Error Probability for Coherent and Differential PSK Over Arbitrary Rician Fading Channels With Multiple Cochannel Interferers

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***Abstract*—This paper discusses the performance of communication systems using binary coherent and differential phase-shift keyed (PSK) modulation, in correlated Rician fading channels with diversity reception. The presence of multiple Rician-faded cochannel users, which may have arbitrary and nonidentical parameters, is modeled exactly. Exact bit error probability (BEP) expressions are derived via the moment generating functions (MGFs) of the relevant decision statistics, which are obtained through coherent detection with maximum ratio combining for coherent PSK modulation, and differential detection with equal gain combining (EGC) for differential modulation. Evaluating the exact expressions requires a complexity that is exponential in the number of interferers. To avoid this potentially time-consuming operation, we derive two low-complexity approximate methods each for coherent and differential modulation formats, which are more accurate than the traditional Gaussian approximation approach. Two new and interesting results of this analysis are: 1) unlike in the case of Rayleigh fading channels, increasing correlation between diversity branches may lead to *better* performance in Rician fading channels and 2) the phase distribution of the line-of-sight or static fading components of the desired user has a significant influence on the BEP performance in correlated diversity channels.**

***Index Terms*—Arbitrary Rician channels, bit error probability, cochannel interferers, coherent and differentially coherent modulations, equal gain combining (EGC), maximum ratio combining (MRC).**

I. Introduction

In cellular mobile communications, the detection of one user is often corrupted by cochannel interference (CCI) as well as background Gaussian noise. In code-division multiple-access (CDMA) systems, CCI is more often known as multi-access interference (MAI). It is generally acknowledged that the MAI problem in CDMA is so severe that sophisticated multi-user detectors are required to achieve good performance. In this paper, however, we concentrate on a scenario where the level of CCI is low enough to allow the use of conventional detectors and study their performance for a very general Rician fading channel, with binary coherent and differential PSK (BPSK and DPSK), and CCI modeled exactly. This framework is suitable for time-division and frequency-division multiple access (T/FDMA) systems, in which guard intervals and guard bands ensure channel orthogonality within a cell, and CCI only comes from cells in the next tiers that are using the same frequency band.

The assumption of the Rice distribution for the fading channel is motivated by the fact that it is used to model propagation paths consisting of one strong direct line-of-sight (LOS) component and many random weaker components [1], [2], a situation that is realistic in most wireless applications such as cellular communications. It includes the Rayleigh fading channel and the unfaded Gaussian noise channel as two special cases, and thus analysis of Rician channels is very useful, but generally more difficult than analysis of Rayleigh channels.

If the fading channel[1] is estimated at the receiver, coherent modulation formats such as PSK can be employed. In this case, coherent detection is carried out at the receiver, and maximal ratio combining (MRC) may be used for diversity combining. On the other hand, if the fading process is difficult to track, differentially coherent modulation such as DPSK, in which data are represented by the phase difference between successive transmitted symbols, can be used in concert with differential detection instead. We should note here that differentially modulated signals can also be *coherently* detected [3], in which case the differential modulation operation helps to resolve the phase reversal problem which is encountered for coherent modulation in fading channels. In this paper, we deal only with coherent BPSK and differential detection of DPSK.

Diversity reception is known to substantially improve performance and is often employed in practice, for instance with multiple receiving antennas. Optimum combining (OC) in fading channels with multiple interferers has been analyzed in many papers, for example, [4]–[6] to name a few. However, the implementation of OC requires channel estimates for all the cochannel interferers as well as for the desired user. Therefore, in practice, it is more common to use MRC for coherent detection [7] and equal gain combining (EGC) for differential detection. These are the combining schemes considered in this paper, which is concerned with performance analysis of *commonly used* systems, that may be suboptimal but are relatively easy to implement.

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[1]Meaning the attenuation and phase shift introduced in each path of a multipath channel.

In previous work on bit error probability (BEP) analysis for cellular mobile systems, a widely used technique is to obtain the probability density function (pdf) of the signal-to-interference-plus-noise ratio (SINR) at the receiver output, and then average the conditional error probability over that pdf to get the average BEP. Since the required pdf is difficult to obtain except in the simple case of Rayleigh fading channels, the analysis performed so far has been limited to such channels [7], [5], [6]. For Nakagami fading channels, a pdf expression for the SINR was derived in [8], but is applicable only to the case of single-channel (nondiversity) reception.

An alternative approach that is valid for many modulation formats and receiver structures is to find the distribution function of the decision variable and then obtain the BEP as the probability that the decision variable is smaller than zero [1], [9]. Using this method, BEP expressions for linear coherent CDMA multiuser detectors in Rayleigh multipath fading channels were derived in [10]. By working on the moment generating functions (MGFs) of the decision variables, we derived in [11] both closed-form expressions and a numerical method to evaluate the BEP for binary and quaternary DPSK (2/4-DPSK) and noncoherent frequency shift keying (NCFSK) with postdetection diversity combining, in arbitrary Rician fading channels. Furthermore, by using the MGF approach, BEP expressions of 2/4 DPSK with post-detection EGC in correlated Nakagami fading channels were derived in [12].

In [11] and [12], only the case of single-user communications is considered. Exact BEP expressions for cellular mobile communication systems, which take into account the effect of CCI, are not available for Rician fading channels with diversity combining and is the topic of this paper.

For analysis of a desired user in the presence of multiple cochannel interferers, we express the decision variable in an indefinite Gaussian quadratic form, derive its MGF conditioned on the interfering users' data, and then obtain the exact conditional BEP expressions. To compute the average BEP, we need to average over all possible interfering bit vectors, and that involves a complexity that increases exponentially with the number of interferers. To evaluate the performance of systems with a large number of interferers, we propose two low-complexity approximate BEP evaluation methods, which are derived using the joint Gaussian distribution analysis framework. This technique is significantly different from the traditional Gaussian approximation (TGA) approach [13]–[15], in which the residual CCI and noise is approximated as a Gaussian random variable, which is assumed to be independent from the desired signal component.

Numerical results show that the approximate schemes match the exact results well and are superior to the TGA approach. The results also show that the phase distribution of the static fading components of the desired signal has a significant impact on the BEP performance in the case of nonindependent diversity branches. Depending on the phases of the static components, the BEP does not necessarily degrade as the fading correlation increases, unlike the case of Rayleigh fading channels. However, increased correlation (for instance, by having antenna elements closer together) does not *always* improve performance, and therefore designing a better receiver by using the new findings will be a nontrivial task.

Finally, it should be mentioned that the analytical framework proposed in this paper is very general, and so the theoretical results are applicable to other modulation schemes like 4- and 8-phase coherent and differential PSK (4-PSK/DPSK, 8-PSK/DPSK) [11], [16], and NCFSK [11], since for all these modulation formats the BEP is given by the probability that the decision variable, which can be written in a Gaussian quadratic form, is smaller than zero (see, for example, [11]). The results are also useful for analyzing linear multiuser receivers [10] in arbitrary Rician faded CDMA channels, after suitable modifications.

Throughout this paper, we use the superscripts $*$, T, H, and -1 to represent the complex conjugate, transpose, conjugate transpose, and matrix inversion operations, respectively.

II. Signal Model

We consider a cellular mobile system where the signal received from a desired user over a total of L diversity branches is corrupted by N cochannel interferers and additive white Gaussian noise (AWGN). After the matched-filtering and sampling operations of the received signal, the complex baseband output signal in the ith bit interval can be expressed conveniently by[2]

$$\mathbf{r}(i) = \sum_{n=0}^{N} \mathbf{c}_n(i) d_n(i) + \boldsymbol{\xi}(i) \tag{1}$$

where $\mathbf{r}(i) = [r_1(i), \ldots, r_L(i)]^\top$, with $r_l(i) = \sum_{n=0}^{N} c_{n,l}(i) d_n(i) + \xi_l(i)$ representing the received signal at the lth antenna, for $l = 1, \ldots, L$. For BPSK, $\{d_n(i)\}$ is an equiprobable ± 1, independent data symbol sequence; while for DPSK, $d_n(i)$ is obtained by differentially modulating the original data bit $b_n(i)$, i.e., $d_n(i) = d_n(i-1) b_n(i)$ for modulation and $b_n(i) = d_n(i-1) d_n(i)$ for demodulation. In (1), $\mathbf{c}_n(i) = [c_{n,1}(i), \ldots, c_{n,L}(i)]^\top$ is the $L \times 1$ complex channel coefficient vector for the nth user, where the subscript 0 represents the desired user, and $n = 1, \ldots, N$ represent the N cochannel interferers. The noise vector $\boldsymbol{\xi}(i) = [\xi_1(i), \ldots, \xi_L(i)]^\top$ is a zero-mean complex Gaussian process, with variance σ_l^2 for the lth element $\xi_l(i)$. Below we assume the noise variance is identical for all the branches, that is, $\sigma^2 = \sigma_1^2 = \cdots = \sigma_L^2$. For all the complex Gaussian processes considered in this paper, including the channel realization and the noise, we assume that they are wide sense stationary (WSS) and circularly symmetric. The receiver models for BPSK modulation with MRC and DPSK modulation with EGC are depicted in Figs. 1 and 2, respectively.

For convenience, let $\mathbf{x} \sim \mathrm{CN}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ denote the vector $\mathbf{x}$ whose elements have the joint complex Gaussian distribution

[2]Here we assume that the received signal is matched-filtered and then sampled at the symbol rate, as in most previous work. For the optimal detection of signals in fading channels [17], the sampling operation no longer provides sufficient statistics. However, optimal detection is not considered in this paper.

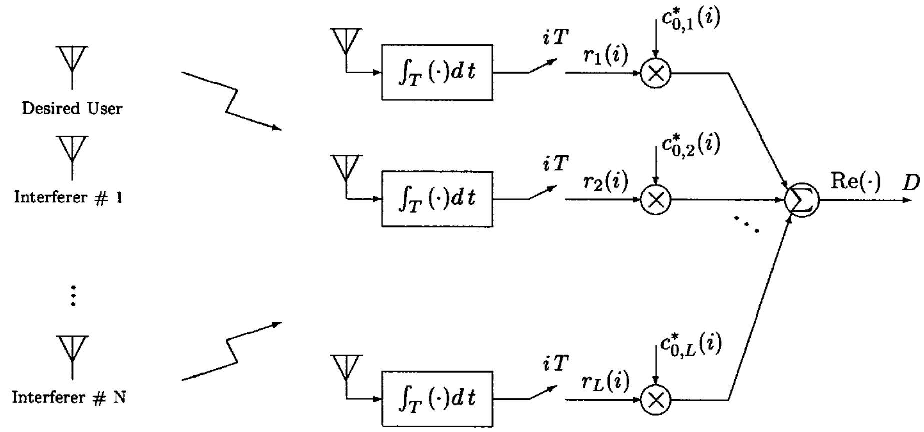

Fig. 1. The signal model for BPSK receiver with L-fold MRC, and in the presence of N cochannel interferers. After the matched filtering and sampling at the symbol rate, the received signals are coherently combined with MRC to form the decision variable D.

