Automatic Adjustment of Transformer and Phase-Shifter Taps in the Newton Power Flow

NORRIS M. PETERSON, MEMBER, IEEE, AND W. SCOTT MEYER, MEMBER, IEEE

***Abstract*—In recent years, Newton's method has been generally accepted as a superior algorithm for solving power flow problems. But the introduction of generator var limits, area-interchange control, load-tap-changing (LTC) transformers, and variable phase shifting transformers has significantly increased the solution times (compared with a fixed system). This paper shows how LTC transformers and variable phase-shifting transformers may be automatically adjusted by the Newton algorithm, thus complementing a previous inclusion of the area interchange control variables within the Newton algorithm [1]. Tests of the new method for adjusting LTC transformers and phase shifters indicate a significant overall saving in solution time.**

Introduction

FOR an N-node power system the nodal admittance power flow equations can be formulated as

$$(P_k + jQ_k) = \boldsymbol{E}_k \sum_{m=1}^{N} \boldsymbol{E}_m{}^* \boldsymbol{Y}_{km}{}^* \tag{1}$$

where

$P_k + jQ_k$	injected power into node k
$\boldsymbol{E}_k$	voltage at node k
$\boldsymbol{Y}_{km}$	negative of admittance between nodes k and m, $k \neq m$
$\boldsymbol{Y}_{kk}$	summation of all admittances terminating on node k
*	complex conjugate
j	$= \sqrt{-1}$.

The well-established Newton power flow method involves repeated direct solution of a system of linear equations derived from (1) for the node voltages $\boldsymbol{E}_k$. Early development and testing were performed by Van Ness and Griffin [2], and their notation will be used throughout. With the development of optimally ordered elimination methods by Sato and Tinney [3], the Newton power flow has become the generally superior power flow method [4].

To review the basic ideas and notation, consider the sample system of Fig. 1. Node 1 is a generation node where real power and voltage magnitude are constrained; nodes 2–5 are load nodes, and node 6 is the swing (reference) node having known voltage magnitude and angle. T_4 and T_5 denote load-tap-changing (LTC) transformers with tap settings t_4 and t_5, respectively; these are to be adjusted to regulate E_4 and E_5 (the voltage magnitudes at nodes 4 and 5) at values $E_4{}^{\text{reg}}$ and $E_5{}^{\text{reg}}$, respectively. Φ_{23} is a phase-shifting transformer with setting ϕ_{23}; it is to be adjusted to regulate the real power flow P_{23} in branch (2,3) to value $P_{23}{}^{\text{reg}}$.

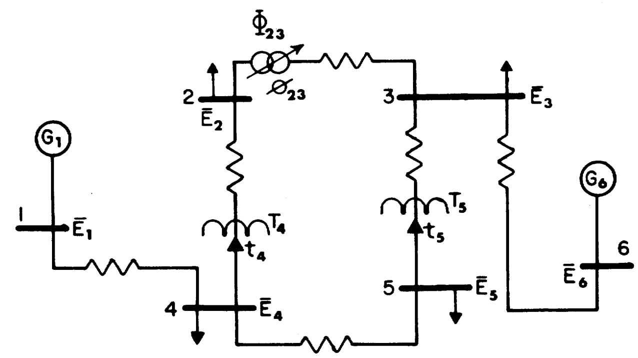

Fig. 1. Sample system.

Present solution methods start by fixing t_4, t_5, and ϕ_{23} at reasonable values, and then solve the linearized power flow equations (2a) for node-voltage corrections ΔE_i and $\Delta\delta_i$.

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta Q_2 \\ \Delta P_3 \\ \Delta Q_3 \\ \Delta P_4 \\ \Delta Q_4 \\ \Delta P_5 \\ \Delta Q_5 \end{bmatrix} = \begin{bmatrix} H_{11} & & & H_{14}N_{14} & \\ & H_{22}N_{22} & H_{23}N_{23} & H_{24}N_{24} & \\ & J_{22}L_{22} & J_{23}L_{23} & J_{24}L_{24} & \\ & H_{32}N_{32} & H_{33}N_{33} & & H_{35}N_{35} \\ & J_{32}L_{32} & J_{33}L_{33} & & J_{35}L_{35} \\ H_{41} & H_{42}N_{42} & & H_{44}N_{44} & H_{45}N_{45} \\ J_{41} & J_{42}L_{42} & & J_{44}L_{44} & J_{45}L_{45} \\ & & H_{53}N_{53} & H_{54}N_{54} & H_{55}N_{55} \\ & & J_{53}L_{53} & J_{54}L_{54} & J_{55}L_{55} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \dfrac{\Delta E_2}{E_2} \\ \Delta\delta_3 \\ \dfrac{\Delta E_3}{E_3} \\ \Delta\delta_4 \\ \dfrac{\Delta E_4}{E_4} \\ \Delta\delta_5 \\ \dfrac{\Delta E_5}{E_5} \end{bmatrix} \tag{2a}$$

$$H_{km} = \frac{\partial P_k}{\partial \delta_m}, \quad N_{km} = \frac{\partial P_k}{\partial E_m} E_m$$

$$J_{km} = \frac{\partial Q_k}{\partial \delta_m}, \quad L_{km} = \frac{\partial Q_k}{\partial E_m} E_m. \tag{2b}$$

