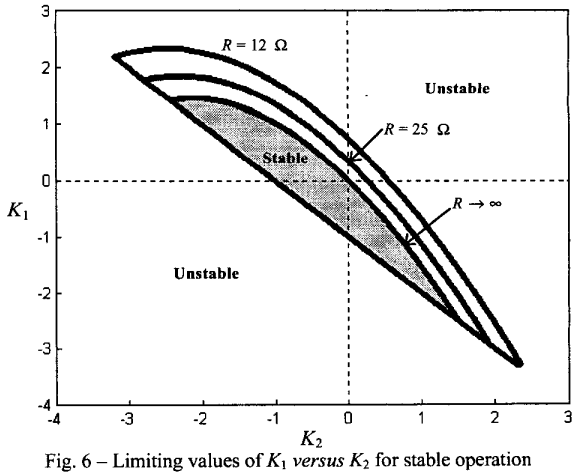

Fig. 6 – Limiting values of K_1 *versus* K_2 for stable operation (L = 1 mH, C = 25 μF, T = 92.6 μs).

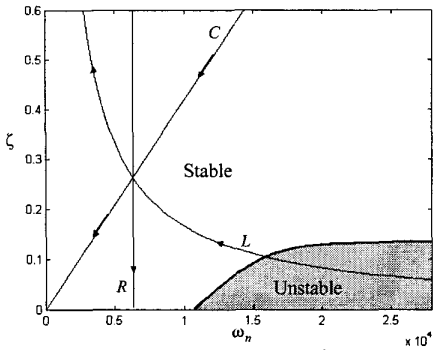

Fig. 7 – Stability region of the predictive PID-feedforward controller in ζ - ω_n plane (Nominal values: R = 12 Ω, C = 25 μF, L = 1 mH).

In Fig. 7 are presented the trajectories of the plant parameters in the $\zeta - \omega_n$ plane. For a given trajectory, one parameter varies along the trajectory and the other two parameters are held constant with their nominal values. This figure indicates that the predictive PID-feedforward controller, by using $K_1 = 0.1033$ and $K_2 = -0.2523$, is stable for any value of ω_n smaller than 10500 rad/s.

With the inclusion of the repetitive control, the transfer function $E(z)/R(z)$ of the system shown in Fig. 3, by using (2) and after z-transforming of (7)–(9), is given by

$$\frac{E(z)}{R(z)} = \frac{(1-G_{PID}(z))(1-z^{-n})}{1-z^{-n}H_{PID}(z)} \tag{15}$$

where the z-transform of the output error is $E(z)$, $R(z)$ is the transformed reference input, and

$$H_{PID}(z) = 1 - \frac{z^{N+2}}{z^2+K_1 z+K_2}(c_1+c_2-c_1 z^{-n})G_{PID}(z) \tag{16}$$

Assuming that (11) is stable, then the stability of the system is determined by the repetitive control. From (15), it is possible to demonstrate that the sufficient condition [9] for the stability is

$$\left|H_{PID}(j\omega)\right| \le 1 \tag{17}$$

where $\omega = 2\pi m f$ $(m = 0, 1, 2, \ldots, n/2)$.

IV. Simulation Results

Table I gives the parameters of the single-phase full-bridge PWM inverter system used in digital computer simulation with MATLAB® to verify the performance of the proposed control scheme.

In Fig. 8 is presented the output voltage $v_c(t)$ and load current $i_L(t)$ waveforms for rated resistive load with phase commutated at angle of 72°. Fig. 9 shows the output voltage $v_c(t)$ and load current $i_L(t)$ waveforms for the same nonlinear cyclic load including an unmodeled zero at -80000 rad/s. This unmodeled stable zero could be physically represented by an unmodeled equivalent series resistance (ESR) of 0.5 Ω of the filter capacitor. Unlike OSAP controller [10], it is observed that the proposed controller presents a good performance even with the inclusion of unmodeled dynamics.