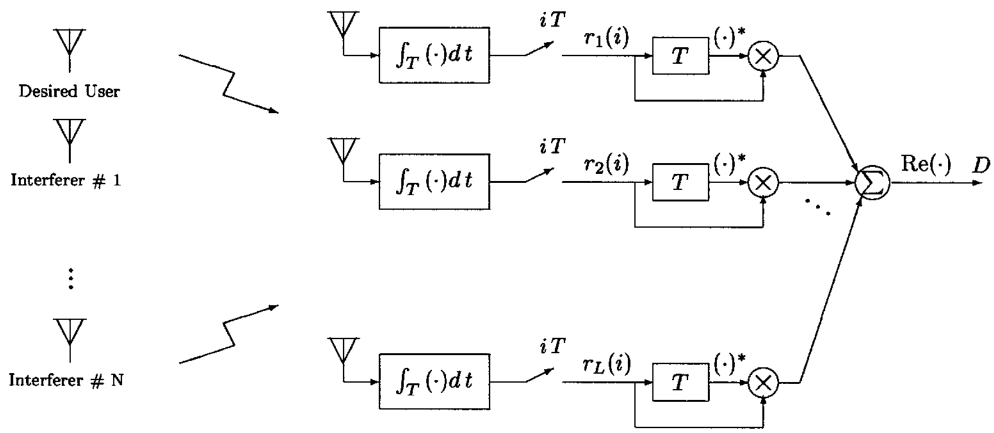

Fig. 2. The signal model for DPSK receiver with L-fold EGC, and in the presence of N cochannel interferers. After the matched filtering and sampling at the symbol rate, the received signal goes through the product detector with EGC, and then the decision variable D is generated.

with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Sigma}$. When $\mathbf{\Sigma} \in \mathbb{C}^{N\times N}$ has full rank, the pdf of $\mathbf{x}$ is defined as [9][3]

$$f(\mathbf{x}) = \frac{1}{\pi^N \det(\mathbf{\Sigma})} \exp\left[-(\mathbf{x}-\boldsymbol{\mu})^{\mathrm{H}}\mathbf{\Sigma}^{-1}(\mathbf{x}-\boldsymbol{\mu})\right]. \quad (2)$$

For Rician fading channels, we assume that the fading process for different users and adjacent symbol intervals has a joint Gaussian distribution. Then the channel vector $\mathbf{c}(i) = [\mathbf{c}_0^\top(i), \ldots, \mathbf{c}_N^\top(i)]^\top$ has the Gaussian distribution $\mathbf{c}(i) \sim \mathrm{CN}(\boldsymbol{\mu}_c, \mathbf{\Sigma}(0))$, where $\boldsymbol{\mu}_c = [\boldsymbol{\mu}_{c,0}^\top, \ldots, \boldsymbol{\mu}_{c,N}^\top]^\top$, with $\boldsymbol{\mu}_{c,n} = E(\mathbf{c}_n(i)) = [\mu_{c,n,1}, \ldots, \mu_{c,n,L}]^\top$, is the mean vector, and $\mathbf{\Sigma}(j) = E[(\mathbf{c}(i)-\boldsymbol{\mu}_c)(\mathbf{c}(i-j)-\boldsymbol{\mu}_c)^{\mathrm{H}}]$ is the fading channel covariance matrix, for $j = 0, \pm 1, \ldots$.

If we make the realistic assumption that the fading processes for different users are independent, then the covariance matrix $\mathbf{\Sigma}(j)$ is block diagonal and can be expressed as $\mathbf{\Sigma}(j) = \mathrm{diag}(\mathbf{\Sigma}_{00}(j), \ldots, \mathbf{\Sigma}_{NN}(j))$, where $\mathbf{\Sigma}_{nn}(j) = E[(\mathbf{c}_n(i) - \boldsymbol{\mu}_{c,n})(\mathbf{c}_n(i-j) - \boldsymbol{\mu}_{c,n})^{\mathrm{H}}]$ is the fading covariance matrix for the L diversity branches of the nth user, $n = 0, \ldots, N$.

For the lth diversity branch ($l = 1, \ldots, L$) of the nth user, the Rician factor is defined as $K_{n,l} = |\mu_{c,n,l}|^2/\gamma_{n,l}$, where $\gamma_{n,l} = E[|c_{n,l}(i) - \mu_{c,n,l}|^2]$ is the power of the scattered components. The temporal fading correlation coefficient between adjacent symbol intervals is then given by $\rho_{n,l} = E[(c_{n,l}(i-1) - \mu_{c,n,l})^*(c_{n,l}(i) - \mu_{c,n,l})]/\gamma_{n,l}$, which depends on the fading power density spectrum and the Doppler fading bandwidth $B_{n,l}$. For example, $\rho_{n,l} = J_0(2\pi B_{n,l}T)$ for Clarke's fading spectrum [18] and $\rho_{n,l} = \exp(-(\pi B_{n,l}T)^2)$ for the Gaussian fading spectrum [19], where T is a bit duration. If for every branch, $K_{n,l} = 0$, the Rician channel is reduced to the Rayleigh fading channel; when $K_{n,l} \to \infty$, it is equivalent to an unfaded Gaussian noise channel.

[3]The difference in the pdf shown here from that shown in [9, Appendix B] arises from the fact that we define the covariance matrix of a complex vector $\mathbf{x}$ as $E(\mathbf{x}\mathbf{x}^{\mathrm{H}})$, rather than $(1/2)E(\mathbf{x}\mathbf{x}^{\mathrm{H}})$ as in [9].

III. MGF of the Decision Variables

In this section, we derive the MGF expressions for the BPSK and DPSK decision variables and then use these to evaluate the probability of error expressions in the following section.

A. BPSK With MRC

Let the data vector of all the users be $\mathbf{d}(i) = [d_0(i), \mathbf{d}_I^\top(i)]^\top$, where $\mathbf{d}_I(i) = [d_1(i), \ldots, d_N(i)]^\top$ is the data vector of the N interferers. For BPSK modulation with MRC diversity, the BEP can be expressed by the probability that a decision variable is smaller than zero. This decision variable for the desired user conditioned on $\mathbf{d}(i)$ is given by[4]

$$D(\mathbf{d}) = \mathrm{Re}(\mathbf{c}_0^{\mathrm{H}}(i)\mathbf{r}(i)) = \mathbf{v}^{\mathrm{H}}(\mathbf{d})\mathbf{Q}\mathbf{v}(\mathbf{d}) \tag{3}$$

where

$$\mathbf{v}(\mathbf{d}) = \begin{bmatrix} \mathbf{c}_0(i) \\ \mathbf{r}(i) \end{bmatrix} \tag{4}$$

$$\mathbf{Q} = \begin{bmatrix} \mathbf{0}_L & 0.5\mathbf{I}_L \\ 0.5\mathbf{I}_L & \mathbf{0}_L \end{bmatrix}. \tag{5}$$

In (5), $\mathbf{0}_L$ and $\mathbf{I}_L$ are the $L \times L$ zero and identity matrices, respectively. The average BEP of BPSK with MRC can be expressed as

$$P_e = 0.5[P_e(d_0(i) = 1) + P_e(d_0(i) = -1)]. \tag{6}$$

For coherent detection,[5] we have $P_e(d_0 = 1) = P_e(d_0 = -1)$, and therefore the average BEP is given by

$$P_e = \sum_{\substack{\text{all } \mathbf{d}(i) \\ d_0(i)=1}} \Pr(D(\mathbf{d}) < 0). \tag{7}$$

To evaluate the cumulative distribution function (cdf) expression $\Pr(D(\mathbf{d}) < 0)$, we will use the moment generating function (MGF) method, and the MGF of $D(\mathbf{d})$ is derived next.

With the assumption that the fading processes for different users are independent, the covariance matrix of $\mathbf{v}(\mathbf{d})$ is independent of $\mathbf{d}_I(i)$ and can be expressed as

$$\mathbf{P}_v = \begin{bmatrix} \mathbf{\Sigma}_{00}(0) & d_0(i)\mathbf{\Sigma}_{00}(0) \\ d_0(i)\mathbf{\Sigma}_{00}(0) & \displaystyle\sum_{n=0}^{N} \mathbf{\Sigma}_{nn}(0) + \mathbf{R}_\xi(0) \end{bmatrix} \tag{8}$$

where $\mathbf{R}_\xi(j) = E[\boldsymbol{\xi}(i)\boldsymbol{\xi}^{\mathrm{H}}(i-j)] \in \mathbb{C}^{L\times L}$ is the noise correlation matrix, for $j = 0, \pm 1, \ldots$.

The mean vector of $\mathbf{v}(\mathbf{d})$ conditioned on a realization of $\mathbf{d}(i)$ is given by

$$\boldsymbol{\mu}_d = E_{c,\xi}[\mathbf{v}(\mathbf{d})] = \begin{bmatrix} \boldsymbol{\mu}_{c,0} \\ \displaystyle\sum_{n=0}^{N} d_n(i)\boldsymbol{\mu}_{c,n} \end{bmatrix} \tag{9}$$

where $E_{c,\xi}(\cdot)$ denotes the expectation operation with respect to the channel vector $\mathbf{c}(i)$ and noise vector $\boldsymbol{\xi}(i)$.

Employing a result for the distribution of a noncentral Gaussian quadratic form [9], [21], [22], the MGF of $D(\mathbf{d})$ in (3) can now be obtained as

$$\Phi_D(s|\mathbf{d}) = \frac{\exp\left(\boldsymbol{\mu}_d^{\mathrm{H}}\mathbf{F}(s)\boldsymbol{\mu}_d\right)}{\det(\mathbf{I} - s\mathbf{\Lambda}_v)} \tag{10}$$

[4]When the noise variance $\{\sigma_l^2\}_{l=1}^{L}$ is not identical at different diversity branches, we shall change vector $\mathbf{c}_0(i)$ in (3) to $[c_{0,1}(i)/\sigma_1^2, \ldots, c_{0,L}(i)/\sigma_L^2]^\top$, as in [9].

[5]For differential modulation, it has been shown [20] that the BEP conditioned on the transmitted symbol is a function of the symbol, and in general $P_e(d_0 = 1) \neq P_e(d_0 = -1)$.

and

$$\mathbf{F}(s) = \mathbf{P}_v^{-1/2}\left(\left(\mathbf{I} - s\mathbf{P}_v^{1/2}\mathbf{Q}\mathbf{P}_v^{1/2}\right)^{-1} - \mathbf{I}\right)\mathbf{P}_v^{-1/2} \tag{11}$$

where $\mathbf{P}_v^{-1/2}$ is the inverse of the symmetric square root of $\mathbf{P}_v$, and $\det(\mathbf{P})$ denotes the determinant of matrix $\mathbf{P}$.

From (9) and (10), some observations are in order. First, if $\boldsymbol{\mu}_{c,n} = \mathbf{0}$ for $n = 1, \ldots, N$, that is, all the interferers are Rayleigh faded, then the MGF expression (10) is independent of $\mathbf{d}_I(i)$, and so the complexity of BEP evaluation is independent of the number of interferers.