Formulas for elements of (2), the Jacobian matrix, are given in Appendix I. Using present methods, problems containing LTC transformers and/or variable phase shifting transformers would be solved as follows. After each solution for ΔE_i and $\Delta\delta_i$, tap settings t_4, t_5, and ϕ_{23} would be adjusted by a displacement technique (reminiscent of Gauss–Seidel methods) to better meet

Paper 70 TP 160-PWR, recommended and approved by the Power System Engineering Committee of the IEEE Power Group for presentation at the IEEE Winter Power Meeting, New York, N. Y., January 25–30, 1970. Manuscript submitted July 29, 1969; made available for printing November 21, 1969.

N. M. Peterson was with the System Planning Department, Northern States Power Company, Minneapolis, Minn. He is now with Systems Control, Inc. Palo Alto, Calif. 94306.

W. S. Meyer is with the University of Minnesota, Minneapolis, Minn. 55455.

the desired operating conditions:

$$t_i^{\text{new}} = t_i^{\text{old}} + c_i(E_i^{\text{reg}} - E_i), \quad i = 4, 5$$
$$\phi_{23}^{\text{new}} = \phi_{23}^{\text{old}} + d_{23}(P_{23}^{\text{reg}} - P_{23}). \tag{3}$$

Acceleration factors c_4, c_5, and d_{23} are empirically determined, often resulting in slow and/or unpredictable overall convergence for the problem.

Automatic Adjustments

Continuing with the sample system, t_4, t_5, and ϕ_{23} can be "automatically" adjusted if included as variables, rather than as constants, in the constraint equations. The two variable types require two procedures: automatic LTC tap changing and automatic phase-shifter adjustments.

Automatic LTC

An LTC such as T_4 is operated so that t_4 is adjusted (within limits) to constrain E_4 at value E_4^{reg}. For this mode of operation E_4 is a constant and t_4 should replace it in (2a) as a variable. Hence, any LTC-regulated node i has its voltage E_i replaced as a variable by t_i. Incorporating these changes, (2a) is modified to become (4a):

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta Q_2 \\ \Delta P_3 \\ \Delta Q_3 \\ \Delta P_4 \\ \Delta Q_4 \\ \Delta P_5 \\ \Delta Q_5 \end{bmatrix} = \begin{bmatrix} H_{11} & & & H_{14}O & \\ & H_{22}N_{22} & H_{23}N_{23} & H_{24}C_{24} & \\ & J_{22}L_{22} & J_{23}L_{23} & J_{24}D_{24} & \\ & H_{32}N_{32} & H_{33}N_{33} & & H_{35}C_{35} \\ & J_{32}L_{32} & J_{33}L_{33} & & J_{35}D_{35} \\ H_{41} & H_{42}N_{42} & & H_{44}C_{44} & H_{45}O \\ J_{41} & J_{42}L_{42} & & J_{44}D_{44} & J_{45}O \\ & & H_{53}N_{53} & H_{54}O & H_{55}C_{55} \\ & & J_{53}L_{53} & J_{54}O & J_{55}D_{55} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \frac{\Delta E_2}{E_2} \\ \Delta\delta_3 \\ \frac{\Delta E_3}{E_3} \\ \Delta\delta_4 \\ \frac{\Delta t_4}{t_4} \\ \Delta\delta_5 \\ \frac{\Delta t_5}{t_5} \end{bmatrix} \tag{4a}$$

$$C_{km} = \frac{\partial P_k}{\partial t_m} t_m$$
$$D_{km} = \frac{\partial Q_k}{\partial t_m} t_m. \tag{4b}$$

The terms C_{km} and D_{km} replace the usual terms N_{km} and L_{km}, respectively, if node m is regulated by LTC transformer T_m whose tap is t_m. Because the variable t_m is not a node variable in the same sense as E_m 1) C_{km} and D_{km} are not only always zero when no branch (k,m) exists for the network, but 2) C_{km} and D_{km} are also zero when branch (k,m) does not represent the LTC transformer that is regulating node m. Each LTC-regulated-node column in (4a) thus usually has only one nonzero off-diagonal C_{km} and/or D_{km} term.

