

Dynamic Model for Linear Induction Motors

Ezio Fernandes da Silva¹ Euler Bueno dos Santos² Paulo C. M. Machado² Marco A. A. de Oliveira²

¹Centro Federal de Educação Tecnológica de Goiás
Rua 75 nº 46, Centro
74055-110 – Goiânia – Goiás - Brazil
efs@cefetgo.br²

Universidade Federal de Goiás
Escola de Engenharia Elétrica
Praça Universitária s/n, Setor Universitário
74605-220 – Goiânia – Goiás - Brazil
ebs@eee.ufg.br

Abstract– This paper presents a mathematical model that describes the dynamic behaviour of a linear induction motor, divided into two portions. The first part represents the model dynamic of conventional induction motor without the end effects while the second portion describes the attenuation caused by the end effects on the linear induction motor.

Nomenclature

v_{xy} voltage.
 i_{xy} current.
 λ_{xy} linkage flux.
 L_x primary inductance.
 L_{lx} leakage inductance.
 L_m magnetizing inductance
 R_x resistance.
 P number of poles.
 D primary length.
 Q factor linked to the primary's length.
 F Thrust force.
 v velocity (m/s).
 τ_p pole pitch.

$x: d, q$ subscripts denoting the synchronous rotating reference frame, d-axis and q-axis respectively.

$y: s, r$ subscripts denoting primary and linor respectively.

1 Introduction

Due to the complexity of the linear induction motor's (LIM) electromagnetic field theory, an analysis of the LIM's equivalent electrical circuit was undertaken, with alterations to the magnetizing inductance and a resistance representing the core losses.

In a LIM with a short primary and infinite linor (secondary), the primary is continuously entering a new linoric region. This new linoric region tends to resist the abrupt increase of the magnetization flux penetration by permitting a gradual accumulation of the magnetizing field density in the gap. The new linor region and its influence

on the magnetic field change the LIM's performance in comparison with the conventional induction motor [2].

To obtain the LIM's equivalent electrical circuit incorporating the end effects, one must quantify these effects during entry and exit from the linor, in respect to the primary.

In the case of no relative movement between the primary and the linor, the LIM's equivalent circuit is equal to the conventional induction motor, since the contribution of the end effects will be relatively small.

When the primary is moving, an analysis of the entry end of the linor, under a uniform magnetic field, reveals that parasite currents will appear in the linor, having the effect of reducing the density of the magnetic field at the entry end of the primary. The linor current is consequently reduced exponentially over time through the linoric time constant.

Since the moving primary passes through the linoric region, the new magnetic field penetrating the linor decreases at the entry end, and increases at the exit end, i.e., the linor currents at the entry end increase very rapidly, mirroring the primary's current, and compensating for the magnetic field caused by the primary, with the effect of zeroing the magnetic flux at the entry end. On the other hand, at the exit end, the linor currents drop very rapidly, with a very short time constant, with respect to the dispersion inductance of the linor.

These transient changes at the entry and exit ends as a function of the linoric time constant, are shown in figure 1.

The spatial distribution of the magnetic flux density along the primary's length depends on the relative velocity between the primary and the linor. For a zero relative velocity, the LIM's primary can be seen as infinite, in which case the end effects may be ignored.

The purpose of this paper is to recommend a technique suitable for describes a dynamic model of the linear LIM, based on the dq model of the equivalent electrical circuit, taking into account the end effects. The proposed method is based in conceptual the primary is simply a rotary-motor primary cut open and rolled flat, this is not simple due to the existence of the end effect.

A velocity inverse function is used to express the effects caused by the LIM's velocity on the magnetization factor of the equivalent electrical circuit [1].

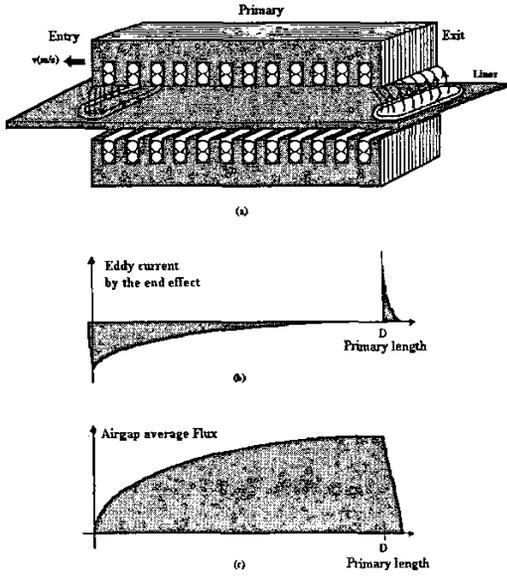


Fig. 1. (a) Linor currents at the entry and exit ends for a given velocity. (b) Linor current values and polarity due to end effects. (c) Gap magnetic flux.