Second, for the single-user channel where only the desired user is present, if there is fading correlation amongst different diversity branches, then the phases of $\boldsymbol{\mu}_{c,0}$ affect the BEP,[6] in contrast to the case of independent diversity branches where the static components can be regarded as real constants [23]. To illustrate, we define the phase vector for the desired user as $\boldsymbol{\theta}_0 = [\theta_{0,1}, \theta_{0,2}, \ldots, \theta_{0,L}]^\top$, then $\boldsymbol{\mu}_{c,0}$ can be expressed as $\boldsymbol{\mu}_{c,0} = \mathrm{diag}(e^{j\theta_{0,1}}, e^{j\theta_{0,2}}, \ldots, e^{j\theta_{0,L}})|\boldsymbol{\mu}_{c,0}|$, where $|\boldsymbol{\mu}_{c,0}|$ denotes the vector obtained after each element of $\boldsymbol{\mu}_{c,0}$ is replaced by its modulus. For independent diversity channels, $\mathbf{P}_v$ in (8) consists of four subblocks which are diagonal matrices, and the phase matrix $\mathrm{diag}(e^{j\theta_{0,1}}, e^{j\theta_{0,2}}, \ldots, e^{j\theta_{0,L}})$ can be absorbed in the exponential term of (10) without changing the MGF expression. However, for the correlated fading case, these submatrices of $\mathbf{P}_v$ will no longer be diagonal, and so variation in $\boldsymbol{\theta}_0$ will result in different MGF expressions. This observation is significant for the BEP evaluation.

Finally, in the presence of multiple interferers, we define the phase vectors for the static components of the interferers as $\boldsymbol{\theta}_n = [\theta_{n,1}, \theta_{n,2}, \ldots, \theta_{n,L}]^\top$, for $n = 1, \ldots, N$. Whether the diversity branches are independent or correlated, $\{\boldsymbol{\theta}_n\}_{n=0}^{N}$ must be taken into account for BEP evaluation.

To represent $\Phi_D(s|\mathbf{d})$ in a more compact form, we define a matrix $\mathbf{A} = \mathbf{P}_v^{1/2}\mathbf{Q}\mathbf{P}_v^{1/2}$ and represent its eigen-decomposition as $\mathbf{A} = \mathbf{U}_v\mathbf{\Lambda}_v\mathbf{U}_v^{\mathrm{H}}$, where

$$\mathbf{\Lambda}_v = \mathrm{diag}(\lambda_1, \lambda_2, \ldots, \lambda_{2L}) \tag{12}$$

is the eigenvalue matrix of $\mathbf{A}$. Also, we define

$$\acute{\boldsymbol{\mu}}_d = [\acute{\mu}_{1,d}, \acute{\mu}_{2,d}, \ldots, \acute{\mu}_{2L,d}]^\top = \mathbf{U}_v^{\mathrm{H}}\mathbf{P}_v^{-1/2}\boldsymbol{\mu}_d. \tag{13}$$

Equation (10) can now be simplified to

$$\Phi_D(s|\mathbf{d}) = \frac{\exp\left(\acute{\boldsymbol{\mu}}_d^{\mathrm{H}}\mathbf{\Lambda}(s)\acute{\boldsymbol{\mu}}_d\right)}{\det(\mathbf{I} - s\mathbf{\Lambda}_v)} = \frac{\exp\left(\displaystyle\sum_{k=1}^{2L}\frac{|\acute{\mu}_{k,d}|^2 s\lambda_k}{1 - s\lambda_k}\right)}{\displaystyle\prod_{k=1}^{2L}(1 - s\lambda_k)} \tag{14}$$

[6]It is furthermore observed from (10) that, for the single-user channel, only the phase differences between the static components will affect the MGF expression. So, without loss of generality, the phase of the first static element may be set to zero.

where $\acute{\boldsymbol{\mu}}_d$ is given by (13), and $\boldsymbol{\Lambda}(s) = (\mathbf{I} - s\boldsymbol{\Lambda}_v)^{-1} - \mathbf{I}$ is a diagonal matrix.

In the case of repeated eigenvalues, the MGF of $D(\mathbf{d})$ is given by

$$\Phi_D(s|\mathbf{d}) = \frac{\exp\left(\sum_{m=1}^{M} \frac{b_{m,d}^2 s\lambda_m}{1 - s\lambda_m}\right)}{\prod_{m=1}^{M} (1 - s\lambda_m)^{v_m}} \quad (15)$$

where v_m is the multiplicity of the eigenvalue λ_m, and M ($0 < M \leq 2L$) is the total number of distinct eigenvalues. In (15)

$$b_{m,d}^2 = \sum_{k \in \mathcal{K}_m} |\acute{\mu}_{k,d}|^2 \quad (16)$$

where $\mathcal{K}_m$ denotes the set of k indices associated with the mth distinct eigenvalue. It is then clear that $(1/\lambda_1, \ldots, 1/\lambda_M)$ are the M distinct poles of the conditional MGF $\Phi_D(s|\mathbf{d})$. For numerical BEP evaluation, (14) will be used for convenience; however, for deriving a closed form BEP expression, we will use (15). For BPSK with MRC in arbitrary Rician channels, there will be 2^N MGF expressions for (14) conditioned on the data pattern of the interferers.

B. DPSK With EGC

For some applications where the channel estimates are not available, differentially coherent modulation, such as DPSK, is often employed [1]. Post-detection equal gain combining (EGC) is a widely used diversity scheme for DPSK signals. Letting $\mathbf{d} = [\mathbf{d}^\top(i-1), \mathbf{d}^\top(i)]^\top$, we give the decision variable for DPSK with post-detection EGC conditioned on $\mathbf{d}$ as [9]

$$D(\mathbf{d}) = \mathrm{Re}(\mathbf{r}^{\mathrm{H}}(i-1)\mathbf{r}(i)) = \tilde{\mathbf{v}}^{\mathrm{H}}(\mathbf{d})\mathbf{Q}\tilde{\mathbf{v}}(\mathbf{d}) \quad (17)$$

where $\tilde{\mathbf{v}}(\mathbf{d}) = \left[\begin{smallmatrix}\mathbf{r}(i-1)\\ \mathbf{r}(i)\end{smallmatrix}\right]$ and $\mathbf{Q}$ is given by (5).

The average BEP can be expressed as

$$P_e = 0.5 \cdot \left(\sum_{\substack{\text{all } \mathbf{d}(i) \\ b_0(i)=1}} \Pr(D(\mathbf{d}) < 0) + \sum_{\substack{\text{all } \mathbf{d}(i) \\ b_0(i)=-1}} \Pr(-D(\mathbf{d}) < 0) \right). \quad (18)$$

If there is noise correlation between adjacent symbol intervals, that is, $\mathbf{R}_\xi(1) \neq \mathbf{0}$, then for $b_0(i) = 1$ and -1 there are usually different conditional BEPs, a phenomenon called channel nonsymmetry[7] that is peculiar to differential detection [20]. So we shall evaluate both the cases $b_0(i) = 1$ and -1.

[7]To exploit this phenomenon to obtain better system performance, it is possible in principle to design an encoding scheme which gives a lower average BEP. However, this is a nontrivial task since the nonuniform *a priori* bit distribution may lower the source entropy.

With the assumption that the fading processes for different users are independent, the covariance matrix of $\tilde{\mathbf{v}}(\mathbf{d})$ is given by

$$\mathbf{P}_v = \begin{bmatrix} \sum_{n=0}^{N} \boldsymbol{\Sigma}_{nn}(0) + \mathbf{R}_\xi(0) & \sum_{n=0}^{N} b_n(i)\boldsymbol{\Sigma}_{nn}^{\mathrm{H}}(1) + \mathbf{R}_\xi^{\mathrm{H}}(1) \\ \sum_{n=0}^{N} b_n(i)\boldsymbol{\Sigma}_{nn}(1) + \mathbf{R}_\xi(1) & \sum_{n=0}^{N} \boldsymbol{\Sigma}_{nn}(0) + \mathbf{R}_\xi(0) \end{bmatrix}. \quad (19)$$

The mean vector of $\tilde{\mathbf{v}}(\mathbf{d})$ is given by

$$\boldsymbol{\mu}_d = \begin{bmatrix} \sum_{n=0}^{N} d_n(i-1)\boldsymbol{\mu}_{c,n} \\ \sum_{n=0}^{N} d_n(i)\boldsymbol{\mu}_{c,n} \end{bmatrix}. \quad (20)$$

The MGF expression for the decision variable can be obtained by using the same method proposed in Section III-A for BPSK. Note that for DPSK there are 2^{2N+1} such MGF expressions depending on the data pattern of the desired user and all the interferers.

IV. EXACT BEP EVALUATION

A. Numerical Method

Using the MGF $\Phi_D(s|\mathbf{d})$, the cdf of $D(\mathbf{d})$ can be evaluated by using the saddle point technique proposed in [24], that is

$$\Pr(D(\mathbf{d}) < 0) = \frac{1}{2\pi j} \int_{c-j\infty}^{c+j\infty} \frac{\Phi_D(-s|\mathbf{d})}{s} \, ds \quad (21)$$

where $j = \sqrt{-1}$ and c is a positive real constant smaller than the minimum positive root of $(\Phi_D(-s|\mathbf{d}))/s$.

The integral in (21) can be computed efficiently by a trapezoidal summation [24]

$$\Pr(D(\mathbf{d}) < 0) = \Delta y \left[\frac{1}{2} I(s_0) + \sum_{m=1}^{\infty} I(s_0 + jm\Delta y) \right] \quad (22)$$

where $I(s) = \mathrm{Re}((\Phi_D(-s|\mathbf{d}))/s\pi)$, s_0 is the saddle point of $I(s)$, and can be computed recursively by Newton's method, that is, $s_0 \leftarrow s_0 - (\phi'(s_0)/\phi''(s_0))$. $\phi'(s_0)$, and $\phi''(s_0)$ are the first- and second-order derivatives of function $\phi(s)$ evaluated at s_0, with $\phi(s)$ being defined as $\phi(s) = \ln(\Phi_D(-s|\mathbf{d})/s)$. In (22), Δy is the step size, whose initial value can be set to $\Delta y = [\phi''(s_0)]^{-(1/2)}$, which roughly measures the width of the integrand in (22) as a function of y [24]. The series in (22) can be truncated to achieve any required precision. The summation in (22) is along a vertical line passing through the saddle point, which is both simple and accurate. A more efficient algorithm that converges faster than trapezoidal summation may be obtained by integrating along the curved path of steepest descent of the MGF [25]. There are also other low-complexity methods for evaluating (21), such as the saddle point approximation technique [25, Sec. 5.3].