The nonzero block pattern of the Jacobian matrix in (4a) is like that of (2a), so elimination may proceed in the usual order (based on network topology) [3], [4]. Appendix II details formulas for the C_{km} and D_{km} elements, and it is seen that only minor changes from the elements in (2a) exist. As a result, programming modifications are minor, and the changing of LTCs between fixed and variable types when LTC limits are encountered during the course of the solution is easily done.

Automatic Phase-Shifter Adjustments

Phase shifter Φ_{23} in Fig. 1 is typically operated to constrain P_{23} at a value P_{23}^{reg}. But since P_{23} is not a regular problem variable, a variable switch as was done for LTCs is not possible. ϕ_{23} enters as an extra problem variable, and the linearized relation $\Delta P_{23} = P_{23}^{\text{reg}} - P_{23}$ is an additional constraint equation. The resulting linearized Newton equations appear in (5a). Note that both LTCs and the phase shifter will now be automatically adjusted.

$$\begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \Delta Q_2 \\ \Delta P_3 \\ \Delta Q_3 \\ \Delta P_4 \\ \Delta Q_4 \\ \Delta P_5 \\ \Delta Q_5 \\ \Delta P_{23} \end{bmatrix} = \begin{bmatrix} H_{11} & & & H_{14}O & & \\ & H_{22}N_{22} & H_{23}'N_{23}' & H_{24}C_{24} & & E_{2,23} \\ & J_{22}L_{22} & J_{23}'L_{23}' & J_{24}D_{24} & & F_{2,23} \\ & H_{32}'N_{32}' & H_{33}N_{33} & & H_{35}C_{35} & E_{3,23} \\ & J_{32}'L_{32}' & J_{33}L_{33} & & J_{35}D_{35} & F_{3,23} \\ H_{41} & H_{42}N_{42} & & H_{44}C_{44} & H_{45}O & \\ J_{41} & J_{42}L_{42} & & J_{44}D_{44} & J_{45}O & \\ & & H_{53}N_{53} & H_{54}O & H_{55}C_{55} & \\ & & J_{53}L_{53} & J_{54}O & J_{55}D_{55} & \\ & H_{23,2}'N_{23,2}' & H_{23,3}'N_{23,3}' & & & E_{23,23} \end{bmatrix} \begin{bmatrix} \Delta\delta_1 \\ \Delta\delta_2 \\ \frac{\Delta E_2}{E_2} \\ \Delta\delta_3 \\ \frac{\Delta E_3}{E_3} \\ \Delta\delta_4 \\ \frac{\Delta t_4}{t_4} \\ \delta\Delta_5 \\ \frac{\Delta t_5}{t_5} \\ \Delta\phi_{23} \end{bmatrix} \tag{5a}$$

$$E_{k,km} = \frac{\partial P_k}{\partial \phi_{km}}$$
$$F_{k,km} = \frac{\partial Q_k}{\partial \phi_{km}}. \tag{5b}$$

Terms in the Jacobian matrix that are associated with the phase shifter are distinguished by primes and as $E_{k,km}$ and $F_{k,km}$ in (5a). Detailed formulas for their calculation are provided in Appendix III.

Test Results

Initial tests of the automatic LTC adjustment scheme indicate typically a 2 to 1 speed advantage over the old (present) method, if generator var limits are not imposed. Fig. 2 shows convergence behavior for different solution techniques applied to a 347-node 572-branch system with 91 generator nodes and 8 LTC transformers. All generators were assumed to have infinite var limits, and no area interchange control was applied. The same problem was solved in three different ways:

1) as a "fixed system," with all transformer tap settings permanently fixed (no LTC transformers);

2) by the old method, using the present NSP Newton power flow program; the 8 LTC transformer taps were adjusted using (3) (tap adjustments started after the third iteration);

