

Blind Carrier Frequency Offset Estimation via Power Spectrum Analysis in MIMO OFDM Systems*

WU Lu (武露), ZHANG Xianda (张贤达)**

State Key Laboratory on Intelligent Technology and Systems, Tsinghua National Laboratory for Information Science and Technology (TNList), Department of Automation, Tsinghua University, Beijing 100084, China

Abstract: As a generalization of orthogonal frequency-division multiplexing (OFDM) systems, multi-input multi-output (MIMO) OFDM systems are very sensitive to carrier frequency offset (CFO). This paper proposes a blind CFO estimation method based on power spectrum analysis, which has high bandwidth efficiency and is much less complex. This method can be used to estimate the residual CFO, which is less than half of the subcarrier spacing. The method uses a cosine cost function to get a closed-form CFO estimate. Simulation results illustrate that the method is effective for MIMO OFDM systems.

Key words: carrier frequency offset (CFO); multi-input multi-output orthogonal frequency-division multiplexing (MIMO OFDM); power spectrum analysis (PSA)

Introduction

Multi-input multi-output (MIMO) orthogonal frequency-division multiplexing (OFDM) systems have emerged as a strong candidate for future fourth generation (4G) communications due to their high capacity, spectral efficiency, and robustness against fading^[1]. Similar to single-input single-output (SISO) OFDM systems, the carrier frequency offset (CFO) estimation is a critical problem in MIMO OFDM systems. The CFO is caused by the Doppler shift and/or the mismatch between transmitter and receiver oscillators. The CFO results in intercarrier interference (ICI), which significantly degrades system performance^[2].

The methods proposed to estimate the CFO in SISO OFDM systems can be categorized as data-aided methods^[3-5] and blind ones^[6-8]. However, few methods

can be directly applied to the much more complicated MIMO OFDM systems. Some data-aided schemes^[9-11] have been proposed for MIMO OFDM systems. Jiang et al.^[11] suggested placing frequency domain training sequences in the signal to estimate the CFO. Generally speaking, blind methods are more attractive due to their higher bandwidth efficiency. The cyclic prefix (CP) method^[8] estimates the CFO by analyzing the cyclic property of the received signal. Null subcarriers^[12] have been placed at specified locations in the signal to estimate the CFO. Furthermore, Yao and Giannakis^[13] proposed the Kurtosis method by optimizing a Kurtosis-type cost function, which gives good performance in MIMO OFDM systems.

This paper presents a blind CFO estimation method for a frequency-selective fading channel. The cost function is proposed based on an analysis of the amplitude of the signals in adjacent subchannels. The cost function is shown to have a cosine shape. Then, the closed-form estimate of the CFO is given to provide a low-complexity method.

1 System Model

Consider a MIMO OFDM system with P transmit

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** To whom correspondence should be addressed.

E-mail: zxd-dau@tsinghua.edu.cn; Tel: 86-10-62794875

antennas and Q receive antennas as shown in Fig. 1. The input data is distributed onto each transmit antenna by space multiplexing. The signal at each transmit antenna is independent of the other signals. The transmitted data on the p -th transmit antenna at time i is denoted by $\mathbf{s}_p(i) = [s_p^0(i), \dots, s_p^{N-1}(i)]^T$, where N is the number of OFDM subcarriers. The entries of $\mathbf{s}_p(i)$ are assumed to be identically independent distribution (i.i.d) complex random variables with zero

mean and $\sigma_{s_p}^2$ variance. The transmitted data is modulated onto the orthogonal subcarriers using an N -point inverse discrete Fourier transform (IDFT). Assume that the channel between the p -th transmit and q -th receive antenna is block fading with the impulse response being $\mathbf{h}_{pq}(i) = [h_{pq}^0(i), \dots, h_{pq}^L(i)]^T$, where L is the channel order. A CP with length $L_{CP} \geq L$ is inserted to avoid intersymbol interference (ISI).