The explicit expressions for $\phi(s)$, $\phi'(s)$, and $\phi''(s)$ are given by,

$$\phi(s) = -\sum_{k=1}^{2L} \frac{\acute{\mu}_{k,d}^2 s\lambda_k}{1+s\lambda_k} - \sum_{k=1}^{2L} \ln|1+s\lambda_k| - \ln s$$

$$\phi'(s) = -\sum_{k=1}^{2L} \frac{\acute{\mu}_{k,d}^2 \lambda_k}{(1+s\lambda_k)^2} - \sum_{k=1}^{2L} \frac{\lambda_k}{1+s\lambda_k} - \frac{1}{s}$$

$$\phi''(s) = \sum_{k=1}^{2L} \frac{2\acute{\mu}_{k,d}^2 \lambda_k^2}{(1+s\lambda_k)^3} + \sum_{k=1}^{2L} \frac{\lambda_k^2}{(1+s\lambda_k)^2} + \frac{1}{s^2}$$

where λ_k and $\acute{\mu}_{k,d}$, for $k = 1, \ldots, 2L$, are given by (12) and (13), respectively. Note that this method also applies to the general case where there are some repeated eigenvalues λ_k.

When the saddle point s_0 is obtained, (21) can also be evaluated by using an efficient Gauss–Chebyshev quadrature (GCQ) formula [26]. Letting $s = s_0 + j\omega$ and $\omega = s_0 \tan(\theta/2)$, (21) is reduced to

$$\begin{aligned}\Pr(D(\mathbf{d}) < 0) &= \frac{1}{\pi}\int_0^\infty \frac{\Phi_D(-s_0 - j\omega|\mathbf{d})}{s_0 + j\omega}\, d\omega \\ &= \frac{1}{2\pi}\int_0^\pi \hat{\Phi}(\theta)\, d\theta \end{aligned} \quad (23)$$

where

$$\hat{\Phi}(\theta) = \Phi_D(-s_0 - js_0\tan(\theta/2)|\mathbf{d})(1 - j\tan(\theta/2)). \quad (24)$$

Next, by defining $x = \cos(\theta)$ in (23), we obtain a GCQ formula for evaluating the conditional BEP as

$$\Pr(D(\mathbf{d}) < 0) = \frac{1}{2N}\sum_{n=1}^{N} \hat{\Phi}\left(\frac{(2n-1)}{2N}\pi\right) + \hat{R}_N \quad (25)$$

where $\hat{\Phi}(x)$ is defined in (24), and $\hat{R}_N$ is a residual term. Our numerical evaluation shows that (25) usually converges faster than (22), and it is also less sensitive to the computational accuracy of the saddle point s_0.

B. Closed-Form Expressions

Assuming that, among the M distinct poles of $\Phi_D(s|\mathbf{d})$ given in (15), M_1 of them denoted by $(\lambda_1, \ldots, \lambda_{M_1})$ are negative, the cdf of $D(\mathbf{d})$ can be evaluated by finding the residues of $\Phi_D(s|\mathbf{d})e^{-sD(\mathbf{d})}$ at these M_1 negative poles. This method is described in detail in [11]. Following the same approach, we obtain a closed-form expression for (21) as

$$\begin{aligned}&\Pr(D(\mathbf{d}) < 0) \\ &= \sum_{k=1}^{M_1}\left(\prod_{m=1,\, m\neq k}^{M}\left(\left(\frac{\lambda_k}{\lambda_k - \lambda_m}\right)^{v_m} \exp\left(\frac{b_{m,d}^2\lambda_m}{\lambda_k - \lambda_m}\right)\right)\right) \\ &\quad \cdot \exp(-b_{k,d}^2)\left(\sum_{n=0}^{\infty}\sum_{r_k=0}^{n+v_k-1}\frac{b_{k,d}^{2n}G_k^{(r_k)}(\lambda)}{n!r_k!(-\lambda_k)^{r_k}}\right)\end{aligned} \quad (26)$$

where

$$\begin{aligned}G_k^{(r)}(\lambda) = &\left[\sum_{r_1=0}^{r-1}\binom{r-1}{r_1} g_k^{(r-1-r_1)}(\lambda)\right. \\ &\left.\cdot \sum_{r_2=0}^{r_1-1}\binom{r_1-1}{r_2} g_k^{(r_1-1-r_2)}(\lambda)\cdots\right].\end{aligned} \quad (27)$$

In (27), $g_k^{(n)}(\lambda)$ is given by

$$\begin{aligned}&g_k^{(n)}(\lambda) = \\ &\sum_{m,\, m\neq k}^{M}\left[n!\left(\frac{\lambda_k\lambda_m}{\lambda_k - \lambda_m}\right)^{n+1}\left(v_m + (n+1)\frac{b_{m,d}^2\lambda_k}{(\lambda_k - \lambda_m)}\right)\right]\end{aligned} \quad (28)$$

where $b_{m,d}$, for $m = 1, \ldots, M$, is given by (16).

Equation (26) gives a new and general closed-form BEP expression for coherent and differentially coherent modulations over arbitrary Rician multipath fading channels with multiple cochannel interferers. Since (26) is derived using the fast-convergent power series expansion of the MGF $\Phi_D(s|\mathbf{d})$ about its negative poles, (26) is expected to converge rapidly with respect to the index n therein. However, a strict convergence rate analysis seems difficult, since it involves some high-order derivatives of the MGF at each of its negative poles. The numerical results for a single-user communications system given in [11] showed that the series (26) converges more slowly as the Rice K-factor increases, therefore it is more suitable for analysis of Rician fading channels with a small K-factor, for example, $K \leq 7$ dB. In this case, for the index n summing from 0 to $n_{\max}$, letting $n_{\max} = 10$ usually gives a very accurate result. In comparison, the numerical scheme [(22) or (25)] which uses the saddle point integration looks more convenient for computations, since it is both easier to program and gives accurate results with a moderate complexity.

The exact evaluation of BEP for coherent and differentially coherent detection in arbitrary Rician channels will have a complexity exponential in the number of interferers. For cellular radio systems with a large number of cochannel interferers, the complexity of an exact BEP expression will be overwhelming. To evaluate the BEP efficiently and with an acceptable accuracy, we use approximations on the MGF expressions of the decision variables, and then obtain approximate BEP expressions that are much simpler. Several approximation techniques are proposed in the next section.

V. Approximate BEP Analysis

A. BPSK With MRC

To reduce the complexity for exact BEP evaluation, we try to average the conditional MGF expression over $\mathbf{d}_I(i)$ to obtain an unconditional MGF expression, that is

$$\Phi_D(s) = E_{d_I}(\Phi_D(s|\mathbf{d})) = \frac{1}{2^N}\sum_{\substack{\text{all } \mathbf{d}_I(i) \\ d_0(i)=1}} \Phi_D(s|\mathbf{d}). \quad (29)$$

Unfortunately, this operation has a complexity increasing exponentially with the number of interferers. In this section, we give an approximate MGF expression $\tilde{\Phi}_D(s)$ for (29), and then an approximate average BEP expression can be obtained as

$$\tilde{P}_e = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty}\frac{\tilde{\Phi}_D(-s)}{s}\,ds \tag{30}$$

which can be evaluated by using the closed-form expression or the numerical method proposed in the previous section.

The procedure to obtain $\tilde{\Phi}_D(s)$ is derived below. In the conditional MGF expression (14), only the exponential term is a function of $\mathbf{d}_I(i)$. Without loss of generality, assuming $d_0(i)=1$ we can decompose $\boldsymbol{\mu}_d$ to two vectors $\boldsymbol{\mu}_d = \overline{\boldsymbol{\mu}}_d + \boldsymbol{\mu}_{d_I}$, where

$$\overline{\boldsymbol{\mu}}_d = \left[\boldsymbol{\mu}_{c,0}^{\top}, \boldsymbol{\mu}_{c,0}^{\top}\right]^{\top} \tag{31}$$

contains the static components of the desired user, and

$$\boldsymbol{\mu}_{d_I} = \begin{bmatrix}\mathbf{0}\\ \sum_{n=1}^{N} d_n(i)\boldsymbol{\mu}_{c,n}\end{bmatrix} \tag{32}$$

is the combination of static components of the N interferers.

Due to the equiprobable, antipodal data modulation $d_n(i)$ in (32), we can apply the Central Limit Theorem and approximate $\boldsymbol{\mu}_{d_I}$ as a zero mean complex Gaussian vector with covariance matrix

$$\mathbf{P}_{\mu_d} = \begin{bmatrix}\mathbf{0_L} & \mathbf{0_L}\\ \mathbf{0_L} & \sum_{n=1}^{N}\boldsymbol{\mu}_{c,n}\boldsymbol{\mu}_{c,n}^{\mathrm{H}}\end{bmatrix}. \tag{33}$$

With this approximation, $\boldsymbol{\mu}_d$ is a complex Gaussian vector, with mean $\overline{\boldsymbol{\mu}}_d$ and covariance matrix $\mathbf{P}_{\mu_d}$ given in (33).

Now we find that the exponential term $\boldsymbol{\mu}_d^{\mathrm{H}}\mathbf{F}(s)\boldsymbol{\mu}_d$ in (10) can be regarded as a noncentral complex Gaussian quadratic form, and thus its MGF expression can be derived. Note that the $2L\times 2L$ matrix $\mathbf{P}_{\mu_d}$ is rank deficient. In this case, we shall decompose $\mathbf{P}_{\mu_d}$ as $\mathbf{P}_{\mu_d}=\mathbf{B}\mathbf{B}^{\mathrm{H}}$, where $\mathbf{B}$ is a $2L\times r$ matrix with rank r ($r\le L$).

Using the result in the Appendix, we can get an approximate average MGF expression $\tilde{\Phi}_D(s)$ for (29) as

$$\tilde{\Phi}_D(s) = \frac{\exp\left(\overline{\boldsymbol{\mu}}_d^{\mathrm{H}}\mathbf{F}(s)\left[\mathbf{I}+\mathbf{B}\left(\mathbf{I}-\mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\right]\overline{\boldsymbol{\mu}}_d\right)}{\det(\mathbf{I}-\mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B})\prod_{k=1}^{2L}(1-s\lambda_k)} \tag{34}$$

where $\overline{\boldsymbol{\mu}}_d$ and λ_k are given by (31) and (12), respectively. We call (33) and (34) Method I from now on. Alternatively, if we consider only the energies of the static components of $\boldsymbol{\mu}_{d_I}$, we can use $\mathrm{diag}(\mathbf{P}_{\mu_d})$ instead of $\mathbf{P}_{\mu_d}$ in (34); this approximation is called Method II. We propose Method II since, at least in independent fading channels, the cross correlation of the static component amongst different branches will not affect the BEP. Furthermore, for convenience, if the phases of the static fading components of the interferers are assumed to be unknown, a good approximation can be obtained by ensemble averaging over the uniformly distributed phases. In this way, the off-diagonal elements in $\mathbf{P}_{\mu_d}$ are set to zeros even for correlated fading channels.

Since analytical differentiation of (34) to obtain the saddle point is difficult, numerical methods can be employed instead.

