High-Performance 64-QAM OFDM via Carrier Interferometry Spreading Codes
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Abstract - OFDM (Orthogonal Frequency Division Multiplexing) is an excellent technology that will almost certainly play a large role in next generation wireless communication systems. However, in OFDM, no frequency diversity is exploited to improve BER performance. Today’s OFDM systems attempt to overcome this limitation by application of channel coding and interleaving, which requires a reduction in throughput. In our earlier work, we showed how Carrier Interferometry (CI) codes may be used to spread OFDM symbols over all N subcarriers to exploit frequency diversity without loss in throughput. However, only BPSK was considered in the earlier work. In this paper, we extend the proposed CI/OFDM (Carrier Interferometry with OFDM) system by updating it for application with QAM modulation schemes. (For example, Minimized Mean Square Error Combining (MMSEC) is derived for CI/OFDM with QAM modulation schemes.) Simulation results over multi-path fading channels show that 64QAM CI/OFDM significantly outperforms 64QAM OFDM at the cost of a small increase in complexity.

1 Introduction
Orthogonal Frequency Division Multiplexing (OFDM) represents a powerful technology with a promising future in next generation wireless communication systems [1][2]. Among OFDM’s successes is its adoption as the standard of choice in Wireless Local Area Network (WLAN) systems (e.g., IEEE 802.11a [3], IEEE 802.11g [4], Hyper-LAN II [5]).

In OFDM, N symbols are serial to parallel converted and sent simultaneously over N orthogonal subcarriers [1][2]. The bandwidth of each subcarrier is carefully selected, ensuring that each is much smaller than the coherence bandwidth of the frequency-selective fading channel. As a result, each subcarrier experiences a flat fade. This translates into simple receiver design and a system that avoids multipath.

However, OFDM is not without its drawbacks. Since each OFDM symbol is transmitted over one subcarrier, the information symbol is likely to be lost when that subcarrier undergoes a deep fade. To account for this, Coded OFDM (COFDM) has been introduced (e.g., [2][3]). Here, incoming information bits are channel coded prior to serial to parallel conversion and carefully interleaved. By doing this, each information bit is effectively sent over more than one subcarrier, creating a frequency diversity benefit that overcomes the flat fading degradation. The drawback is, of course, (1) a reduced throughput and (2) the increased complexity due to the addition of a channel decoder (typically a Viterbi decoder).

In our earlier work of [6][7], we proposed a novel OFDM architecture capable of exploiting frequency diversity (improving BER performance) without any throughput loss and with a very low increase in complexity. The novel system, referred to as CI/OFDM (Carrier Interferometry OFDM), spreads each information symbol across all N subcarriers using orthogonal Carrier Interferometry (CI) spreading codes. A performance analysis of the proposed CI/OFDM system shows that, in a BPSK system and at a bit error rate (BER) of $10^{-3}$, CI/OFDM gains approximately 10 dB relative to OFDM.

However, in our earlier work, only BPSK modulation is considered. For higher modulation schemes, e.g., 16QAM or 64QAM, it is possible that the increased MAI (multiple access interference) in CI/OFDM may cancel some of the performance benefits achieved via frequency diversity.

In this paper, we extend the proposed CI/OFDM architecture to study its operation with 16QAM and 64QAM constellations. Optimal Minimized Mean Square Error Combining (MMSEC) schemes for both modulation formats are presented. Performance analysis and simulation results show that, even with QAM modulation schemes, CI/OFDM still significantly outperforms OFDM, gaining 5-7 dB at a fixed $BER = 10^{-3}$.

Section II introduces the QAM CI/OFDM transmitter and receiver structures. Section III presents the MMSE combining schemes for CI/OFDM with QAM modulation. Section IV presents the performance results over frequency-selective fading channels, demonstrating the significant performance gain over OFDM.