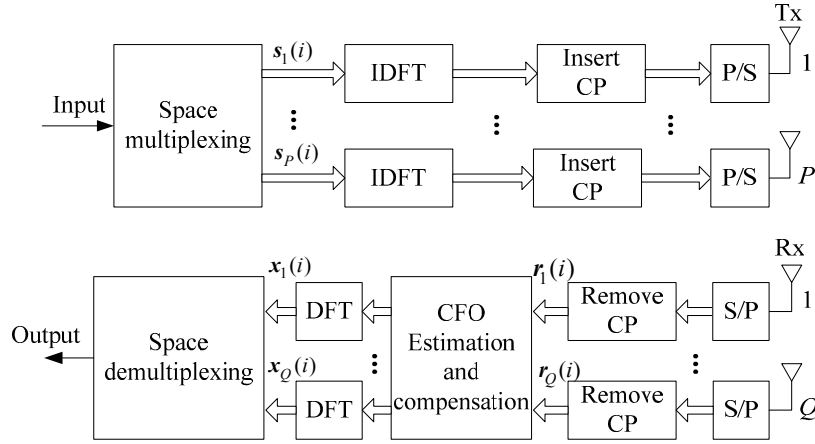


Fig. 1 MIMO OFDM system model

In practice, the differences among the CFOs for all the transmit-receive antenna pairs are usually negligible^[13], so this analysis considers only a single common CFO. After discarding the CP, the received OFDM signal of the q -th receive antenna can be expressed as

$$\mathbf{r}_q(i) = e^{j\theta(i)} \mathbf{D}(\varepsilon) \sum_{p=1}^P \mathbf{H}_{pq}(i) \mathbf{F}^H \mathbf{s}_p(i) + \mathbf{n}_q(i) \quad (1)$$

where ε is the normalized CFO, $\theta(i) = \frac{2\pi}{N} \varepsilon [i(N + L_{CP}) + L_{CP}]$, $\mathbf{D}(\varepsilon) = \text{diag}(1, e^{j2\pi\varepsilon/N}, \dots, e^{j2\pi\varepsilon(N-1)/N})$, and $\mathbf{n}_q(i) \sim \text{CN}(\mathbf{0}, \sigma_{n_q}^2 \mathbf{I})$ is the channel noise. The residual CFO is considered in this paper, i.e., $\varepsilon \in [-0.5, 0.5)$. \mathbf{F} is the $N \times N$ discrete Fourier transform (DFT) matrix with the (m, n) -th element being $e^{-j2\pi mn/N} / \sqrt{N}$. In Eq. (1), $\mathbf{H}_{pq}(i)$ is the channel matrix, whose (m, n) -th element is $h_{pq}^{\text{mod}(m-n, N)}(i)$.

Using an estimated μ to compensate for the CFO, the received signal in the frequency domain is

$$\begin{aligned} \mathbf{x}_q(i) &= \mathbf{FD}(-\mu) \mathbf{r}_q(i) = \\ &= e^{j\theta(i)} \mathbf{FD}(\tilde{\varepsilon}) \sum_{p=1}^P \mathbf{H}_{pq}(i) \mathbf{F}^H \mathbf{s}_p(i) + \mathbf{z}_q(i) = \end{aligned}$$

$$e^{j\theta(i)} \mathbf{FD}(\tilde{\varepsilon}) \mathbf{F}^H \sum_{p=1}^P \mathbf{A}_{pq}(i) \mathbf{s}_p(i) + \mathbf{z}_q(i) \quad (2)$$

where $\mathbf{A}_{pq}(i) \triangleq \mathbf{F} \mathbf{H}_{pq}(i) \mathbf{F}^H = \text{diag}(H_{pq}^0(i), H_{pq}^1(i), \dots, H_{pq}^{N-1}(i))$ with $H_{pq}^k(i) \triangleq \sum_{n=0}^L h_{pq}^n(i) e^{-j2\pi nk/N}$, $\mathbf{z}_q(i) \sim \text{CN}(\mathbf{0}, \sigma_{n_q}^2 \mathbf{I})$, and $\tilde{\varepsilon} = \varepsilon - \mu$ is the CFO estimation error. Equation (2) shows that the CFO destroys the orthogonality of the subcarriers and introduces ICI. However, if the CFO is estimated correctly, i.e., $\tilde{\varepsilon} = 0$, Eq. (2) can be simplified as

$$\mathbf{x}_q(i) = e^{j\theta(i)} \sum_{p=1}^P \mathbf{A}_{pq}(i) \mathbf{s}_p(i) + \mathbf{z}_q(i) \quad (3)$$

where the ICI is completely eliminated. This analysis provides an effective CFO estimation method to suppress the ICI.