Fig. 10 presents the response of PID-feedforward controller with repetitive controller for a rectifier load. In Fig. 11 is shown the output voltage and load current waveforms under a step load change from no load to full load, demonstrating the good transient response of this controller for a step load change.

TABLE I

Parameters of PWM inverter.

Filter inductance	L = 1 mH
Filter capacitance	C = 25 μF
DC input voltage	V_B = 200 V
Reference voltage	r = 110 V_{RMS}, f = 60 Hz
Linear load	R = 12 Ω
Nonlinear load	R_L = 25 Ω C_L = 330 μF
Sampling frequency	f_S = 10800 Hz
Sampling time	T = 92.6 μs

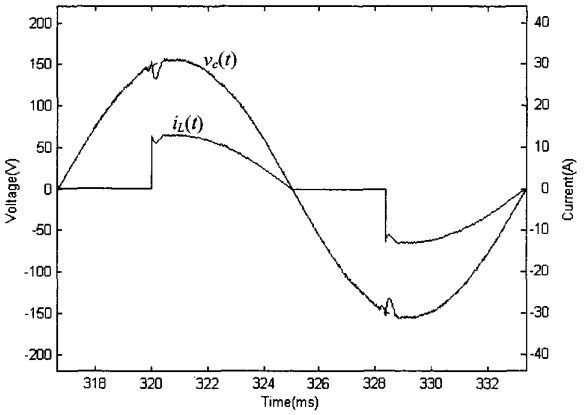

Fig. 8 – Output voltage $v_c(t)$ and load current $i_L(t)$ waveforms for rated resistive load with phase commutated at angle of 72°.

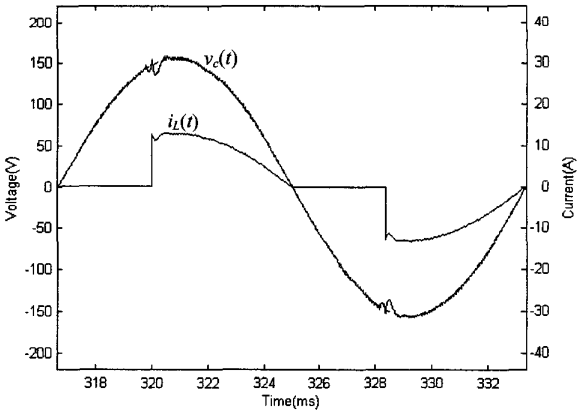

Fig. 9 – Output voltage $v_c(t)$ and load current $i_L(t)$ waveforms for rated resistive load with phase commutated at angle of 72° with the inclusion of a unmodeled stable zero at -80000 rad/s.

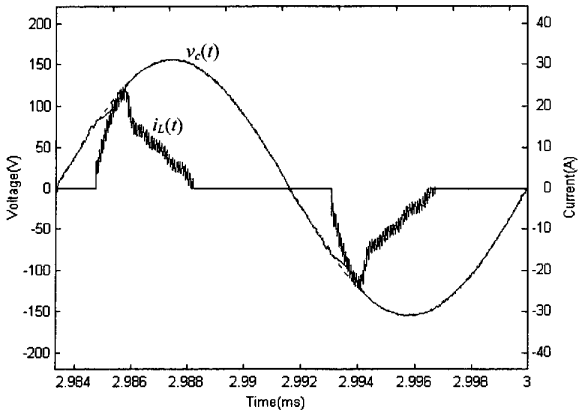

Fig. 10 – Output voltage $v_c(t)$ and load current $i_L(t)$ waveforms for a rectifier-RC load.