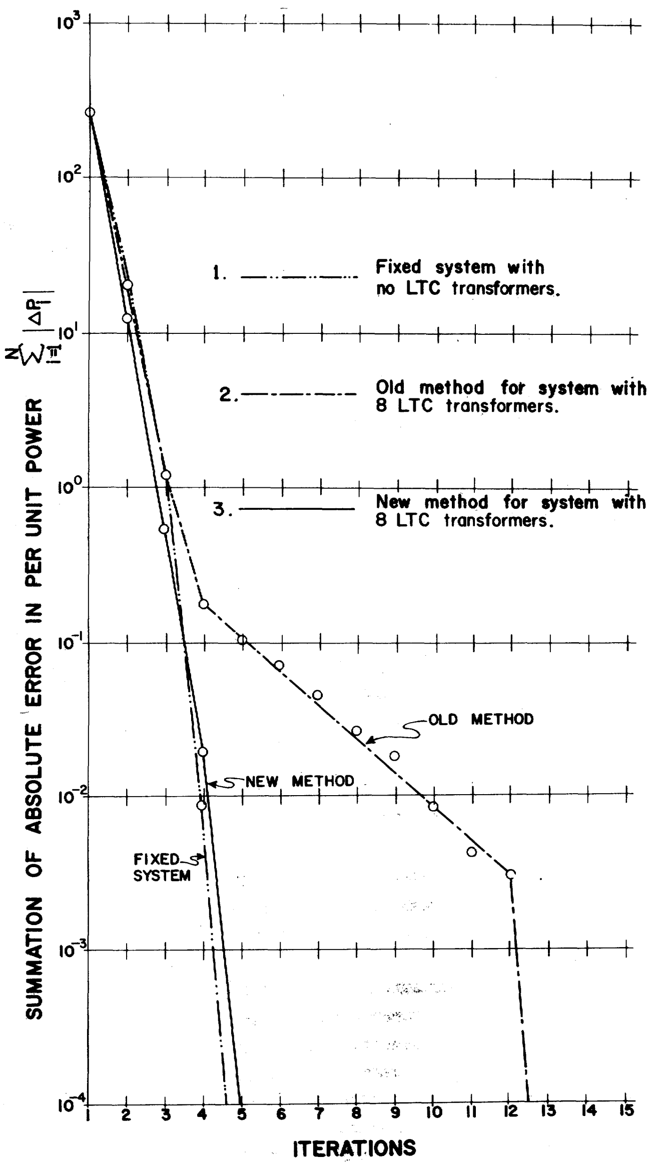

Fig. 2. Solution plot for 347-node system.

3) by the new method, with automatic adjustment of the 8 LTC transformer taps as in (4a).

An examination of Fig. 2 shows that convergence using the new LTC adjustment scheme is virtually identical to the well-known quadratic convergence for the fixed system. This is contrasted with the much slower, linear convergence of the old method (curve 2). The exactness of the new method is also worth emphasizing; LTC-regulated nodes have their voltage maintained at "exactly" the desired values. This is in contrast to the old method (curve 3), where taps are continually changed after each Newton iteration until all changes in (3) are less than 10^{-3} pu; then taps are permanently fixed (iteration 12 of Fig. 2), and a final solution is obtained after an additional iteration (iteration 13). Although of course taps on the actual transformers are discrete, at least one is now able to "exactly" solve the problem being formulated.

As of June 1969, the testing of automatic phase-shifter adjustments by the new method has been confined to small test systems only. By inserting four phase shifters into the 20-node NSP test system and finding solutions for different values of regulated line flows, it was observed that convergence behavior is similar to that for the corresponding fixed system. Solutions are found in four or five iterations from a flat start.

Summary and Conclusions

A new method has been presented for the automatic adjustment of LTC and phase-shifting transformers within Newton's method. Experimentally observed convergence rates appear similar to those for fixed systems (requiring about five iterations), thereby representing typically a 2-to-1 improvement over existing displacement procedures. Such improvement is based on ignoring generator var limits—the inclusion of which would further complicate any comparison in unpredictable, system-dependent ways—but it is believed that the new method offers substantial improvement for many if not most power flow problems. It is especially practical because its incorporation into existing Newton power flow programs requires (generally) only minor modifications.

Appendix I

Formulation of Conventional Jacobian Terms

For an electrical network in the steady state, phasor node voltages, injected currents, and admittances are complex values written respectively as

$$\begin{aligned} \boldsymbol{E}_m &= e_m + jf_m \\ \boldsymbol{I}_m &= a_m + jb_m \\ \boldsymbol{Y}_{km} &= G_{km} + jB_{km}. \end{aligned} \tag{6}$$

The injected node-power equation is

$$(P_k + jQ_k) = \boldsymbol{E}_k \sum_{m=1}^{N} \boldsymbol{E}_m{}^* \boldsymbol{Y}_{km}{}^*. \tag{7}$$

Elements of the Jacobian matrix (2a) are, for $k \neq m$

$$\begin{aligned} a_m + jb_m &= (e_m + jf_m)(G_{km} + jB_{km}) \\ H_{km} = L_{km} &= a_m f_k - b_m e_k \\ N_{km} = -J_{km} &= a_m e_k + b_m f_k \end{aligned} \tag{8}$$

and the diagonal elements are

$$\begin{aligned} H_{kk} &= -Q_k - B_{kk}E_k{}^2 \\ L_{kk} &= Q_k - B_{kk}E_k{}^2 \\ N_{kk} &= P_k + G_{kk}E_k{}^2 \\ J_{kk} &= P_k - G_{kk}E_k{}^2. \end{aligned} \tag{9}$$