2 Mathematical Model

The end effects are not very noticeable in conventional induction motors. On the other hand, in LIM, these effects become increasingly relevant with the increase in the relative velocity between the primary and the linor. Therefore, the end effects will be analysed as a function of the LIM's velocity.

2.1 Resistance and inductance in the magnetization branch

The end effects behave differently at the entry and exit ends of the LIM. The currents induced in the linor at the entry end decay more slowly than at the exit end, due to its larger time constant.

The Q factor is associated with the length of the primary, and to a certain degree, quantifies the end effects as a function of the velocity v as described by equation (1).

$$Q = \frac{DR_r}{(L_m + L_r)v} \quad (1)$$

Note that the primary's length is inversely dependent on the velocity, i.e., for a zero velocity the primary's length may be considered infinite, and the end effects may be ignored.

As the velocity increases, the primary's length decreases, increasing the end effects, which causes a reduction of the LIM's magnetization current. This effect may be quantified in terms of the magnetization inductance with the equation:

$$L'_m = L_m(1 - f(Q)) \quad (2)$$

where

$$f(Q) = \frac{1 - e^{-Q}}{Q} \quad (3)$$

The resistance in series with the inductance (L'_m) in the magnetization branch of the equivalent electrical circuit of the d-axis, is determined in relation to the increase in losses occurring with the increase of the currents induced at the entry and exit ends of the linor. These losses may be represented as the product of the linor resistance R_r by the factor $f(Q)$, ie, $R_r f(Q)$.

2.2 Equivalent dq Circuit

The linear induction motor's equivalent dq circuit is different from that of a conventional induction motor. The LIM's equivalent electrical circuit is shown in fig.2. The linor entry current on the q-axis maintains a null reference q-axis linor flux, within a narrow error band. The equivalent q-axis electrical circuit for the LIM is identical to that of a conventional induction motor. In this case, the parameters are not changed by the end effects. The gap flux is influenced by the linor d-axis entry currents. Therefore the equivalent d-axis electrical circuit associated with a conventional induction motor can not be used in the analysis of a linear induction motor, if the end effects, as a function of velocity, are to be considered.

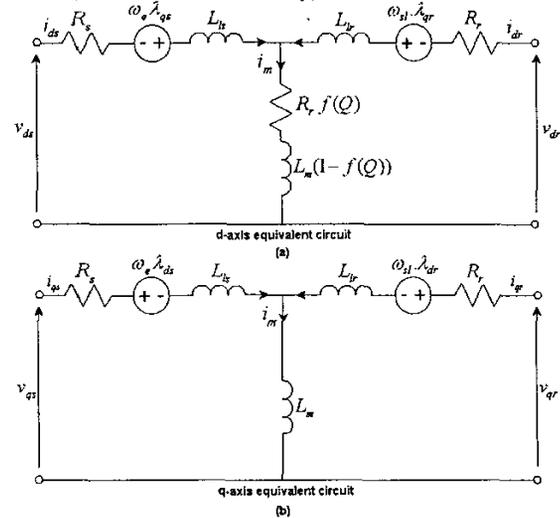


Fig. 2 - The LIM equivalent electrical circuit taking into account end effects.

2.2.1 Linor Voltage and Primary dq Voltage

The primary and linor voltage equations, using a synchronous reference system are given in [1]

$$v_{ds} = R_s i_{ds} + R_r f(Q)(i_{ds} + i_{dr}) + \frac{d\lambda_{ds}}{dt} - \omega_e \lambda_{qs} \quad (4)$$

$$v_{qs} = R_s i_{qs} + \frac{d\lambda_{qs}}{dt} + \omega_e \lambda_{ds} \quad (5)$$

$$v_{dr} = R_r i_{dr} + R_r f(Q)(i_{ds} + i_{dr}) + \frac{d\lambda_{dr}}{dt} - \omega_{sl} \lambda_{qr} \quad (6)$$

$$v_{qr} = R_r i_{qr} + \frac{d\lambda_{qr}}{dt} + \omega_{sl} \lambda_{dr} \quad (7)$$