II QAM CI/OFDM Transmitter and Receiver Structures
A typical OFDM transmitter is shown in Figure 1(a), and the CI/OFDM transmitter is depicted in Figures 1(b) and 1(c). In both OFDM and CI/OFDM, input bits are serial to parallel converted. In OFDM, each information symbol is modulated onto a single (unique) carrier; in CI/OFDM,
each information symbol is modulated onto all of the $N$ carriers. To ensure the separation of information symbols at the receiver side, the transmitter applies a unique orthogonal spreading code to each information symbol (where spreading is applied in the frequency domain, i.e., across carriers (Figure 1 (c))). In CI/OFDM, orthogonal Carrier Interferometry (CI) codes, first proposed for MC-CDMA systems (CI/MC-CDMA [8][9]), are applied to ensure the orthogonality among all transmitted information symbols. These spreading codes correspond to the application of (to the $k^{th}$ symbol)

$$c^{(k)}(t) = \sum_{i=0}^{N-1} \beta_i^{(k)} e^{j2\pi ft^{(k)}i} \cdot g(t) \quad (1)$$

where (1) $\Delta f$ is the carrier separation ($\Delta f = 1/T_s$ to ensure carrier orthogonality); (2) $g(t)$ is a rectangular pulse shape of duration $T_s$ (where $T_s$ is OFDM symbol length); and (3) $\{\beta_i^{(k)}, i = 0,1,\cdots,N-1\}$ refers to $k^{th}$ symbol’s spreading sequence characterized by

$$\{\beta_0^{(k)}, \beta_1^{(k)}, \cdots, \beta_{N-1}^{(k)}\} = \left\{ e^{-\frac{2\pi}{N} k0}, e^{-\frac{2\pi}{N} k1}, \cdots, e^{-\frac{2\pi}{N} k(N-1)} \right\} \quad (2)$$

It is important to note that CI spreading codes defined in equation (3) are a group of orthogonal spreading codes, i.e.,

$$\text{Re} \left[ \frac{1}{N} \sum_{i=0}^{N-1} e^{j \frac{2\pi}{N} k(i-l)} \right] = \delta_{kl} = \begin{cases} 1 & k = l \\ 0 & k \neq l \end{cases} \quad (3)$$

The transmitted signal for the $k^{th}$ symbol in CI/OFDM system is

$$s^{(k)}(t) = \text{Re} \left[ A \cdot s^{(k)} \cdot c^{(k)}(t) \cdot e^{j2\pi ft^{(k)}i} \right] \quad (4)$$

$$s^{(k)}(t) = \text{Re} \left[ \sum_{i=0}^{N-1} A \cdot s^{(k)} \cdot e^{j(2\pi f - \Delta f)i} \cdot e^{j\frac{2\pi}{N} ki} \cdot e^{j2\pi ft^{(k)}i} \cdot g(t) \right] \quad (5)$$

In equation (5), $A$ is a constant that ensures bit energy of unity (i.e., $A = \frac{1}{\sqrt{N}}$ for BPSK, $A = \frac{1}{\sqrt{5 \cdot N}}$ for 16QAM, and $A = \frac{1}{\sqrt{21 \cdot N}}$ for 64QAM) and, $s^{(k)}$ is the $k^{th}$ information symbol, and corresponds to
\[ s^{(k)} = s_I^{(k)} + j s_Q^{(k)} \]  \hspace{1cm} (6)

where \( s_I^{(k)} \) is the so-called in-phase component of \( s^{(k)} \) and \( s_Q^{(k)} \) is the quadrature component. For BPSK modulation, \( s_I^{(k)} \in \{-1,+1\} \) and \( s_Q^{(k)} = 0 \). For 16QAM, \( s_I^{(k)} \in \{-3,-1,+1,+3\} \). For 64QAM, \( s_I^{(k)} \in \{-7,-5,-3,-1,+1,+3,+5,+7\} \). Also in equation (5), \( f_c \) is the carrier frequency.

The total transmitted signal for one entire CI/OFDM symbol (considering all \( N \) transmit symbols) corresponds to

\[ S(t) = \text{Re} \left[ \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} A \cdot s^{(k)}_i \cdot e^{j(2\pi i \cdot k \cdot f_c)} \cdot e^{\frac{2\pi}{N} k i} \cdot e^{j2\pi f_c t} \cdot g(t) \right] \]  \hspace{1cm} (7)

\[ S(t) = S_I(t) + S_Q(t) \]

\[ = \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} A \cdot s_I^{(k)} \cdot \cos(2\pi f_c t + 2\pi f_c t + \frac{2\pi}{N} k \cdot i) \cdot g(t) \]

\[ - \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} A \cdot s_Q^{(k)} \cdot \sin(2\pi f_c t + 2\pi f_c t + \frac{2\pi}{N} k \cdot i) \cdot g(t) \]  \hspace{1cm} (8)