The MGF in (34) is obtained by averaging $\exp(sD(\mathbf{d}))$ firstly over the distribution of the scattered components of $\mathbf{c}(i)$ and the noise $\boldsymbol{\xi}(i)$, and then over the distribution of $\boldsymbol{\mu}_{d_I}$, with the multivariate Gaussian distribution approximation of $\boldsymbol{\mu}_{d_I}$. In fact, since $\mathbf{P}_v$ and $\mathbf{P}_{\mu_d}$ are covariance matrices of two independent joint Gaussian distributions, these two steps can be merged into one. As a result, we obtain a form equivalent to (34) as

$$\tilde{\Phi}_D(s) = \frac{\exp\left(\overline{\boldsymbol{\mu}}_d^{\mathrm{H}}\acute{\mathbf{P}}_v^{-1/2}\left(\left(\mathbf{I}-s\acute{\mathbf{P}}_v^{1/2}\mathbf{Q}\acute{\mathbf{P}}_v^{1/2}\right)^{-1}-\mathbf{I}\right)\acute{\mathbf{P}}_v^{-1/2}\overline{\boldsymbol{\mu}}_d\right)}{\det\left(\mathbf{I}-s\acute{\mathbf{P}}_v^{1/2}\mathbf{Q}\acute{\mathbf{P}}_v^{1/2}\right)} \tag{35}$$

where (for Method I)

$$\acute{\mathbf{P}}_v = \mathbf{P}_v + \mathbf{P}_{\mu_d} \tag{36}$$

$$= \begin{bmatrix}\boldsymbol{\Sigma}_{00}(0) & d_0(i)\boldsymbol{\Sigma}_{00}(0)\\ d_0(i)\boldsymbol{\Sigma}_{00}(0) & \sum_{n=0}^{N}\boldsymbol{\Sigma}_{nn}(0)+\mathbf{R}_{\xi}(0)+\sum_{n=1}^{N}\boldsymbol{\mu}_{c,n}\boldsymbol{\mu}_{c,n}^{\mathrm{H}}\end{bmatrix} \tag{37}$$

and $\mathbf{P}_v$ is given by (8). For Method II, $\mathrm{diag}(\mathbf{P}_{\mu_d})$ is used instead of $\mathbf{P}_{\mu_d}$ in (36). The equivalence between (34) and (35) is verified by our numerical comparisons.

It is interesting to note that (35) is the MGF expression for a Gaussian quadratic form $\mathbf{x}^{\mathrm{H}}\mathbf{Q}\mathbf{x}$, where $\mathbf{x}\sim \mathrm{CN}(\overline{\boldsymbol{\mu}}_d, \acute{\mathbf{P}}_v)$. So it is clear that, in this approximation, the system is changed to a single-user diversity channel, where $\mathbf{P}_{\mu_d}$ or $\mathrm{diag}(\mathbf{P}_{\mu_d})$, the covariance matrix of $\boldsymbol{\mu}_{d_I}$, is directly incorporated into the covariance matrix $\acute{\mathbf{P}}_v$. This means that the Rician-faded interferers are approximated to be Rayleigh faded, with modified covariance matrix $\acute{\mathbf{P}}_v$ instead of $\mathbf{P}_v$. In addition to the reduced complexity for BEP evaluation, Method II also avoids the need to know $\boldsymbol{\theta}_1,\ldots,\boldsymbol{\theta}_N$, the phases of the static components $\boldsymbol{\mu}_{c,n}$ (for $n=1,\ldots,N$) of the interferers.

With the new approximate MGF expressions, we can evaluate the approximate average BEP of coherent modulation with MRC and multiple cochannel interferers, by using the closed-form expression or the numerical method proposed in Section IV. By using the numerical method, these schemes need to evaluate only one integral rather than 2^N integrals required for the exact expression.

B. DPSK With EGC

This case is slightly different from that of BPSK; we need to first average (19) over the distribution of $\{b_n(i)\}_{n=1}^{N}$ and obtain

$$\hat{\mathbf{P}}_v = \begin{bmatrix} \sum_{n=0}^{N} \boldsymbol{\Sigma}_{nn}(0) + \mathbf{R}_\xi(0) & b_0(i)\boldsymbol{\Sigma}_{00}^{\mathrm{H}}(1) + \mathbf{R}_\xi^{\mathrm{H}}(1) \\ b_0(i)\boldsymbol{\Sigma}_{00}(1) + \mathbf{R}_\xi(1) & \sum_{n=0}^{N} \boldsymbol{\Sigma}_{nn}(0) + \mathbf{R}_\xi(0) \end{bmatrix}. \tag{38}$$

Next, using the same method as in the previous subsection for BPSK, we approximate $\boldsymbol{\mu}_d$ as a complex Gaussian vector with mean $\overline{\boldsymbol{\mu}}_d$, and covariance matrix $\mathbf{P}_{\mu_d}$, which are respectively given by

$$\overline{\boldsymbol{\mu}}_d = \left[d_0(i-1)\boldsymbol{\mu}_{c,0}^{\top},\, d_0(i)\boldsymbol{\mu}_{c,0}^{\top}\right]^{\top} \tag{39}$$

$$\mathbf{P}_{\mu_d} = \begin{bmatrix} \sum_{n=1}^{N} \boldsymbol{\mu}_{c,n}\boldsymbol{\mu}_{c,n}^{\mathrm{H}} & \mathbf{0}_L \\ \mathbf{0}_L & \sum_{n=1}^{N} \boldsymbol{\mu}_{c,n}\boldsymbol{\mu}_{c,n}^{\mathrm{H}} \end{bmatrix}. \tag{40}$$

Assuming that $\mathbf{P}_{\mu_d}$ has full rank, we can use a derivation similar to the one used to obtain (10), and get an approximate average MGF expression

$$\tilde{\Phi}_D(s) = \frac{\exp\left[\overline{\boldsymbol{\mu}}_d^{\mathrm{H}}\mathbf{P}_{\mu_d}^{-1/2}\left(\left(\mathbf{I} - \mathbf{P}_{\mu_d}^{1/2}\mathbf{F}(s)\mathbf{P}_{\mu_d}^{1/2}\right)^{-1} - \mathbf{I}\right)\mathbf{P}_{\mu_d}^{-1/2}\overline{\boldsymbol{\mu}}_d\right]}{\det(\mathbf{I} - \mathbf{P}_{\mu_d}\mathbf{F}(s))\prod_{k=1}^{2L}(1 - s\lambda_k)} \tag{41}$$

where, in the expression for $\mathbf{F}(s)$, $\mathbf{P}_v$ is replaced by $\hat{\mathbf{P}}_v$ in (38). We call this scheme Method I for DPSK. If we use the diagonal matrix $\mathrm{diag}(\mathbf{P}_{\mu_d})$ rather than $\mathbf{P}_{\mu_d}$ in (41), we call the resulting expression Method II. When $\mathbf{P}_{\mu_d}$ is rank deficient, (34) can be used with the parameters replaced by those for DPSK.

Similarly to the case of BPSK, we can obtain a simpler form that is equivalent to (41) as

$$\tilde{\Phi}_D(s) = \frac{\exp\left(\overline{\boldsymbol{\mu}}_d^{\mathrm{H}}\acute{\mathbf{P}}_v^{-1/2}\left(\left(\mathbf{I} - s\acute{\mathbf{P}}_v^{1/2}\mathbf{Q}\acute{\mathbf{P}}_v^{1/2}\right)^{-1} - \mathbf{I}\right)\acute{\mathbf{P}}_v^{-1/2}\overline{\boldsymbol{\mu}}_d\right)}{\det\left(\mathbf{I} - s\acute{\mathbf{P}}_v^{1/2}\mathbf{Q}\acute{\mathbf{P}}_v^{1/2}\right)} \tag{42}$$

where $\overline{\boldsymbol{\mu}}_d$ is given by (39). In (42), we set $\acute{\mathbf{P}}_v = \hat{\mathbf{P}}_v + \mathbf{P}_{\mu_d}$ for Method I and $\acute{\mathbf{P}}_v = \hat{\mathbf{P}}_v + \mathrm{diag}(\mathbf{P}_{\mu_d})$ for Method II, where $\hat{\mathbf{P}}_v$ and $\mathbf{P}_{\mu_d}$ are given by (38) and (40), respectively. By using a numerical method, this scheme needs to evaluate only two integrals rather than 2^{2N+1} integrals for the exact evaluation scheme.

Note that the approximation technique proposed in this subsection and the previous one is carried out in a multivariate analysis framework, thus it is significantly different from traditional Gaussian approximation (TGA) technique for BEP analysis [13]–[15], in which the residual CCI and noise is approximated as a Gaussian random variable.

C. Comparison With the Traditional Gaussian Approximation

To illustrate the differences between our technique and the popular TGA method, we develop new results by applying the latter method to the signal model studied in this paper.

Let the instantaneous SINR of the receiver output be denoted by γ_{SINR}. The average BEP is given by

$$P_b = \int_0^\infty P_b(\gamma_{\mathrm{SINR}}) f(\gamma_{\mathrm{SINR}})\, d\gamma_{\mathrm{SINR}} \tag{43}$$

where $f(\gamma_{\mathrm{SINR}})$ is the probability density function (pdf) of γ_{SINR} and $P_b(\gamma_{\mathrm{SINR}})$ is the conditional BEP. For BPSK modulation $P_b(\gamma_{\mathrm{SINR}}) = Q(\sqrt{\gamma_{\mathrm{SINR}}})$, where $Q(x) = \int_x^\infty (1/\sqrt{2\pi})e^{-(y^2/2)}\, dy$ is the Gaussian-Q function. For DPSK with L-fold diversity, the conditional BEP is given in [1].

For MRC diversity reception of the signal model given in (3), the output signal at the lth branch, D_l, can be expressed as (here we drop the bit index for simplicity)

$$D_l = c_{0,l}^* r_l = |c_{0,l}|^2 d_0 + \sum_{n=1}^{N} c_{0,l}^* c_{n,l} d_n + c_{0,l}^* \xi_l \tag{44}$$

where the first term on the right-hand side is the desired signal component, and the second and third terms are due to the CCI and the noise, respectively. The variance of D_l conditioned on $\mathbf{c}_0$ is given by

$$\sigma_{D_l}^2 = |c_{0,l}|^2 \left(\sum_{n=1}^{N} E[|c_{n,l}|^2] + \sigma^2\right).$$

The SINR for D_l is then given by

$$\gamma_{D_l} = \frac{|c_{0,l}|^2}{\left(\sum_{n=1}^{N} E[|c_{n,l}|^2] + \sigma^2\right)}.$$

To proceed further, we assume that the signals for all the interferers are i.i.d. at the L diversity branches, then the denominator of γ_{D_l} is independent of l ($l = 1, \ldots, L$), and the SINR of the MRC output is given by

$$\gamma_{\mathrm{SINR}} = \frac{(\mathbf{c}_0^{\mathrm{H}}\mathbf{c}_0)}{\sigma_I^2} \tag{45}$$

where $\sigma_I^2 = \sum_{n=1}^{N} E[|c_{n,l}|^2] + \sigma^2 = (1/L)\sum_{n=1}^{N} w_n + \sigma^2$ is the average power of all the interferers and the noise per diversity branch, and $w_n = E[\mathbf{c}_n^{\mathrm{H}}\mathbf{c}_n]$ is defined as the mean signal power for the nth interferer, for $n = 1, \ldots, N$.

Deriving the pdf expression $f(\gamma_{\mathrm{SINR}})$ is both tedious and unnecessary. Instead, we can use the MGF method to evaluate (43) more conveniently. The MGF of γ_{SINR}, $\Phi_{\gamma_{\mathrm{SINR}}}(s)$, can be evaluated by using the method proposed in [27], as shown below.