Appendix II

Terms Associated with LTC Transformers

If node m is regulated by LTC transformer T_m (with tap t_m), the conventions shown in Fig. 3 can be assumed. T_m affects the self- and mutual admittances of nodes k and m by

$$\begin{aligned} \boldsymbol{Y}_{km}{}^0 &= (G_{km}{}^0 + jB_{km}{}^0) = -\boldsymbol{Y}^0 \\ \boldsymbol{Y}_{km} &= \boldsymbol{Y}_{mk} = t_m \boldsymbol{Y}_{km}{}^0 \\ \boldsymbol{Y}_{mm} &= \boldsymbol{Y}_{mm}{}' + t_m{}^2 \boldsymbol{Y}^0 \\ \boldsymbol{Y}_{kk} &= \boldsymbol{Y}_{kk}{}' + \boldsymbol{Y}^0. \end{aligned} \tag{10}$$

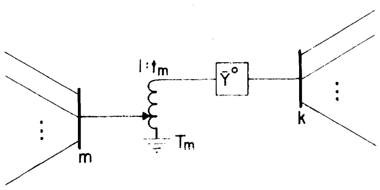

Fig. 3. Representation of LTC transformers.

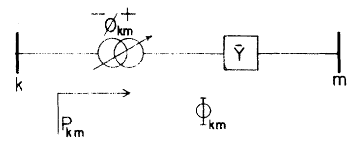

Fig. 4. Variable phase-shifting transformer representation.

The power flow equations for nodes k and m are

$$(P_k + jQ_k) = E_k^2 Y_{kk}^* + E_k E_m^* t_m Y_{km}^{0\,*} + E_k \sum_{\substack{i=1\\ i\neq k\\ i\neq m}}^{N} E_i^* Y_{ki}^*$$

$$(P_m + jQ_m) = E_m^2(t_m^2 Y^{0\,*} + Y_{mm}'^{\,*})$$
$$+ E_m E_k^* t_m Y_{km}^{0\,*} + E_m \sum_{\substack{i=1\\ i\neq m\\ i\neq k}}^{N} E_i^* Y_{mi}^*. \quad (11)$$

Differentiating (11) as in (4b), the off-diagonal terms where $k \neq m$ are

$$a_m^0 + jb_m^0 = (e_m + jf_m)(G_{km}^0 + jB_{km}^0)$$
$$C_{km} = (a_m^0 e_k + b_m^0 f_k)t_m = N_{km}$$
$$D_{km} = (a_m^0 f_k - b_m^0 e_k)t_m = L_{km} \quad (12)$$

and the diagonal terms are

$$C_{mm} = -2G_{km}^0 E_m^2 t_m^2 + (a_k^0 e_m + b_k^0 f_m)t_m$$
$$D_{mm} = 2B_{km}^0 E_m^2 t_m^2 + (a_k^0 f_m - b_k^0 e_m)t_m. \quad (13)$$

Appendix III

Terms Associated with Variable Phase-Shifting Transformer

If nodes k and m are the terminal of the km phase-shifting transformer and if the phase shifter is on the k side of the Y admittance, the phase-shifting transformer can be represented as shown in Fig. 4.

The admittance relationships are

$$Y = Y(\cos\theta_{km} + j\sin\theta_{km})$$
$$Y_{km} = G_{km} + jB_{km}$$
$$= -Y[\cos(\theta_{km} - \phi_{km}) + j\sin(\theta_{km} - \phi_{km})]$$
$$Y_{mk} = G_{mk} + jB_{mk}$$
$$= -Y[\cos(\theta_{mk} + \phi_{km}) + j\sin(\theta_{mk} + \phi_{km})]$$
$$Y_{kk} = Y_{kk}' + Y$$
$$Y_{mm} = Y_{mm}' + Y. \quad (14)$$

The calculation of the terms of the Jacobian matrix (5a) that are affected by the presence of a phase-shifting transformer can be divided into three categories.

1) The terms in the k and m rows of the Jacobian matrix shown as primes in (5a) are calculated by (8) using the admittances given in (14).

2) The off-diagonal terms in the row of the P_{km}^{reg} constraint equation are

$$H_{km,m} = -H_{km,k} = H_{km}$$
$$N_{km,m} = N_{km}.$$
$$N_{km,k} = N_{km} + 2E_k^2 Y\cos\theta_{km} \quad (15)$$

3) The terms in the column of the P_{km}^{reg} constraint equation are calculated using (5b). The off-diagonal terms are

$$a_m + jb_m = (e_m + jf_m)(G_{km} + jB_{km})$$
$$E_{k,km} = -a_m f_k + b_m e_k = -H_{km}$$
$$F_{k,km} = +a_m e_k + b_m f_k = J_{km}$$
$$a_k + jb_k = (e_k + jf_k)(G_{mk} + jB_{mk})$$
$$E_{m,km} = +a_k f_m - b_k e_m = H_{mk}$$
$$F_{m,km} = -a_k e_m - b_k f_m = J_{mk} \quad (16)$$

