

SIMULATION OF INTEGRATING ZERO CROSSING AND QUASISYNCHRONOUS SAMPLE INTERPOLATION IN MATLAB ENVIRONMENT

V. Backmutsky, M. Gankin, E. Kolmakov, G. Vaisman

Center for Technological Education, Golomb Street 52, Holon 58102, Israel

ABSTRACT

Simulation results of Integrating Zero Crossing (IZC) and Conventional Zero Crossing (CZC) with its modifications (including Level Zero Crossing-LZC) obtained in Matlab environment are given. Affects of different methods of numerical differentiation and integration as well as instrumental errors (including round-off and other dynamic errors) of Sampling and Hold (S&H) and A/D - Conversion (ADC) devices are investigated. Simulation results of Quasynchronous Sample Interpolation (QSI), based on the IZC, and Discrete or Fast Fourier Transform (named shortly Conventional Fourier Transform - CFT) also obtained in Matlab environment are described. Leakage errors accompanied with CFT estimation of harmonic magnitudes are especially investigated. General considerations concerning further application of above methods (IZC, QSI) in real-time are discussed.

1. INTRODUCTION

Most Power System applications and many other industrial applications of Digital Signal Processing (DSP) are based on the precise measurement of the actual power-line frequency.

The variable fundamental frequency is the most important characteristic of power system behavior, and its accurate estimation and interpretation is a challenging theoretical and practical problem [1]. Deviation of the power-line frequency from its stationary value is a measure of the imbalance between the load and generation. Therefore, it gives an oblique description of all of the interactions between consumers and suppliers in a power system.

Generally, there are the average drift of the system frequency and superimposed frequency oscillations. In compact or local power system the average behavior of the frequency is dominant, but its oscillations are also

informative from the viewpoint of power supply quality and its influence on the precise measuring instruments or computer systems. Regular power-line frequency variations are 0.01 - 0.1%, but sometimes, under emergency conditions, they may reach 1 - 2% and more.

There are several DSP methods for tracking and estimating the actual power-line frequency: Discrete and Fast Fourier Transforms (DFT and FFT) of the power-line voltage [2], Kalman filtering [3], the least squares error technique [4], the bilinear form approach [5], Newton's iterative procedure [6], the orthogonal component decomposition technique [7], etc.

Some of these methods and their modifications are used especially for frequency oscillation analysis with selective description of the average and oscillating behavior [8], as well as for distinguishment between long-term and short-term averaging [9]. The disadvantages of these methods are their sensitivity to power-line distortions and the round-off errors of A/D-converters, as well as their limited dynamic performance and computational complexity.

Recently some combined methods have been proposed for their improvement: 1) combining conventional zero-crossing, DFT (FFT) or synchronous demodulation with a polynomial fitting technique [10]; 2) combining conventional anti-aliasing prefiltering prior to the sample-and-hold action with improved synchronous demodulation (by the Hilbert transform), or conventional DFT (FFT) with sample interpolation over the actual or predicted power-line period [11].

As to the zero-crossing technique, its combination with sample averaging is more successful than its polynomial fitting extension and this simple idea has been realized in Integrating Zero-Crossing (IZC) [12] which is presented here in the wider context of Data Acquisition applications (some of them are described in [13]). Concerning the DFT (FFT) technique, it should be noted that power-line frequency deviations lead to desynchronization with the sample frequency, and result in significant errors in estimating DFT (FFT) parameters,

due to the well-known "leakage effect". To decrease such errors, some special windows and interpolation algorithms have been proposed [14] which, however, are very complicated and do not eliminate all the errors. In particular, using advanced windows, while reducing systematic errors, increase DFT (FFT) random errors caused by various noise and signal distortions. Because of this, a DFT (FFT) analysis has recently been proposed which is based on the synchronous sampling technique [15] and uses actual frequency tracking by Phase - Locked - Loop (PLL) and frequency multiplication. Unfortunately, the dynamic properties of PLL for relatively low power - line frequencies seem to be far from perfect. Therefore, it is desirable to find a faster quasisynchronous sampling technique. One possible solution to the problem is the Quasisynchronous Sample Interpolation (QSI), described below.