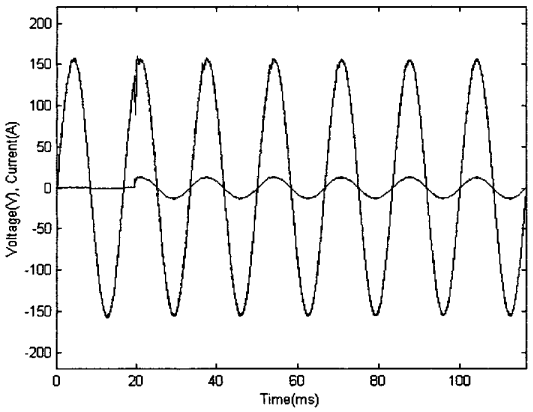

Fig. 11 – Output voltage and load current waveforms under a step load change from no load to full load.

V. Experimental Results

An IGBT prototype of the single-phase PWM inverter has been built in laboratory to verify the performance of the predictive PID-feedforward controller with repetitive controller. The component values of the inverter system and rectifier-RC load are given in Table I.

The simplified block diagram of the experimental setup is shown in Fig. 12. The controller has been implemented using an eight bits wide data word microcontroller (PIC17C756 of Microchip Technology Inc.). It has an embedded 10 bits A/D converter and a PWM signal generator. These features reduce significantly the PWM inverter control circuitry without penalizing the cost.

Fig. 13 shows the output voltage $v_c(t)$ (THD = 1.49%) and load current $i_L(t)$ waveforms for nominal resistive load (12 Ω/1 kVA). Fig. 14 presents the reference voltage $r(t)$, which is generated by the microcontroller and a D/A converter (DAC0800), and output voltage $v_c(t)$ waveforms for nominal resistive load, demonstrating the tracking capability of this controller.

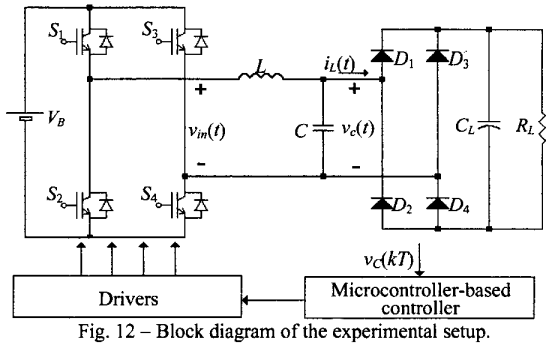

Fig. 12 – Block diagram of the experimental setup.

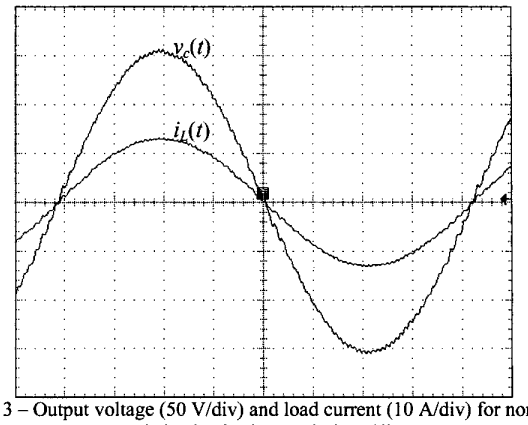

Fig. 13 – Output voltage (50 V/div) and load current (10 A/div) for nominal resistive load. Time scale 2 ms/div.

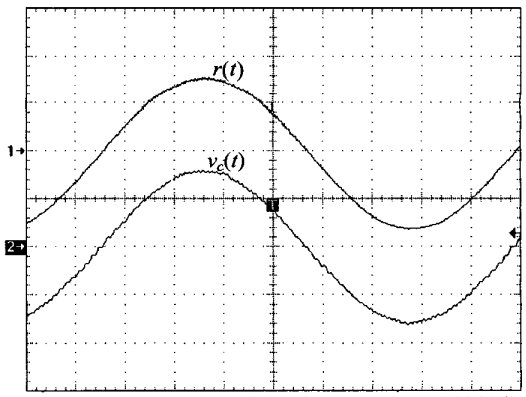

Fig. 14 – Reference voltage (5 V/div) and output voltage (100 V/div) for nominal resistive load. Time scale 2 ms/div.