and the diagonal term is

$$E_{km,km} = E_{k,km}. \quad (17)$$

References

[1] J. P. Britton, "Improved area-interchange control for Newton's method load flows," *IEEE Trans. Power App. Syst.*, vol. PAS-88, pp. 1577–1581, October 1969.
[2] J. E. Van Ness and J. H. Griffin, "Elimination methods for load flow studies," *AIEE Trans. (Power App. Syst.)*, vol. 80, pp. 299–304, June 1961.
[3] N. Sato and W. F. Tinney, "Techniques of exploiting the sparsity of the network admittance matrix," *IEEE Trans. Power App. Syst.*, vol. 82, pp. 944–950, December 1963.
[4] W. F. Tinney and C. E. Hart, "Power flow solution by Newton's method," *IEEE Trans. Power App. Syst.*, vol. PAS-86, pp. 1449–1460, November 1967.

Discussion

D. K. Subramanian (Indian Institute of Science, Bangalore, India): The authors have improved Newton's method of incorporating additional automatic facilities that had been hampering the convergence. This will aid in the use of these modified methods for on-line load flows. The authors' opinions are sought on the following points.

1) When the voltages and powers corrected by tap changers and phase shifters are not fixed at particular values but are kept within specified limits, can this method be suitably modified to incorporate the changes? Also, can minimization methods minimizing losses be used in conjunction with these methods?

2) From Fig. 2 it is found that both old and new methods converge with nearly the same speed up to an accuracy of 0.1. So for

Manuscript received February 9, 1970.

a system with ranges specified for controlled voltages and powers, if the old method is used up to an accuracy of 0.1 and then the new method is resorted to, will the computations be reduced?

3) In cases where phase shifters are employed on a line with the voltages of the buses connected to that line kept at fixed values, can phase shifters then be eliminated completely from the system equations?

I thank the authors for making available a copy of the paper.

W. F. Tinney and **W. L. Powell** (Bonneville Power Administration, Portland, Oreg.): This important paper describes an improved scheme for the control of transformers and phase shifters in Newton's method of power flow solution. A similar scheme has been implemented in the Bonneville Power Administration's new 2000-node power flow program. This new scheme, which reduces the overall solution time for average problems by about 50-percent, further establishes Newton's method as the definitive algorithm for power flow solution.

The paper shows the modifications of the Jacobian matrix for the transformer control scheme. A practical implementation must also consider tap limits, simultaneous control of a bus by both a transformer and a reactive source, and the orientation of a transformer's variable-tap side with respect to the controlled quantity.

Our experience indicates that the handling of tap limits deserves careful attention. The derivation of the Jacobian matrix does not take tap limits into account. As a result, the Newton adjustment may cause transformer taps to go beyond their limits. If these excessive adjustments are curtailed, the compensating nature of the remaining dependent variable adjustments is destroyed. The aggregate effect of such curtailments may induce greater error in the system than existed prior to the adjustment. This adverse effect may be partially alleviated during the back substitution by replacing the computed adjustments with smaller truncated adjustments and allowing their effects to be propagated to the remaining adjustments. Limiting the size of the error to be corrected in one Newton adjustment cycle is another way to prevent tap ratios from exceeding their limits by large amounts.

The control of a bus voltage by the simultaneous adjustment of a transformer and a reactive source is an actual operating situation that can cause difficulty in the power flow solution. Such controls operate under priorities: the reactive source is used first, then the transformer tap adjustment. In order to circumvent difficulties in this situation it has been found expedient to introduce a temporary constraint on reactive flow through the transformer (similar to the phase-shifter control of real power through itself). This causes the system to first utilize the reactive sources most effectively. When a reactive source reaches its limit, the transformer reactive flow constraint is removed and direct transformer control is assumed.

The transformer and phase-shifter adjustments are examples of single-criterion controls. A control variable is adjusted to maintain another functionally dependent variable at a specified value. Examples other than those used thus far can be envisioned and may be of potential value. The equations for the direct control of such adjustments through the Newton algorithm can be derived quite readily, but their implementation can become exceedingly difficult.

Implementation strategies depend upon whether the controlled variable is a regular system variable or a new variable and whether the control equation has a nonzero diagonal term or not. If the controlled variable is a system variable and the control equation has a nonzero diagonal, then the control equation can be substituted for the normal equation. If its diagonal is zero, the equation can be substituted but its order of elimination may have to be changed from that of the equation it replaces and the strategy for its elimination can become very complicated. This would be the situation for a transformer controlling a remote node voltage. If the controlled variable is not a regular system variable, the original system must be augmented by the control equation. If the augmented equation

Manuscript received February 12, 1970.