where \( S_I(t) \) is the in-phase component of the transmit signal and \( S_Q(t) \) is the quadrature component. Similarly, after transmission over a frequency-selective fading channel, the received CI/OFDM signal, assuming the transmit signal in (8), corresponds to

\[ r(t) = r_I(t) + r_Q(t) \]

\[ = \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} A \cdot \alpha_i \cdot s_I^{(k)} \cdot \cos(2\pi f_c t + 2\pi f_c t + 2\pi f_c t + \frac{2\pi}{N} k \cdot i + \phi) \cdot g(t) + n(t) \]

\[ - \sum_{k=0}^{N-1} \sum_{i=0}^{N-1} A \cdot \alpha_i \cdot s_Q^{(k)} \cdot \sin(2\pi f_c t + 2\pi f_c t + 2\pi f_c t + \frac{2\pi}{N} k \cdot i + \phi) \cdot g(t) + n(t) \]  \hspace{1cm} (9)

where \( \alpha_i \) and \( \phi_i \) are the fading gain and phase offset, respectively, introduced into the \( i^{th} \) carrier by the frequency selective Rayleigh fading channel, and (2) \( n(t) \) is additive white Gaussian noise (AWGN). We assume perfect phase synchronization for ease in presentation.

The receiver structure for the \( k^{th} \) symbol in CI/OFDM (with a QAM constellation) is illustrated conceptually in Figure 2. Here, both the in-phase and quadrature components of the received signal are decomposed into their \( N \) carrier components and recombined to minimize the interference from other symbols (inter-symbol interference) and the noise. A hard decision devices follows to create the symbol estimation \( \hat{s}_I^{(k)} \) and \( \hat{s}_Q^{(k)} \). In practice, the frequency decomposition is better implemented (i.e., implemented at a reduced cost) by application of a single FFT.

![Figure 2. Receiver for CI/OFDM with QAM](image-url)

**III Minimized Mean Square Error Combining for CI/OFDM with QAM**

We will present the combiner (see Figure 2) for the in-phase component of the received signal (the presentation is analogous for the quadrature component combining). Considering in-phase component, the decision statistics correspond to (Figure 2)
\[ \mathbf{r}_i^{(k)} = (r_{i,0}^{(k)}, r_{i,1}^{(k)}, \cdots, r_{i,N-1}^{(k)}) \]  

where

\[ r_{i,j}^{(k)} = A\alpha_j s_i^{(l)} + \sum_{l \neq k} A\alpha_l s_l^{(l)} \cos \left( \frac{2\pi}{N} k \cdot i - \frac{2\pi}{N} l \cdot i \right) \]

\[ + \sum_{l \neq k} A\alpha_l s_l^{(l)} \sin \left( \frac{2\pi}{N} k \cdot i - \frac{2\pi}{N} l \cdot i \right) + n_i \]

\[ (9) \]

In equation (9), the first term represents the desired signal on the \( i^{th} \) carrier, the second and the third terms represent inter-symbol interference from the remaining \( N-1 \) symbols, and the fourth term (a zero mean Gaussian random variable with variance \( N_0/2 \)) represents the contribution of additive Gaussian noise. In an AWGN or a flat fading channel, i.e., \( \alpha_i = C \) where \( C \) is a constant, a simple equal gain combining across carriers (index \( i \)) causes the second and third terms (ISI terms) to sum to zero (due to the orthogonality between CI spreading codes) (see equation (3))). However, in a frequency selective fading channel, a carefully designed combiner (across carriers) needs to be employed to counter the loss of orthogonality between CI spreading codes (due to the carrier dependent gain, \( \alpha_i \)). The general form of the combiner corresponds to

\[ R_i^{(k)} = \sum_{j=0}^{N-1} W_j \cdot r_{i,j}^{(k)} \]  

\[ (10) \]

We propose the design of weights \( W_i \) based on Minimized Mean Square Error Combining (MMSEC) since this scheme has been shown (in the MC-CDMA literatures e.g., [10]) (1) to exploit the frequency diversity available in a frequency-selective fading channel and (2) to jointly minimize the inter-symbol interference (the second and third terms in (9)) and the additive noise (the fourth term in (9)).