2.2.2 Primary and Linor Linkage Fluxes

The linkage fluxes are given by the following equations:

$$\lambda_{ds} = L_{ls}i_{ds} + L'_m(i_{ds} + i_{dr}) \quad (8)$$

$$\lambda_{qs} = L_{ls}i_{qs} + L'_m(i_{qs} + i_{qr}) \quad (9)$$

$$\lambda_{dr} = L_{lr}i_{dr} + L'_m(i_{ds} + i_{dr}) \quad (10)$$

$$\lambda_{qr} = L_{lr}i_{qr} + L'_m(i_{qs} + i_{qr}) \quad (11)$$

2.2.3 Thrust Force

The thrust force is given by [1]:

$$F_e = \frac{3\pi}{2\tau_p} \frac{P}{2} (\lambda_{ds}i_{qs} - \lambda_{qs}i_{ds}) \quad (12)$$

2.3 Primary and Linor dq Currents

The d-axis and q-axis currents in the LIM's primary and linor are derived from equations (8) and (10).

Therefore, isolating i_{ds} from (8) and i_{dr} from (10), results in:

$$i_{ds} = \frac{\lambda_{ds} - L'_m i_{dr}}{L_{lr} + L'_m} \quad (13)$$

$$i_{dr} = \frac{\lambda_{dr} - L'_m i_{ds}}{L_{lr} + L'_m} \quad (14)$$

Isolating i_{ds} from (9) and i_{dr} from (11), we have:

$$i_{qs} = \frac{\lambda_{qs} - L'_m i_{qr}}{L_s} \quad (15)$$

$$i_{qr} = \frac{\lambda_{qr} - L'_m i_{qs}}{L_r} \quad (16)$$

Substitution of (14) into (13) results in:

$$i_{ds} = \frac{(L_r - L'_m f(Q))\lambda_{ds} - L'_m(1-f(Q))\lambda_{dr}}{(L_s - L'_m f(Q))(L_r - L'_m f(Q)) - L_m^2(1-f(Q))^2} \quad (17)$$

Substitution of (13) into (14) gives:

$$i_{dr} = \frac{(L_s - L'_m f(Q))\lambda_{ds} - L'_m(1-f(Q))\lambda_{ds}}{(L_s - L'_m f(Q))(L_r - L'_m f(Q)) - L_m^2(1-f(Q))^2} \quad (18)$$

Substituting (16) into (15) results in:

$$i_{qs} = \frac{L_r\lambda_{qs} - L'_m\lambda_{qr}}{L_\sigma} \quad (19)$$

Substituting (15) into (16) gives:

$$i_{qr} = \frac{L_s\lambda_{qs} - L'_m\lambda_{qs}}{L_\sigma} \quad (20)$$

where

$$L_\sigma = L_s L_r - L_m^2 \quad (21)$$

$$L_l = L_{ls} + L_{lr} \quad (22)$$

2.4 Separating the Primary and Linor dq Currents

The LIM's primary and linor dq currents are separable into two portions, of which the first is independent of the end

effects, and the second dependent of the end effects. With this approach, the first portion behaves as a conventional induction motor current and the second portion as a attenuation function due to the LIM's end effects.

To derive these currents, note that the linkage fluxes are also separated into two parts: the first is independent of the end effects and will be indicated by the index "1", while the second describes the linkage flux dependent on the end effects, indicated by the index "2". These fluxes are given by the following equations [3]:

$$\lambda_{ds} = \lambda_{ds1} + \lambda_{ds2} \quad (23)$$

$$\lambda_{qs} = \lambda_{qs1} + \lambda_{qs2} \quad (24)$$

$$\lambda_{dr} = \lambda_{dr1} + \lambda_{dr2} \quad (25)$$

$$\lambda_{qr} = \lambda_{qr1} + \lambda_{qr2} \quad (26)$$

Inserting equations (23) through (26) into (17) through (20), and after some mathematical operations, the following equations emerge:

$$i_{ds} = i_{ds1} + i_{ds2} \quad (27)$$

$$i_{qs} = i_{qs1} + i_{qs2} \quad (28)$$

$$i_{dr} = i_{dr1} + i_{dr2} \quad (29)$$

$$i_{qr} = i_{qr1} + i_{qr2} \quad (30)$$

where:

$$i_{ds1} = \frac{L_r\lambda_{ds1} - L'_m\lambda_{dr1}}{L_\sigma} \quad (31)$$

$$i_{ds2} = A_1 + A_2 + A_3 \quad (32)$$

$$i_{dr1} = \frac{L_s\lambda_{dr1} - L'_m\lambda_{ds1}}{L_\sigma} \quad (33)$$

$$i_{dr2} = A_4 + A_5 + A_6 \quad (34)$$

$$i_{qs1} = \frac{L_r\lambda_{qs1} - L'_m\lambda_{qr1}}{L_\sigma} \quad (35)$$

$$i_{qs2} = \frac{L_r\lambda_{qs2} - L'_m\lambda_{qr2}}{L_\sigma} \quad (36)$$

$$i_{qr1} = \frac{L_s\lambda_{q1s} - L'_m\lambda_{qs1}}{L_\sigma} \quad (37)$$

$$i_{qr2} = \frac{L_s\lambda_{q2s} - L'_m\lambda_{qs2}}{L_\sigma} \quad (38)$$

$$A_1 = \frac{L_r\lambda_{ds2} - L'_m\lambda_{dr2}}{L_\sigma} \quad (39)$$

$$A_2 = \frac{L'_m L_l f(Q) (L_r\lambda_{ds} - L'_m\lambda_{dr})}{L_\sigma (L_\sigma - L'_m L_l f(Q))} \quad (40)$$

$$A_3 = \frac{L'_m L_\sigma f(Q) (\lambda_{ds} - \lambda_{dr})}{L_\sigma (L_\sigma - L'_m L_l f(Q))} \quad (41)$$

$$A_4 = \frac{L_s\lambda_{dr2} - L'_m\lambda_{ds2}}{L_\sigma} \quad (42)$$

$$A_5 = \frac{L'_m L_l f(Q) (L_s\lambda_{dr} - L'_m\lambda_{ds})}{L_\sigma (L_\sigma - L'_m L_l f(Q))} \quad (43)$$

$$A_6 = \frac{L_m L_\sigma f(Q)(\lambda_{dr} - \lambda_{ds})}{L_\sigma (L_\sigma - L_m L_f(Q))} \quad (44)$$

2.5 Separating the Thrust Forces

The LIM's thrust force may be separated into two components, where the first is similar in expression to the conventional induction motor's thrust force and will be indicated by the index "1", while the second portion represents the attenuation caused by the end effect and will be denoted by the index "2". Therefore, the LIM thrust force may be represented by [3][5][6]:

$$F_e = F_{e1} + F_{e2} \quad (53)$$

The component that is similar to the thrust force of a conventional induction motor is given by :

$$F_{e1} = \frac{3\pi}{2\tau_p} \frac{P}{2} (\lambda_{ds1} i_{qs1} - \lambda_{qs1} i_{ds1}) \quad (54)$$

The attenuation thrust force which arises from the end effect is given by:

$$F_{e2} = F_{e2a} - F_{e2b} \quad (55)$$

where:

$$F_{e2a} = \frac{3\pi}{2\tau_p} \frac{P}{2} (\lambda_{ds1} i_{qs2} + \lambda_{ds2} i_{qs1} + \lambda_{ds2} i_{qs2}) \quad (56)$$

$$F_{e2b} = \frac{3\pi}{2\tau_p} \frac{P}{2} (\lambda_{qs1} i_{ds2} - \lambda_{qs2} i_{ds1} - \lambda_{qs2} i_{ds2}) \quad (57)$$

3 Simulation

The LIM parameters used in the dynamics simulation are given in table I and were determined experimentally using the methodology described in [4]. We were then able to work with actual, more accurate values obtained from the motor terminals, using a data acquisition system. Examples include linor and primary resistances and reactances.

TABLE I
Linear Induction Motor Parameters Used

Parameters	Values (Units)
Primary length - D	210 mm
Primary Width	45 mm
Number of Poles - P	2
Pole Pitch - τ_p	105 mm
Linor thickness	4.5 mm
Number of slots	12
Air Gap length	8 mm
Primary resistance- R_s	5.348 Ω
Linor resistance - R_r	11.603 Ω
Primary Inductance - L_s	0.1073 mH
Linor inductance - L_r	0.094618 mH
Magnetizing inductance - L_m	0.09213 mH
Inertia Moment - J	0.00247 Kg \cdot m ²

The SIMULINK® dynamics model used for our simulation of the LIM is shown in fig. 3.