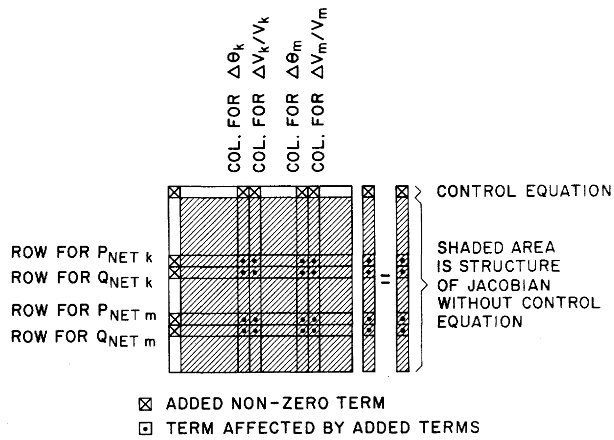

Fig. 5. Structure of Jacobian matrix with one control equation.

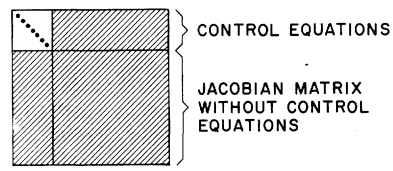

Fig. 6. Jacobian matrix with several control equations.

has a nonzero diagonal, its elimination can take place anywhere in the ordering that is advantageous from the standpoint of programming convenience or efficiency.

The programming for single-criterion control equations that augment the original system and have nonzero diagonal terms (e.g., a phase shifter controlling the power flow through itself) can be accomplished with very little disturbance of the basic algorithm. One control equation of this type is shown added to the original system in Fig. 5. Note that its row and column have symmetrically located nonzero diagonal terms only in positions corresponding to regular problem variables associated with nodes k and m. The significance of this structure is twofold: 1) since the added equation has a nonzero diagonal term its elimination can always be performed without pivot search; 2) only those elements with a dot are directly affected by the elimination of the control equation. Since these dotted elements are nonzero from the beginning, no new entries are created when the control equation is first in the order of elimination. These properties are true for any number of control equations of this type. Within the system of control equations, no elimination process is necessary since the relevant submatrix in the Jacobian matrix is diagonal, as shown in Fig. 6. The matrix need not be actually augmented. Upon reaching the rows for $P_{NET\ k}$, $Q_{NET\ k}$, $P_{NET\ m}$, $Q_{NET\ m}$, the dotted elements are modified to eliminate the control variable. The correction for this variable is found after the completed back substitution from the control equation.

Slobodan T. Despotović (Electrotechnical Research Institute Nikola Tesla, Belgrade, Yugoslavia): The introduction of LTC transformers and variable phase shifting transformers into any current method of solving power flow problems significantly increases the solution time. This paper gives a new and very simple method for the automatic adjustment of such devices within the Newton power flow algorithm and indicates that significant overall saving in solution time is thereby possible.

The experimentally obtained results given in Fig. 2 show that convergence is almost identical to that for the fixed system having no LTC transformers. In this way, the new method for automatic

Manuscript received February 13, 1970.

adjustment of LTC transformers presents significant improvement for the solution of power flow problems.

Incorporation of the new technique into the Newton algorithm requires only minor modifications, yet gives significant saving in solution time. I compliment the authors for incorporating the new technique into existing Newton power flow programs.

Norris M. Peterson and **W. Scott Meyer**: The authors wish to thank the discussers for their constructive comments and interest.

Mr. Subramanian's suggestion of using the proposed procedure to maintain node voltage or power flows within certain predetermined limits would require a reversal of the type switching and priority schedules usually employed. The LTC transformer and phase-shifter taps would then remain fixed until voltage or power flow inequality constraints were violated, at which point the proper control equations would be substituted or added to adjust the taps to maintain the appropriate limits. It must be noted that such a scheme permits a wide range of solutions for a given power flow problem and appears to be unnecessary due to the rapid convergence of the proposed new method.

The proposed method can be used in conjunction with minimization or optimization techniques, provided that the voltage and/or power flow regulation thereby provided is indeed a constraint to the problem. For such cases, the proposed Newton power flow solutions fit naturally into gradient optimization procedures [5]. But if such regulation is not desired, the proposed adjustment procedure is of course inapplicable. The LTC regulation of node voltage magnitude will not, in general, produce the minimum possible system loss, for example. The same would generally be true of constraining the real power flow through phase shifters, which is generally not compatible with general economic dispatch.

A question was also raised by Mr. Subramanian about the comparative rates of convergence during early stages of the solution. The old method is similar to that described in [4], where the problem is treated as a fixed system (LTC taps fixed) until after the third iteration. It is thus only from the fourth iteration on that the old method is actually solving the problem that was posed, and comparisons must be made for this portion of the solution process. It should be remembered that the computational effort per iteration for the new and old methods is nearly identical.