It is easy to show that the \( i^{th} \) combining weight, derived via the MMSE criteria, corresponds to

\[ W_i = \frac{A\alpha_i}{E[(i_{i,j}^{(k)})^2]} = \frac{A\alpha_i}{NA^2\alpha_i^2E[(s_i^{(l)})^2] + \frac{N_0}{2}} \]  

\[ (11) \]

It is obvious that for 16QAM

\[ E[(s_i^{(l)})^2] = 0.5 \cdot (1)^2 + 0.5 \cdot (3)^2 = 5 \]  

\[ (12) \]

and for 64QAM

\[ E[(s_i^{(l)})^2] = 0.25 \cdot (1)^2 + 0.25 \cdot (3)^2 + 0.25 \cdot (5)^2 + 0.25 \cdot (7)^2 = 21 \]  

\[ (13) \]

Hence, the MMSEC weight for CI/OFDM with 16QAM is

\[ W_i = \frac{1}{\sqrt{5N}}\alpha_i = \frac{\alpha_i}{N \cdot \frac{1}{5N}\alpha_i^2 \cdot 5 + \frac{N_0}{2}} \]  

\[ (14) \]

and for CI/OFDM with 64QAM is

\[ W_i = \frac{1}{\sqrt{21N}}\alpha_i = \frac{\alpha_i}{N \cdot \frac{1}{21N}\alpha_i^2 \cdot 21 + \frac{N_0}{2}} \]  

\[ (15) \]

IV Channel Model and Simulation Results

In this section, we test the performance of the CI/OFDM system with QAM modulation schemes and compare these with the performance of traditional OFDM systems. Here, simulations are performed over frequency selective Rayleigh fading channels, where both CI/OFDM and OFDM systems employ \( N=48 \) carriers to transmit 48 data bearing symbols.

To model realistic wireless environments, the Rayleigh fading channel employed in our simulation demonstrates frequency selectivity over the entire bandwidth, \( BW \), but flat fading over each of the \( N \) carriers. Specifically, we assumed a channel model with coherence bandwidth, \( (\Delta f)_c \), characterized by

\[ (\Delta f)_c / BW = 0.07 \]  

\[ (16) \]

which is common for outdoor OFDM based Wireless LAN (WLAN) systems.

As a result, the \( \alpha_i \)'s in the 48 carriers are correlated according to

\[ \rho_{i,j} = \frac{1}{1 + ((f_i - f_j)/(\Delta f)_c)^2} \]  

\[ (17) \]

where \( \rho_{i,j} \), is the correlation between the \( i^{th} \) carrier and the \( j^{th} \) carrier, and \((f_i - f_j)\) is the frequency separation between these two carriers. Generation of correlated fades, for purposes of simulation, has been discussed in [11].

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Figure 3 illustrates the bit error rate (BER) versus signal to noise ratio (SNR) results for OFDM and CI/OFDM with 16QAM and Figure 4 plots the same axis for 64QAM. In both figures, the curve marked with circles represents the performance of OFDM, while the curve marked with stars represents the performance of the proposed CI/OFDM. It is evident from these figures that CI/OFDM significantly outperforms OFDM in both cases. Specifically, for a 16QAM modulation scheme and a $BER = 10^{-3}$, we observe a 7 dB gain for CI/OFDM relative to OFDM. With 64QAM at $BER = 10^{-3}$, we observe a 5 dB gain for CI/OFDM relative to OFDM. The performance benefits of CI/OFDM over traditional OFDM (evident in Figures 3 and 4) are a direct consequence of the spreading of each information symbol over all $N$ carriers. The frequency domain spreading via orthogonal CI codes brings frequency diversity benefits to CI/OFDM systems, providing gain over traditional OFDM systems (where no frequency diversity is achieved).

V Conclusions

In this paper, the performance of CI/OFDM with QAM modulation schemes is analyzed. In frequency selective fading channels, CI/OFDM outperforms OFDM by, e.g., 7 dB (for 16QAM) and 5 dB (for 64QAM) at a BER of $10^{-3}$ (without any cost in throughput, and with little cost in complexity). This is a direct consequence of CI/OFDM’s inherent ability to exploit the frequency diversity available in frequency-selective channels.

REFERENCES


[4] IEEE P802.11 - TASK GROUP G - Project IEEE 802.11g Standard for Higher Rate (20+ Mbps) Extensions in the 2.4GHz Band