TABLE I
BEP VALUES (TRUNCATED TO FOUR MOST SIGNIFICANT DIGITS) COMPUTED BY USING THE ACCURATE AND THE TGA METHODS FOR BPSK WITH MRC ($L = 3$) IN INDEPENDENT AND CORRELATED FADING CHANNELS. $K_0 = 7$ dB, $K_I = -\infty$ dB, $N = 6$, AND $\overline{\gamma}_{\rm SNR} = 30$ dB. $x(-n)$ DENOTES $x \times 10^{-n}$

	Independent Rayleigh channel ($\rho = 0$)				
γ_{SIR} (dB)	0	6	12	18	24
Accurate	1.226(-2)	7.133(-5)	5.957(-8)	1.982(-10)	5.222(-12)
TGA [(46), (47)]	1.226(-2)	7.133(-5)	5.957(-8)	1.982(-10)	5.222(-12)
	Correlated Rayleigh channel ($\rho = 0.4$)				
γ_{SIR} (dB)	0	6	12	18	24
Accurate	4.370(-2)	1.735(-3)	1.457(-5)	1.513(-7)	5.168(-9)
TGA [(46), (47)]	1.681(-2)	3.895(-4)	4.630(-6)	8.927(-8)	4.320(-9)

Define the eigen-decomposition of the covariance matrix of $\mathbf{c}_0$ as $\mathbf{\Sigma}_{00}(0) = \mathbf{U}\mathbf{\Lambda}_0\mathbf{U}^{\rm H}$, with $\mathbf{\Lambda}_0 = {\rm diag}(\lambda_1, \ldots, \lambda_L)$. Then we define a transform of the static fading vector of $\mathbf{c}_0$, that is $\acute{\boldsymbol{\mu}} = [\acute{\mu}_1, \ldots, \acute{\mu}_L]^\top = \mathbf{U}^{\rm H}[\mathbf{\Sigma}_{00}(0)]^{-(1/2)}\boldsymbol{\mu}_{c,0}$. The MGF is given by

$$\Phi_{\gamma_{\rm SINR}}(s) = \exp\left(\sum_{k=1}^{L} \frac{s\lambda_k/\sigma_I^2}{1 - s\lambda_k/\sigma_I^2} |\acute{\mu}_k|^2\right) \prod_{k=1}^{L} (1 - s\lambda_k/\sigma_I^2)^{-1}. \quad (46)$$

By using the alternative form of the Gaussian-Q function [28], for BPSK demodulation the average BEP expression in (43) is given by

$$P_b = \frac{1}{\pi}\int_0^{\pi/2} \Phi_{\gamma_{\rm SINR}}(-1/\sin^2\theta)\, d\theta \quad (47)$$

where $\Phi_{\gamma_{\rm SINR}}(s)$ is given by (46). To the best of our knowledge, (47) and (46) are new and exact results for MRC diversity of a Rician-faded signal in the presence of multiple Rayleigh-faded interferers with arbitrary mean signal powers. In this fading channel case, the TGA method [(46) and (47)] is equivalent to our exact approach (7), and also to our low-complexity Methods I and II [(30), (35)], which is verified by our numerical evaluations. This can be explained by two facts: 1) when the interferers are Rayleigh faded, $\sum_{n=1}^{N} \mathbf{c}_{n,l}(i)d_n(i) + \xi_l(i)$, the sum of the N interferers' signal and the noise at each lth branch ($l = 1, \ldots, L$), is a zero-mean Gaussian variable with variance σ_I^2 and 2) the sum of the interference components and the noise is independent between different branches.

Based on a different approach, the work in [7] presents another expression for the distribution of the SINR of one Rayleigh-faded signal in the presence of multiple Rayleigh-faded interferers with L-fold MRC diversity. In [7], a pdf expression $f(\gamma_{\rm SINR})$ was derived by analyzing the probability density distributions of both the interferers and the desired signal. In comparison, our TGA result [(47), (46)] applies to more general cases. First, our result applies to the case when the desired signal is Rician-faded with nonidentical statistics at different branches, while in [7] the desired signal was limited to be Rayleigh faded with i.i.d. statistics. Second, for the case of different mean powers w_n of the CCI, [7] gives a result for up to $N = 3$ interferers, while our result is valid for an arbitrary number of interferers with distinct w_n. Equation (46) can also be used to obtain the outage probability expression for the desired signal, in the form of a single integral. Furthermore, by using (46), the analysis can be extended to many coherent and noncoherent modulation formats (see [29]).

Although the new result using the TGA approach is quite useful, it is still less versatile than the low-complexity Methods I and II [(30) and (35)] in that the former only applies to the independent fading channels, while the latter methods are applicable to correlated fading channels as well. Some numerical comparison of the TGA approach and our low-complexity methods are given in the next section.

VI. NUMERICAL RESULTS

Numerical results on BEP performance of coherent and differentially coherent detection are presented, including the effect of diversity reception, fading correlation, and multiple cochannel interferers, etc. For the Rician fading channel, the Rician factors at all the diversity branches that belong to the same user are assumed to be identical, that is $K_n = K_{n,1} = \cdots = K_{n,L}$ for $n = 0, 1, \ldots, N$. Furthermore, when the fading correlation is considered, we assume a constant correlation model for all the users. For example, for $L = 3$, we set

$$\mathbf{\Sigma}_{nn}(0) = \frac{w_n}{(1 + K_n)L}\begin{bmatrix} 1 & \rho & \rho \\ \rho & 1 & \rho \\ \rho & \rho & 1 \end{bmatrix}$$

for all n, where w_0 and $w_1 = w_2 = \cdots = w_N$ are the normalized mean signal powers of the desired user and the cochannel interferers, respectively. The iid fading channel model is obtained by setting $\rho = 0$. The average signal-to-noise ratio (SNR) at the combiner output is defined as $\overline{\gamma}_{\rm SNR} = w_0/\sigma^2$, and the average signal-to-interference ratio (SIR) is defined as $\overline{\gamma}_{\rm SIR} = w_0/\sum_{n=1}^{N} w_n$.

First, we compare the TGA approach [(47), (46)] with the exact result for BPSK modulation with MRC diversity ($L = 3$), when the desired user is Rician faded with the SNR $\overline{\gamma}_{\rm SNR} = 30$ dB, and in the presence of $N = 6$ Rayleigh-faded interferers. The desired user has the Rician factor $K_0 = 7$ dB and real static fading components $\boldsymbol{\mu}_{c,0}$ (that is $\boldsymbol{\theta}_0 = \mathbf{0}$). The BEP values computed by using the exact and approximate expressions for different operating SIRs are presented in Table I.

For independent diversity with Rayleigh-faded interferers, the TGA gives accurate results as expected. However, in the presence of correlated diversity branches ($\rho = 0.4$), the TGA approach is quite inaccurate. Note that, in this case, the approximate schemes using (35) is accurate.

TABLE II
BEP VALUES (TRUNCATED TO THE THREE MOST SIGNIFICANT DIGITS) COMPUTED BY USING THE ACCURATE AND THE APPROXIMATE METHODS FOR BPSK WITH MRC ($L = 3$) IN INDEPENDENT AND CORRELATED FADING CHANNELS. $K_0 = 7$ dB, $K_I = 3$ dB, $N = 6$, AND $\overline{\gamma}_{\rm SNR} = 30$ dB. $x(-n)$ DENOTES $x \times 10^{-n}$

	Independent Rician channel ($\rho = 0$, $K_I = 3$ dB)				
γ_{SIR} (dB)	0	6	12	18	24
Accurate	1.61(-2)	6.63(-5)	5.03(-8)	1.85(-10)	5.14(-12)
Method I	1.44(-2)	9.80(-5)	7.97(-8)	2.25(-10)	5.43(-12)
Method II	1.23(-2)	7.13(-5)	5.96(-8)	1.98(-10)	5.22(-12)
	Correlated Rician channel ($\rho = 0.4$, $K_I = 3$ dB))				
γ_{SIR} (dB)	0	6	12	18	24
Accurate	2.25(-2)	1.67(-4)	1.77(-7)	6.66(-10)	1.81(-11)
Method I	1.93(-2)	2.04(-4)	2.50(-7)	7.87(-10)	1.91(-11)
Method II	1.72(-2)	1.70(-4)	2.19(-7)	7.47(-10)	1.88(-11)

TABLE III
BEP VALUES (TRUNCATED TO THE THREE MOST SIGNIFICANT DIGITS) COMPUTED BY USING THE ACCURATE AND THE APPROXIMATE METHODS FOR BPSK WITH MRC ($L = 3$) IN INDEPENDENT AND CORRELATED FADING CHANNELS. $K_0 = 7$ dB, $K_I = 6$ dB, $N = 6$, AND $\overline{\gamma}_{SNR} = 30$ dB. $x(-n)$ DENOTES $x \times 10^{-n}$

	Independent Rician channel ($\rho = 0$, $K_I = 6$ dB)				
γ_{SIR} (dB)	0	6	12	18	24
Accurate	1.64(-2)	5.35(-5)	4.16(-8)	1.71(-10)	5.03(-12)
Method I	1.48(-2)	1.05(-4)	8.55(-8)	2.33(-10)	5.50(-12)
Method II	1.23(-2)	7.13(-5)	5.96(-8)	1.98(-10)	5.22(-12)
	Correlated Rician channel ($\rho = 0.4$, $K_I = 6$ dB))				
γ_{SIR} (dB)	0	6	12	18	24
Accurate	2.14(-2)	1.23(-4)	1.35(-7)	5.95(-10)	1.76(-11)
Method I	1.84(-2)	1.92(-4)	2.44(-7)	7.86(-10)	1.91(-11)
Method II	1.59(-2)	1.55(-4)	2.12(-7)	7.42(-10)	1.88(-11)

Next, we show the accuracy of our low-complexity methods (35) for the case of a Rician-faded signal in multiple Rician-faded interferers. We assume the static components of the desired user $\boldsymbol{\mu}_{c,0}$ are complex, with the phase vector $\boldsymbol{\theta}_0 = [0, \pi/4, \pi/2]^\top$. The phases of the static components of the interferers, $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_N$, are independent and uniformly distributed between $[0, 2\pi)$. We set $K_0 = 7$ dB and $\overline{\gamma}_{SNR} = 30$ dB. Let $K_I = K_1 = \cdots = K_N$ denote the Rician factor of all the interferers. The BEP results using the exact analysis (7) and the approximate methods (35) for $K_I = 3$ dB and 6 dB are given in Tables II and III, respectively.

The results show that our low-complexity methods give quite a good approximation to the accurate result, and the accuracy improves when the SIR becomes high. In comparison, Method II is more accurate than Method I for medium to high SIRs.