Mr. Subramanian's final point concerns the removal of phase shifters. If voltage magnitude is maintained constant at one terminal this could indeed be accomplished by opening the element across the ideal phase shifter and inserting two generators having opposite injected power outputs. The reactive generation requirement could be calculated to maintain the voltage magnitude at the voltage-controlled node. But such elimination is not completely general in that it requires that a phase shifter control both the real power flow and either the reactive flow or the voltage magnitude at the terminal node. Further, such an approach would not significantly affect problem storage requirements (see comments by Mr. Tinney and Mr. Powell), and would not in general converge any faster than the proposed scheme. Remote regulation also would not be possible using the suggested approach.

The authors are especially grateful to Mr. Tinney and Mr. Powell for their very relevant comments. We are gratified to find that the Bonneville Power Administration has experienced solution time reductions similar to those stated in the paper and confirmed by our experimental and production runs. The authors concur that tap limits and the other points mentioned must be efficiently handled in order to make the method practical for production programs; these points are considered in the following paragraphs.

Certainly the first and most important consideration is that of constraint limits. As the discussers state, Newton's method cannot directly take limits into account; if large tap changes Δt occur during the back substitution, it is possible for the resulting transformer taps to greatly exceed their predefined limits, thereby resulting in large ΔP and ΔQ residuals when the tap limits are actually imposed.

Manuscript received March 30, 1970.

In the NSP program we have chosen to limit the magnitude of all variable changes during the back-substitution step, with present bounds being $|\Delta V|$ and $|\Delta t| \lesssim 0.1$ pu, and $|\Delta\delta| \lesssim 30°$. For certain systems these restrictions might retard convergence during the early stages of the solution; but they seem to increase the probability of solving other cases that would otherwise diverge or oscillate due to the imposition of excessive variable corrections. Naturally the encountering of (or backing off from) tap limits requires the use of transformer-type switching analogous to the case for generators, where reactive power limits are involved.

We have chosen to handle the regulation priority question (where both a generator and an LTC transformer regulate the same node voltage magnitude) a little differently from the way suggested by the discussers. Although we assume that a generator will be given first priority to regulate the node voltage magnitude, we have chosen to hold the associated LTC transformer tap constant so long as generator reactive limits are not reached (rather than constrain the reactive power flow). The LTC transformer tap would only become a variable if and when the generator reaches its reactive limit.

Mr. Tinney and Mr. Powell are absolutely correct concerning the ordering of the phase-shifter equations; this recognition, included in [6], is important for maximum elimination efficiency. The suggested order is clearly preferable to that shown in (5a), where fill-in during the elimination generally will occur.

It has been found necessary to develop logic for the NSP program to handle special system configurations such as series and parallel LTC transformer connections. The series condition occurs when two or more LTC transformers are connected in series and one LTC transformer regulates the nonregulated terminal of another LTC transformer. The parallel condition occurs when a node is regulated by two or more LTC transformers that are connected to different nodes. The inclusion of such features makes it possible to handle general topology.

A matter which we feel requires investigation is the area of remote control by the proposed method (i.e., the control of a line flow or a sum of line flows by a remote phase shifter, or the control of a node voltage magnitude by a remote transformer). Although there is no theoretical reason why remote regulation by the proposed method should not be possible, we have not investigated its practicality and feel that three areas may cause difficulty.

1) The inclusion of remote regulation requires that the renumbering strategy be modified to insure that a nonzero diagonal exists (following elimination to the left of the diagonal). For the case of remote voltage regulation by a transformer, nodes forming a path between the remotely controlled node and the controlling transformer, plus the transformer terminal that terminates this path, must all be eliminated before the remotely controlled node. Analogous considerations apply to the phase-shifter case. The development of efficient program logic for the general case is simply stated in words, but it appears to be quite involved.

2) The altered node numbering (as above) will degrade the solution efficiency. Inclusion of remote control by phase shifters does not allow the elimination order as outlined by Mr. Tinney and Mr. Powell, which is most efficient; the general case is similar to that associated with area-interchange equations [1]. The use of transformers to remotely regulate voltage magnitude adds two additional nonzero blocks to the original Jacobian matrix for each such transformer; remote regulation in both cases thus results in an asymmetrical nonzero block pattern for the Jacobian matrix. The inclusion of many remote controls might significantly degrade the problem sparsity and the efficiency of the overall algorithm.

3) Remote regulation should in general require changes in the control variable larger than would be required for local regulation. This might increase convergence difficulties normally associated with constraint limits.

References

[5] H. W. Dommel and W. F. Tinney, "Optimal power flow solutions," *IEEE Trans. Power App. Syst.*, vol. PAS-87, pp. 1866–1876, October 1968.

[6] H. W. Dommel, W. F. Tinney, and W. L. Powell, "Further developments in Newton's method for power system applications," Paper 70 CP 161-PWR, presented at the IEEE Winter Power Meeting, New York, N. Y., January 25–30, 1970.