Figure 4 shows the LIM thrust force characteristics, with the free acceleration as a function of slippage, considering a synchronous velocity of 1.95 m/s and a synchronous reference system.

In figures 5 to 7, the reference system has changed, as follows: in the [0,1] second interval, the reference is synchronous, in the (1,2] second interval, the reference is half the synchronous velocity, and between 2 and 3 seconds, the reference is stationary. Note that in the above figures, the thrust forces F_e e F_{e2} have oscillations due to the end effects, in the [1,3] second interval. In figure 7, the thrust force F_{e1} does not present oscillations because it is independent of the end effect. The thrust force (F_e and F_{e2}) oscillations caused by the end effect are not noticeable when a synchronous reference system is used.

Figure 8 contains a graph of the thrust forces F_e , F_{e1} e F_{e2} over time, using a synchronous reference system. A load of 5.5 N was applied from t=1.5 s to t=2.15 s. Note that the thrust force F_{e1} becomes greater than F_e due to compensation required for the thrust force F_{e2} , caused by the end effects.

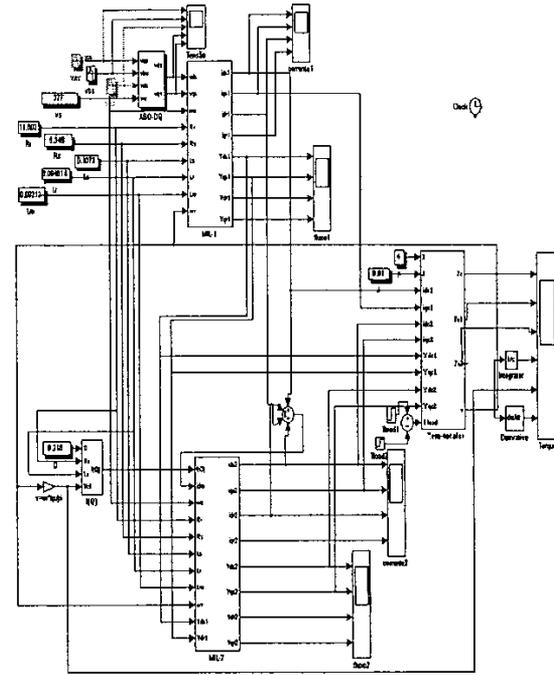


Fig. 3 – LIM dynamics model, simulated using SIMULINK®.

4 Conclusions

We have shown that the dynamics model of the linear induction motor (LIM) can be separated into end-effect-dependent and independent parts, which can be interpreted as representation of the dynamics model of the conventional induction motor (end effects ignored) plus the attenuation due to a thrust force caused by the end effects.

The thrust force oscillations present in the LIM are caused by the end effects, as can be seen in fig. 6. The thrust force behaviour, without the contribution of the end effects, does not present oscillations, as shown in fig. 7.

The research described here is being extended by an investigation of vectorial control methods that take into account the influences upon the LIM of end effects.

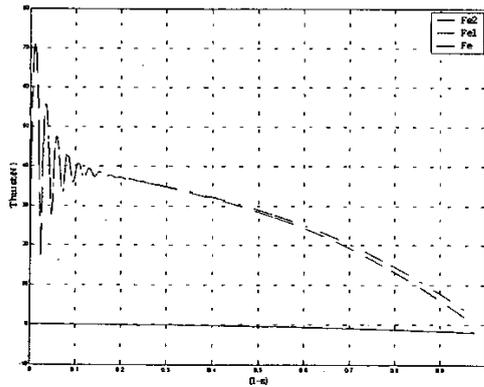


Fig. 4–Thrust forces F_e (dashed line), F_{e1} (dash-dotted line) and F_{e2} (solid line) as a function of slippage, in a synchronous reference system.