2 Estimation Method

2.1 Power spectrum analysis (PSA) theory

In practical OFDM systems, the channel order, L , is much less than the OFDM block size, N , thus, the signals in adjacent subchannels are related as

$$|H_{pq}^{k+1}(i)|^2 = \sum_{n_1=0}^L \sum_{n_2=0}^L (h_{pq}^{n_1}(i)h_{pq}^{n_2*}(i)e^{-j2\pi k(n_1-n_2)/N}) \cdot e^{-j2\pi(n_1-n_2)/N} \approx |H_{pq}^k(i)|^2 \quad (4)$$

Therefore, the signals on adjacent subchannels are similar in amplitude.

Equation (3) shows that when the CFO is totally compensated, the mean square of the received signal, $x_q^k(i)$, can be given by

$$E(|x_q^k(i)|^2) = \sum_{p=1}^P \sigma_{s_p}^2 E(|H_{pq}^k(i)|^2) + \sigma_{n_q}^2 \quad (5)$$

Equations (4) and (5) can be combined to show that the mean square values of adjacent subcarrier signals are more approximate when ε equals zero than for $\varepsilon \neq 0$. An objective function can be defined as follows using the differences between adjacent subcarrier signals as a measure:

$$J_{\text{PSA}}(\mu) = \sum_{i=1}^M \sum_{q=1}^Q \sum_{k=0}^{N-1} (|x_q^k(i)|^2 - |x_q^{k+1}(i)|^2)^2 \quad (6)$$

where M is the number of OFDM blocks employed to estimate the CFO. Equation (6) indicates that a smoother received signal power spectrum can be obtained when the CFO is more accurately estimated. Therefore, the CFO estimate of the current method, called power spectrum analysis (PSA), is given by

$$\hat{\varepsilon} = \arg \min_{\mu} J_{\text{PSA}}(\mu) \quad (7)$$

Figure 2 shows how J_{PSA} varies with the CFO estimate candidate μ averaged over 5000 Monte Carlo trials. J_{PSA} is minimized when the CFO is accurately estimated, which confirms the validity of the power spectrum analysis concept.

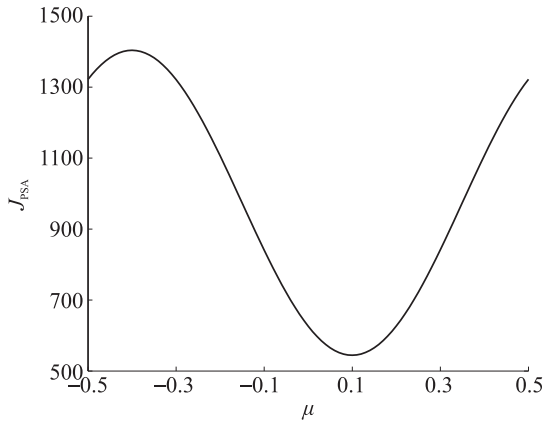


Fig. 2 Cost function for the PSA method when $P=Q=2$, $N=64$, $M=1$, $\varepsilon=0.1$, and $\text{SNR}=20$ dB

2.2 Closed-form CFO estimation

The properties of the cost function of the PSA method were investigated to find a simple, closed-form CFO estimation method.

Let $M=1$ without loss of generality. The cost function can be written as (dropping the time index i for brevity):

$$J_{\text{PSA}}(\mu) = 2 \sum_{q=1}^Q \sum_{k=0}^{N-1} |x_q^k|^4 - 2 \sum_{q=1}^Q \sum_{k=0}^{N-1} |x_q^k|^2 |x_q^{k+1}|^2 \quad (8)$$

From Eq. (2), $|x_q^k|^2$ can be expressed by

$$|x_q^k|^2 = [(\mathbf{FD}(-\mu)\mathbf{r}_q) \odot (\mathbf{FD}(-\mu)\mathbf{r}_q)^*]_k = \frac{1}{\sqrt{N}} \sum_{n=-N+1}^{N-1} R_q(n) e^{-j2\pi nk/N} = \frac{1}{\sqrt{N}} R_q(0) + \frac{1}{\sqrt{N}} \sum_{n=1}^{N-1} (R_q(n) + R_q(n-N)) e^{-j2\pi nk/N} \quad (9)$$

where

$$R_q(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} [\mathbf{D}(-\mu)\mathbf{r}_q]_{n+k} [\mathbf{D}(-\mu)\mathbf{r}_q]_n^* = \frac{1}{\sqrt{N}} e^{-j2\pi\mu k/N} \sum_{n=0}^{N-1} r_q^{n+k} r_q^{n*} \quad (10)$$