After showing the accuracy of our low-complexity methods, we apply them to the analysis of BEP performance of coherent and differentially coherent modulations in a cellular radio system and study in detail the effects of correlated diversity branches and nonidentical phases in $\boldsymbol{\theta}_0$. In the system, the normalized reuse distance R is defined as the ratio of the distances between the centers of two nearby cochannel cells and the cell radius of the desired station. As in [30], the fourth power path loss law is assumed for the user and all the interferers; we also use the approximation that the distances

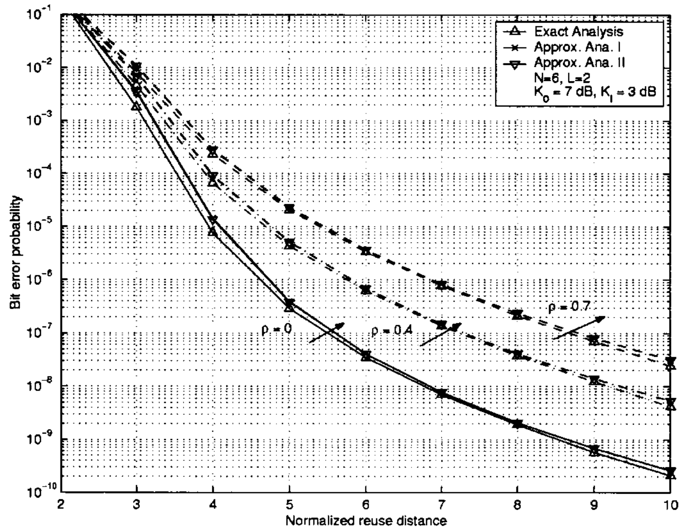

Fig. 3. BEP versus the normalized reuse distance for BPSK with MRC in independent and correlated Rician fading channels, with $L = 2$, $K_0 = 7$ dB, $K_I = 3$ dB, and with $N = 6$ cochannel interferers. $\boldsymbol{\mu}_{c,0}$ is real and $\{\boldsymbol{\mu}_{c,n}\}_{n=1}^N$ are complex.

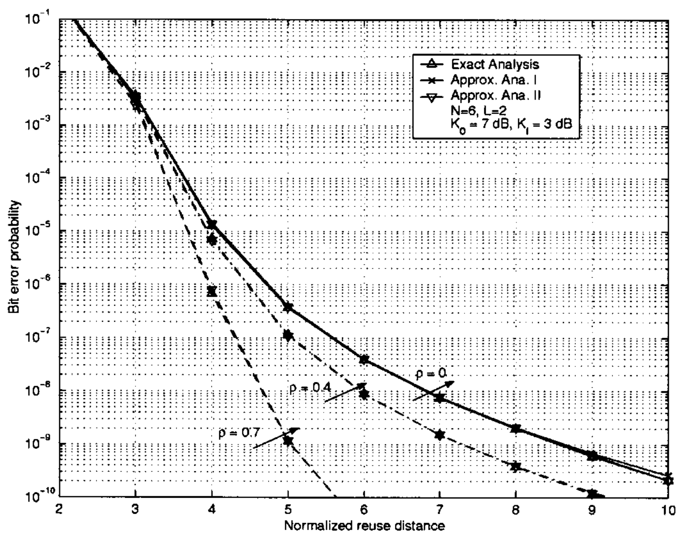

Fig. 4. BEP versus the normalized reuse distance for BPSK with MRC in independent and correlated Rician fading channels, with $L = 2$, $K_0 = 7$ dB, $K_I = 3$ dB, and with $N = 6$ cochannel interferers. $\boldsymbol{\mu}_{c,0}$ is complex with phases $\boldsymbol{\theta}_0 = [0, \pi/2]^\top$, and $\{\boldsymbol{\mu}_{c,n}\}_{n=1}^N$ are also complex.

from the different interfering base stations to the mobile are equal. Since the interference is considered to be overwhelming over the background noise in cellular radio systems, we ignore the effect of the additive noise. We set $K_0 = 7$ dB and $K_I = 3$ dB below.

In Figs. 3 and 4, we plot BEP curves against R for BPSK with MRC diversity ($L = 2$) over independent and correlated Rician fading channels, and with $N = 6$ cochannel interferers.

In Fig. 3, the static fading components of the desired user are assumed to be cophased (i.e., $\boldsymbol{\theta}_0 = \mathbf{0}$), while in Fig. 4 the static components have the same powers but different phases which are arbitrarily set to $\boldsymbol{\theta}_0 = [0, \pi/2]^\top$. We use this set of values for $\boldsymbol{\theta}_0$ just to demonstrate that $\boldsymbol{\theta}_0$ have a significant impact on the BEP performance, nevertheless it should be understood that $\boldsymbol{\theta}_0$ may be arbitrarily distributed in reality. The phases of the static

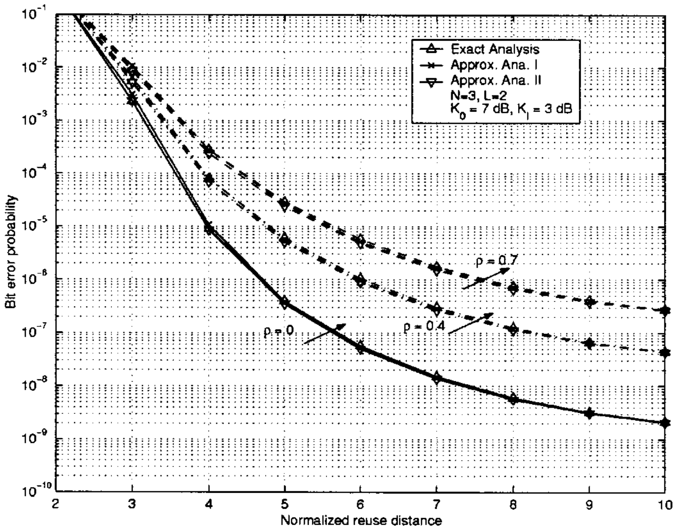

Fig. 5. BEP versus the normalized reuse distance for DPSK with EGC in independent and correlated Rician fading channels, with $L = 2$, $K_0 = 7$ dB, $K_I = 3$ dB, and with $N = 3$ cochannel interferers. $\boldsymbol{\mu}_{c,0}$ is real and $\{\boldsymbol{\mu}_{c,n}\}_{n=1}^{N}$ are complex.

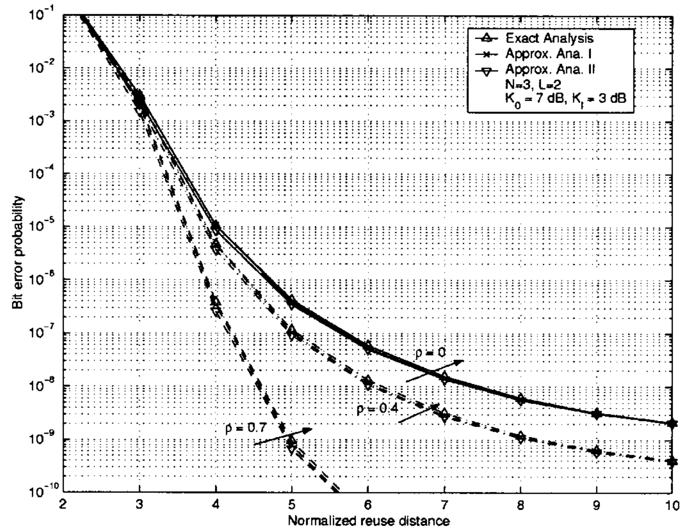

Fig. 6. BEP versus the normalized reuse distance for DPSK with EGC in independent and correlated Rician fading channels, with $L = 2$, $K_0 = 7$ dB, $K_I = 3$ dB, and with $N = 3$ cochannel interferers. $\boldsymbol{\mu}_{c,0}$ is complex with the phase vector $\boldsymbol{\theta}_0 = [0,\ \pi/2]^\top$, $\{\boldsymbol{\mu}_{c,n}\}_{n=1}^{N}$ are complex.

components of the interferers, $\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_N$, are still assumed to be independent and uniformly distributed between $[0, 2\pi)$.

As the reuse distance R increases, the BEP in all cases decreases, and the approximate methods give a good matching to the exact results in the whole reuse distance range studied. For independent channels, the BEP curves are insensitive to the phase vector $\boldsymbol{\theta}_0$, which can be observed by comparing the curves for $\rho = 0$ in Figs. 3 and 4. However, for correlated fading channels, $\boldsymbol{\theta}_0$ has a substantial impact on the BEP performance. It is interesting to observe that, when $\boldsymbol{\theta}_0 = \mathbf{0}$, the BEP curves degrade significantly as the fading correlation ρ increases. On the contrary for $\boldsymbol{\theta}_0 = [0, \pi/2]^\top$ that we have set, the BEP curves improves as ρ increases. Our extensive computations (not shown here) also demonstrate that channels with cophased $\boldsymbol{\mu}_{c,0}$ normally cause higher BEPs than those with noncophased $\boldsymbol{\mu}_{c,0}$. In other words, static fading components with phase differences for the desired user will in many cases give lower BEPs, as shown in Fig. 4.

Next we consider DPSK modulation with EGC diversity, with $L = 2$ and $N = 3$ cochannel interferers. The exact result for DPSK is computed from (18) and (22), by using the MGF expression (10) and (11), where $\mathbf{P}_v$ and $\boldsymbol{\mu}_d$ are given by (19) and (20). BEP versus R is plotted in Fig. 5 for real $\boldsymbol{\mu}_{c,0}$, and in Fig. 6 for complex $\boldsymbol{\mu}_{c,0}$, respectively. The temporal fading correlation coefficients are assumed to be identical for all the branches and for all the users, that is, $\boldsymbol{\Sigma}_{nn}(1) = \rho_t \boldsymbol{\Sigma}_{nn}(0)$, for $n = 0, 1, \ldots, N$. We set $\rho_t = 0.995$: it is known that a larger ρ_t corresponds to a smaller Doppler frequency bandwidth for a given fading power spectrum [1], [9], thus a better performance will result for differential detection.

As R increases, the approximate BEP curves become more accurate and merge with the exact BEP curves, and they all give almost identical error floors for the same spatial correlation ρ. The appearance of error floors when the CCI vanishes is due to the fact that $\rho_t < 1$ (or the nonzero Doppler fading bandwidth) for the desired signal. This also demonstrates that the Rayleigh-fading approximation for the interferers becomes more accurate for larger reuse distance. When ρ increases the same trend as the coherent modulation is observed, that is, the performance degrades for larger ρ with real static components, while it improves for complex static components with the phases $\boldsymbol{\theta}_0$ that we set. It is interesting to see that with complex $\boldsymbol{\mu}_{c,0}$ and higher ρ, the BEP curves and their error floors are significantly lower than the others.

VII. Conclusion

In this paper, we proposed a general mathematical framework to determine the BEP performance for binary coherent and differentially coherent modulation formats with diversity reception in arbitrary Rician fading channels, and in the presence of multiple cochannel interferers. The effects of arbitrary and distinct fading correlation and noise correlation between different diversity branches for the desired user and the interferers were modeled exactly. By expressing the decision variables in Gaussian quadratic forms and using the MGF approach, we derived the exact BEP expressions, which however entails a complexity that increases exponentially with the number of cochannel interferers. To facilitate performance evaluation for systems with a large number of cochannel interferers, we proposed two low-complexity approximate BEP evaluation methods which yield accurate results and are far superior to the TGA method.