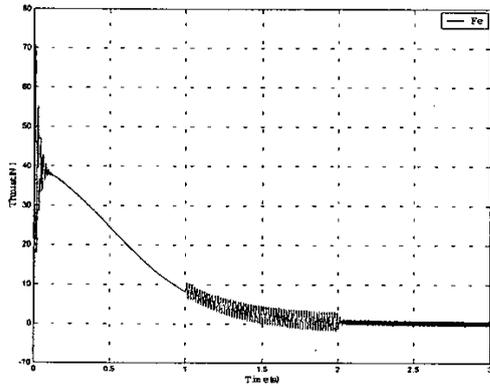


Fig. 5 – Thrust force F_e over time for, respectively, a synchronous ([0,1] seconds), half synchronous ((1,2] seconds) and stationary ((2,3] seconds) reference system.

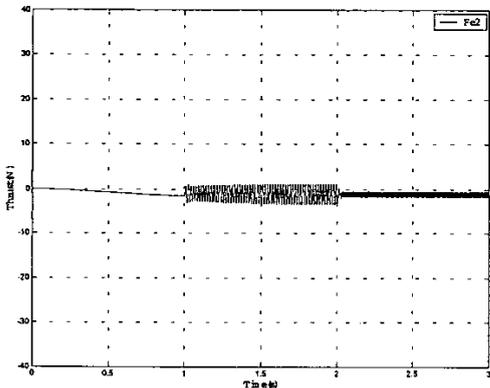


Fig. 6 – Thrust force F_{e2} over time for, respectively, a synchronous ([0,1] seconds), half synchronous ((1,2] seconds) and stationary ((2,3] seconds) reference system.

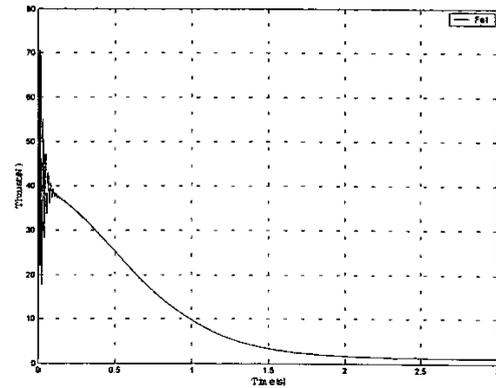


Fig. 7 – Thrust force F_{e1} over time for, respectively, a synchronous ([0,1] seconds), half synchronous ((1,2] seconds) and stationary ((2,3] seconds) reference system.

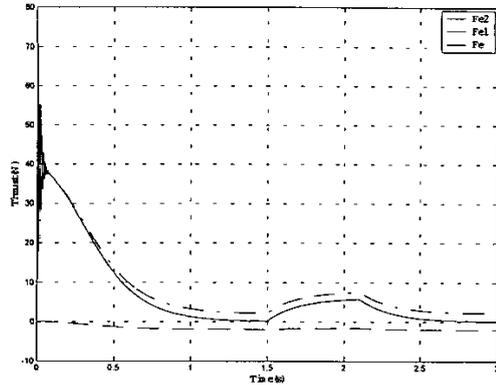


Fig. 8 – Thrust forces F_e (solid line), F_{e1} (dash-dotted line) and F_{e2} (dashed line) over time for a synchronous reference system, with a load force of 5.5 N applied during the the interval between 1.5 and 2.15 seconds.

References

- [1] K.Nam, J. H Sung, "A New Approach to Vector Control for Linear Induction Motor Considering End Effects," *IEEE IAS annual meeting*, 3-7 Oct., in Phoenix, Arizona, pp. 2284-2289, 1999.
- [2] J. Duncan, "Linear Induction Motor – Equivalent Circuit Model," in *IEE Proc.*, Vol. 130, pt. B, n° 1, pp. 51-57, January, 1983.
- [3] E.F. Silva, E. B. Santos, M. A. A. Oliveira, "A Novel Dynamic For Linear Induction Motors," in *7°COBEP*, Fortaleza-CE, Brasil, 2003, pp. 713-719.
- [4] E. B. Santos, "Análise do Motor de Indução Linear baseada na equivalência com o motor assimétrico rotativo," *Ph.D. Thesis, UFU*, Uberlândia, MG, Brasil, 2000.
- [5] J.F. Gieras, *Linear Induction Drives*, University of Tokyo Press, Japan, 1994.
- [6] R.C. Creppe, *Uma contribuição à modelagem de Máquinas de Indução Lineares*, *Ph.D. Thesis, UNICAMP*, Campinas, São Paulo, Brasil, 1997.