Rewriting Eq. (9) and defining $\mathbf{z}_q = [z_q^0, \dots, z_q^{N-1}]^T$ as

$$\mathbf{z}_q \triangleq \mathbf{F}^H(\mathbf{x}_q \odot \mathbf{x}_q^*) =$$

$$[R_q(0), R_q(1) + R_q(1-N), \dots, R_q(N-1) + R_q(-1)]^T \quad (11)$$

where $z_q^0 = R_q(0)$ and $z_q^k = R_q(k) + R_q(k-N)$ for $1 \leq k \leq N-1$.

According to Parseval's theorem, the first term of Eq. (8) can be given by

$$\sum_{q=1}^Q \sum_{k=0}^{N-1} |x_q^k|^4 = \sum_{q=1}^Q \sum_{k=0}^{N-1} |z_q^k|^2 \quad (12)$$

Let $R_q(k) \triangleq a_k e^{j(\alpha_k - 2\pi\mu k/N)}$, $R_q(k-N) \triangleq b_k e^{j(\beta_k - 2\pi\mu(k-N)/N)}$, and $\theta_k = \beta_k - \alpha_k$, where a_k and b_k are positive. a_k , b_k , α_k , and β_k are all independent of μ . Thus,

$$|z_q^k|^2 = a_k^2 + b_k^2 + 2a_k b_k \cos(2\pi\mu + \theta_k) \quad (13)$$

Then Eq. (12) can be written as

$$\sum_{q=1}^Q \sum_{k=0}^{N-1} |x_q^k|^4 = \sum_{q=1}^Q \left[|R_q(0)|^2 + \sum_{k=1}^{N-1} (a_k^2 + b_k^2 + 2a_k b_k \cos(2\pi\mu + \theta_k)) \right] = T_1 \cos(2\pi\mu + T_2) + T_3 \quad (14)$$

where T_1 , T_2 , and T_3 are independent of μ . The last equation in Eq. (14) results because the sum of

various cosine functions with the same period is also a cosine function.

Using Eqs. (9) and (11), the second term of Eq. (8) becomes

$$\begin{aligned} & \sum_{q=1}^Q \sum_{k=0}^{N-1} |x_q^k|^2 |x_q^{k+1}|^2 = \\ & \sum_{q=1}^Q \sum_{k=0}^{N-1} \frac{1}{\sqrt{N}} \sum_{n_1=0}^{N-1} z_q^{n_1} e^{-j2\pi n_1 k/N} \frac{1}{\sqrt{N}} \sum_{n_2=0}^{N-1} z_q^{n_2} e^{-j2\pi n_2 (k+1)/N} = \\ & \frac{1}{N} \sum_{q=1}^Q \sum_{n_1, n_2=0}^{N-1} z_q^{n_1} z_q^{n_2} e^{-j2\pi n_2/N} \left(\sum_{k=0}^{N-1} e^{-j2\pi k(n_1+n_2)/N} \right) = \\ & \sum_{q=1}^Q \left(R_q^2(0) + \sum_{n_1=1}^{N-1} z_q^{n_1} z_q^{-n_1} e^{j2\pi n_1/N} \right) \end{aligned} \quad (15)$$

From Eq. (10), $R_q(-k) = R_q^*(k)$. Therefore, $z_q^{N-k} = z_q^{k*}$.

Utilizing this equation and Eq. (13), Eq. (15) can be further written as

$$\begin{aligned} & \sum_{q=1}^Q \sum_{k=0}^{N-1} |x_q^k|^2 |x_q^{k+1}|^2 = \sum_{q=1}^Q \left(R_q^2(0) + \sum_{k=1}^{N-1} |z_q^k|^2 e^{j2\pi k/N} \right) = \\ & \sum_{q=1}^Q \left(R_q^2(0) + \sum_{k=1}^{N-1} (a_k^2 + b_k^2 + 2a_k b_k \cos(2\pi\mu + \theta_k)) e^{j2\pi k/N} \right) = \\ & T_4 \cos(2\pi\mu + T_5) + T_6 \end{aligned} \quad (16)$$

where T_4 , T_5 , and T_6 are independent of μ . The last equation in Eq. (16) comes from the fact that

$$\sum_{q=1}^Q \sum_{k=0}^{N-1} |x_q^k|^2 |x_q^{k+1}|^2 \text{ is a real value and the sum of various cosine functions with the same period is also a cosine function.}$$