To check the response of the proposed control scheme for a nonlinear load, the resistive load was replaced by a rectifier with RC filter, as shown in Fig. 12. Fig. 15 shows the output voltage $v_c(t)$ (THD = 2.41%) and load current $i_L(t)$ waveforms for a rectifier-RC load. Fig. 16 presents the reference voltage $r(t)$ and output voltage $v_c(t)$ waveforms for the same rectifier-RC load, to demonstrate the tracking capability of this controller for nonlinear loads. Fig. 17 shows the output voltage $v_c(t)$ and the input filter voltage $v_{in}(t)$ waveforms for this nonlinear load.

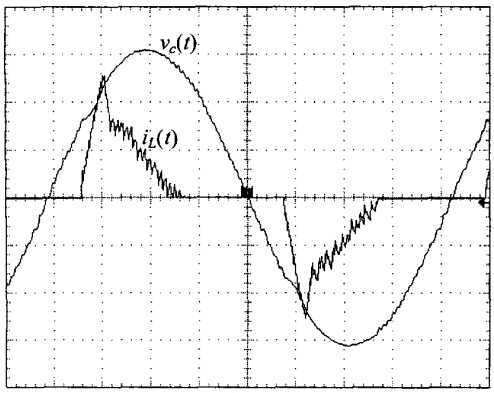

Fig. 15 – Output voltage (50 V/div) and load current (10 A/div) for a rectifier-RC load. Time scale 2 ms/div.

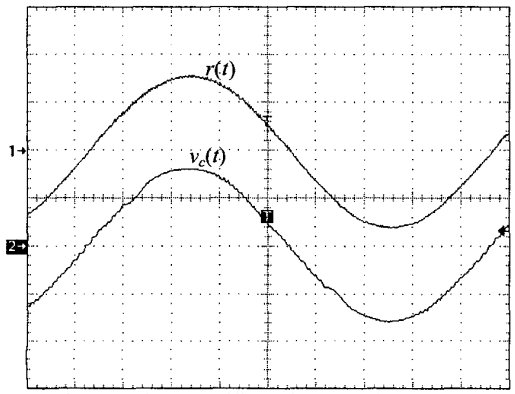

Fig. 16 – Reference voltage (5 V/div) and output voltage (100 V/div) for a rectifier-RC load. Time scale 2 ms/div.

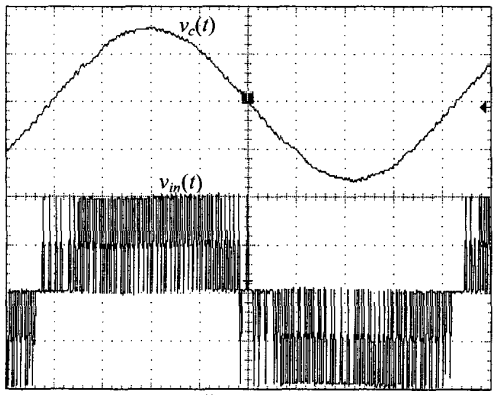

Fig. 17 – Output voltage (100V/div) and filter input voltage (100V/div) for a rectifier-RC load. Time scale 2 ms/div.

VI. Conclusions

This paper describes a predictive PID controller including a feedforward and a repetitive control actions for UPS applications. Simulation results show that the proposed control scheme can minimize distortions resulted by nonlinear cyclic loads and unmodeled dynamics. Stability analysis accomplished in this paper demonstrate that the closed-loop system is stable for any resistive load. Besides, the use of a predictive controller makes possible the implementation of the control law on a low cost microcontroller at low and medium switching frequencies. Even with the limitations of the processing speed and fixed-point routines, this control algorithm presents a good performance. Furthermore, it has been experimentally demonstrated the good performance of the proposed controller under different load conditions.

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