In independent Rician fading channels, the BEP performance is insensitive to the phase distribution of static fading components for the desired user. However in correlated diversity channels, the phase distribution plays an important role and may result in significantly different BEP performance. Numerical results also showed that increasing fading correlation will not necessarily degrade the detection performance in Rician fading channels, but on the contrary may significantly improve it, unlike the case for the Rayleigh fading channels.

With slight modifications, the results in this paper are applicable to many other modulation formats, including 4- and 8-PSK/DPSK, NCFSK. The proposed analytical technique is also useful for the analysis of linear multiuser receivers in arbitrary Rician faded CDMA channels, which is an ongoing research subject.

APPENDIX
DERIVATION OF THE APPROXIMATE AVERAGE MGF EXPRESSION FOR THE DECISION VARIABLE

We approximate $\boldsymbol{\mu}_d$ as a Gaussian vector, that is, $\boldsymbol{\mu}_d \sim CN(\overline{\boldsymbol{\mu}}_d, \mathbf{P}_{\mu_d})$. After decomposing $\mathbf{P}_{\mu_d}$ to $\mathbf{P}_{\mu_d} = \mathbf{B}\mathbf{B}^{\mathrm{H}}$, we can express $\boldsymbol{\mu}_d$ as $\boldsymbol{\mu}_d = \overline{\boldsymbol{\mu}}_d + \mathbf{B}\mathbf{y}$, where $\mathbf{y} \sim CN(\mathbf{0}, \mathbf{I}_r)$ is a r-variate complex Gaussian vector, and its pdf is

$$f(\mathbf{y}) = \frac{1}{\pi^r} \exp\left(-\mathbf{y}^{\mathrm{H}}\mathbf{y}\right). \qquad \text{(A1)}$$

The derivation here follows similarly to that of [21, Theorem 3.2a.3] for real Gaussian quadratic forms, but our result is more general and is applicable to more complex Gaussian quadratic forms. Using (10) and (A1), the average MGF expression (29) can be approximated by

$$\tilde{\Phi}_D(s) = \int_{\mathbf{y}} \frac{\exp\left(\boldsymbol{\mu}_d^{\mathrm{H}}\mathbf{F}(s)\boldsymbol{\mu}_d - \mathbf{y}^{\mathrm{H}}\mathbf{y}\right)}{\pi^r \det(\mathbf{I} - s\boldsymbol{\Lambda}_v)}\, d\mathbf{y}. \qquad \text{(A2)}$$

Here $\int_{\mathbf{y}} d\mathbf{y}$ denotes an r-dimensional integral over vector $\mathbf{y}$. Obviously $\mathbf{B}^{\mathrm{H}}\mathbf{F}(\mathbf{s})\mathbf{B}$ is not a zero matrix, then it can be shown that

$$\begin{aligned}\boldsymbol{\mu}_d^{\mathrm{H}}\mathbf{F}(s)\boldsymbol{\mu}_d - \mathbf{y}^{\mathrm{H}}\mathbf{y} &= \overline{\boldsymbol{\mu}}_d^{\mathrm{H}}\mathbf{F}(s)\overline{\boldsymbol{\mu}}_d + \overline{\boldsymbol{\mu}}_d^{\mathrm{H}}\mathbf{F}(s)\mathbf{B}\left(\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\overline{\boldsymbol{\mu}}_d \\ &\quad - (\mathbf{y} - \mathbf{c})^{\mathrm{H}}\left(\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B}\right)(\mathbf{y} - \mathbf{c}) \qquad \text{(A3)}\end{aligned}$$

where $\mathbf{c} = (\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B})^{-1}\mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\overline{\boldsymbol{\mu}}_d$. Using the property of Gaussian density function that $\int_{\mathbf{y}} \exp(-\mathbf{y}^{\mathrm{H}}\boldsymbol{\Sigma}^{-1}\mathbf{y})\, d\mathbf{y} = \pi^r \det(\boldsymbol{\Sigma})$ [see (2)], we have

$$\int_{\mathbf{y}} \exp\left(-(\mathbf{y} - \mathbf{c})^{\mathrm{H}}\left(\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B}\right)(\mathbf{y} - \mathbf{c})\right) d\mathbf{y} = \frac{\pi^r}{\det(\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B})}. \qquad \text{(A4)}$$

It is notable that, since variable s can be in an arbitrarily small neighborhood of the origin, without loss of generality both matrices $\mathbf{F}(s)$ and $\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B}$ can be assumed Hermitian symmetric, and in addition $\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B}$ can be assumed to be positive definite. By substituting (A4) and (A3) into (A2), finally we get a new average MGF expression

$$\tilde{\Phi}_D(s) = \frac{\exp\left[\overline{\boldsymbol{\mu}}_d^{\mathrm{H}}\mathbf{F}(s)\left(\mathbf{I} + \mathbf{B}\left(\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B}\right)^{-1}\mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\right)\overline{\boldsymbol{\mu}}_d\right]}{\det(\mathbf{I} - \mathbf{B}^{\mathrm{H}}\mathbf{F}(s)\mathbf{B})\det(\mathbf{I} - s\boldsymbol{\Lambda}_v)}. \qquad \text{(A5)}$$

REFERENCES

[1] J. G. Proakis, *Digital Communications*, 3rd ed. New York: McGraw-Hill, 1995.
[2] M. K. Simon and M.-S. Alouini, "A unified approach to the probability of error for noncoherent and differentially coherent modulations over generalized fading channels," *IEEE Trans. Commun.*, vol. 46, pp. 1625–1638, Dec. 1998.
[3] L. Bin and P. Ho, "Data-aided linear prediction receiver for coherent DPSK and CPM transmitted over Rayleigh flat-fading channels," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1229–1236, July 1999.
[4] J. H. Winters, "Optimum combining in digital mobile radio with cochannel interference," *IEEE J. Select. Areas Commun.*, vol. SAC-2, pp. 528–539, July 1984.
[5] A. Shah and A. M. Haimovich, "Performance analysis of optimum combining in wireless communications with Rayleigh fading and cochannel interference," *IEEE Trans. Commun.*, vol. 46, pp. 473–479, Apr. 1998.
[6] E. Villier, "Performance analysis of optimum combining with multiple interferers in flat Rayleigh fading," *IEEE Trans. Commun.*, vol. 47, pp. 1503–1510, Oct. 1999.
[7] J. Cui and A. U. H. Sheikh, "Outage probability of cellular radio systems using maximum ratio combining in the presence of multiple interferers," *IEEE Trans. Commun.*, vol. 47, pp. 1121–1124, Aug. 1999.
[8] M. Abdel-Hafez and M. Safak, "Performance analysis of digital cellular radio systems in Nakagami fading and correlated shadowing environment," *IEEE Trans. Veh. Technol.*, vol. 48, pp. 1381–1391, Sept. 1999.
[9] M. Schwartz, W. Bennett, and S. Stein, *Communication Systems and Techniques*. New York: McGraw Hill, 1966.
[10] M. J. Juntti and M. Latva-aho, "Bit-error probability analysis of linear receivers for CDMA systems in frequency-selective fading channels," *IEEE Trans. Commun.*, vol. 47, pp. 1788–1791, Dec. 1999.
[11] Y. Ma and T. J. Lim, "Bit error probability for MDPSK and NCFSK over arbitrary Rician fading channels," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 2179–2189, Nov. 2000.
[12] Y. Ma, C. C. Chai, and T. J. Lim, "Bit error probability for 2 and 4 DPSK in general Nakagami fading channels with correlated Gaussian noise," *IEE Proc.—Commun.*, vol. 147, pp. 155–162, June 2000.
[13] T. Eng and L. B. Milstein, "Coherent DS-CDMA performance in Nakagami multipath fading," *IEEE Trans. Commun.*, vol. 43, pp. 1134–1143, Feb./Mar./Apr. 1995.
[14] G. P. Efthymoglou, V. A. Aalo, and H. Helmken, "Performance analysis of coherent DS-CDMA systems in a Nakagami fading channel with arbitrary parameters," *IEEE Trans. Veh. Technol.*, vol. 46, pp. 289–297, May 1997.
[15] K. Cheun, "Performance of direct-sequence spread-spectrum Rake receivers with random spreading sequences," *IEEE Trans. Commun.*, vol. 45, pp. 1130–1143, Sept. 1997.
[16] P. Y. Kam, "Bit error probabilities of MDPSK over the nonselective Rayleigh fading channel with diversity reception," *IEEE Trans. Commun.*, vol. 39, pp. 220–224, Feb. 1991.
[17] U. Hansson and T. Aulin, "Aspects on single symbol signaling on the frequency flat Rayleigh fading channel," *IEEE Trans. Commun.*, vol. 47, pp. 874–883, June 1999.
[18] R. H. Clarke, "A statistical theory of mobile-radio reception," *Bell Syst. Tech. J.*, vol. 47, pp. 957–1000, July/Aug. 1968.
[19] W. C. Jakes, Jr., *Microwave Mobile Communications*. New York: Wiley, 1974.
[20] Y. Ma, T. J. Lim, and C. C. Chai, "Systematic study on channel nonsymmetry for differentially coherent detection in general Rician fading channels," *Electron. Lett.*, vol. 36, pp. 261–262, Feb. 2000.
[21] A. M. Mathai and S. B. Provost, *Quadratic Forms in Random Variables: Theory and Applications*. New York: Marcel Dekker, 1992.
[22] Y. Ma, "Diversity reception in fading channels and CDMA multiuser detection," Ph.D. dissertation, National University of Singapore, 2000.
[23] W. C. Lindsey, "Error probability for Rician fading multichannel reception of binary and N-ary signals," *IEEE Trans. Inform. Theory*, vol. IT-10, pp. 339–350, Oct. 1964.
[24] C. W. Helstrom, "Calculating error probabilities for intersymbol and cochannel interference," *IEEE Trans. Commun.*, vol. COM-34, pp. 430–435, May 1986.
[25] ——, *Elements of Signal Detection & Estimation*. Englewood Cliffs, NJ: Prentice-Hall, 1995.
[26] M. Abramowitz and I. A. Stegun, *Handbook of Mathematical Functions With Formulas, Graphs, and Mathematical Tables*, 9th ed. New York: Dover, 1970.
[27] Y. Ma, C. C. Chai, and T. J. Lim, "Unified analysis of error probability for MRC in correlated fading channels," *Electron. Lett.*, vol. 35, pp. 1314–1315, Aug. 1999.
[28] M. K. Simon and D. Divsalar, "Some new twists to problems involving the Gaussian probability integral," *IEEE Trans. Commun.*, vol. 46, pp. 200–210, Feb. 1998.
[29] Y. Ma and C. C. Chai, "Unified error probability analysis for generalized selection combining in Nakagami fading channels," *IEEE J. Select. Areas Commun.*, vol. 18, pp. 2198–2210, Nov. 2000.
[30] Q. T. Zhang, "Co-channel interference analysis for mobile radio suffering lognormal shadowed Nakagami fading," *IEE Proc.—Commun.*, vol. 146, no. 1, pp. 49–54, 1999.

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