Combining Eqs. (14) and (16), the result of Eq. (8) can be obtained as

$$J_{\text{PSA}}(\mu) = A \cos(2\pi\mu + B) + C \quad (17)$$

where A , B , and C are independent of μ . Although Eq. (17) was derived for $M=1$, it is also true for $M>1$. Thus, the power spectrum analysis cost function has the form of a cosine which suggests the existence of a closed-form CFO estimate. By evaluating $J_{\text{PSA}}(\mu)$ at three selected points, such as 0, $-1/4$, $-1/2$, the CFO estimate, $\hat{\varepsilon}$, can be formulated as

$$\hat{\varepsilon} = \begin{cases} -\frac{1}{2\pi} \tan^{-1}(f/g), & g \leq 0; \\ \frac{1}{2} - \frac{1}{2\pi} \tan^{-1}(f/g), & g > 0 \text{ and } f > 0; \\ -\frac{1}{2} - \frac{1}{2\pi} \tan^{-1}(f/g), & g > 0 \text{ and } f \leq 0 \end{cases} \quad (18)$$

where $f = J_{\text{PSA}}(-1/4) - \frac{J_{\text{PSA}}(0) + J_{\text{PSA}}(-1/2)}{2}$ and

$g = J_{\text{PSA}}(0) - \frac{J_{\text{PSA}}(0) + J_{\text{PSA}}(-1/2)}{2}$. Thus, the computations for this method are quite simple.

3 Simulation Results

Simulation results are presented to demonstrate the efficiency of the PSA method. The MIMO OFDM system employs a quaternary phase-shift keying constellation with 64 subcarriers. A Rayleigh fading channel is used with the power delay profile given by

$$E(|h_{pq}^l(i)|^2) = e^{-l/3} \sum_{n=0}^4 e^{-n/3}, \quad l = 0, 1, \dots, 4.$$

The CP length is set to 4 and the CFO is equal to 0.1. All the results are averaged over 5000 Monte Carlo trials.

The mean square errors (MSEs) of the current method are compared with the results for the CP^[8] method and the Kurtosis^[13] method in Fig. 3 for various antennas. With a single transmit antenna, the current method has significantly better estimates than the CP and Kurtosis methods. With multiple transmit antennas, the performance of the current method is almost the same as that of the Kurtosis method. With multiple transmit antennas, the received signal is the sum of the signals from all the transmit antennas, so the power spectrum of the received signal is much more complicated than with a single antenna. Hence, the property of the smooth power spectrum of the received signal is less obvious than for a single antenna. As shown in Fig. 3, the current PSA method is much better in SISO OFDM systems than in MIMO OFDM systems.

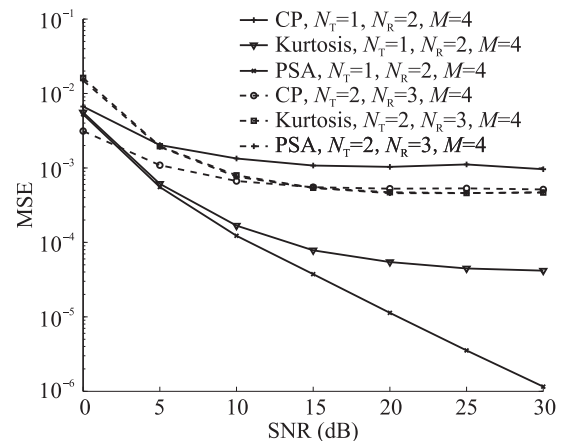


Fig. 3 MSEs of the CP, Kurtosis, and PSA methods

4 Conclusions

This paper presents a simple CFO estimation method based on an analysis of the power spectrum of the received signal for MIMO OFDM systems using a closed-form estimate of the CFO. This power spectrum analysis method significantly outperforms the CP and Kurtosis methods with just a single transmit antenna. The performances of the power spectrum analysis and Kurtosis methods are nearly the same with multiple transmit antennas.

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