The main Data Acquisition application of the combined IZC/QSI method is power system monitoring and control including precise measurement of voltage, current, power and other AC parameters under dynamic conditions. Furthermore, direct DC - measurement application of IZC in Digital Multimeters and Data Acquisition Systems (DMM, DAS) with different transducers and a basic A/D - converter of the weight integrating type [16, 17] gives an essential improvement in their power - line related noise rejection. This means that by IZC, developed at first for power system applications, a high dynamic precision of different DC and AC instruments may be achieved.

2. THEORETICAL CONSIDERATIONS

If the input voltage (from the power - line) is

$$u_p = U_p \sin(2\pi f_p t + \psi), \quad (1)$$

the corresponding output value of the A/D - converter is

$$u_p(i) = \text{integer}[u(iT_s) / U_b], \quad (2)$$

where $i = 1, \dots, k$ are the numbers of successive samples through the period $T_p = 1/f_p$; f_p is the estimated power - line frequency; n is the number of bits of the A/D - converter, including the first bit for the sign; T_s is the sample period; $U_b = U_p/2^{n-1}$ is the voltage value of the least significant bit (LSB) of the A/D - converter.

The round - off error in each A/D - converter estimate is:

$$e(i) = u_p(iT_s) - U_b u_p(i). \quad (3)$$

This error is a random variable uniformly distributed on the segment $[0, U_b]$. If we use the n - bit A/D - converter, the relative voltage estimation error is

$$\delta U_p = e(i) / U_p \leq U_b / U_p = 1/2^{n-1},$$

i.e. less than 0.05% for $n = 12$. It can be shown that, by linear interpolation of samples $u_p(i)$ adjacent to a zero-crossing of u_p , the actual power-line frequency f_p can be estimated with a relative error $\delta f_p \leq 1/\pi \cdot 2^n$, i.e. about 0.01% or 100 ppm for $n = 12$ [12]. In practice, this precision is not attainable because of various noise and signal distortions.

The following study shows that Integrating Zero Crossing makes it possible to decrease the frequency estimation error to 0.001 - 0.003% or 10 - 30 ppm by using the same A/D - converter ($n=12$) and real signal distortions.

2.1 Integrating Zero - Crossing (IZC)

In this method we produce a digital sum which is close to the integral of the input voltage and use this sum for further processing. If the input voltage is sinusoidal, this sum is also sinusoidal with the same period, and its zero-crossing (with interpolation) makes it possible to determine the required period of the power - line voltage T_p with a smaller error due to smoothing round - off errors and other distortions. In the rectangle rule approximation we use an algebraic sum of samples:

$$S_k = \sum_{i=0}^K u(t_i) = \sum_{i=0}^k U_p \cdot \sin(2\pi f_p i T_s + \psi) \\ = \frac{U_p \sin[(k+1)\pi f_p T_s] \sin(k\pi f_p T_s + \psi)}{\sin(\pi f_p T_s)}. \quad (4)$$

This sum is close to zero, if

$$k\pi f_p T_s \approx \pi. \quad (5)$$

From two adjacent values of this sum,

$$S_k < 0, S_{k+1} > 0,$$

we can determine the floating point increment to k by interpolation:

$$\Delta k = S_k / (S_k - S_{k+1}), \quad (6)$$

and finally,

$$f_p = 1 / (k + \Delta k) T_s. \quad (7)$$

By using the linear interpolation between neighboring sum values on different sides of zero level, we estimate the required period $T_p = 1/f_p$ with a time error:

$$\Delta T_p = 0,5 T_s / (S_{k+1} - S_k) = 0,5 T_s / u(t_{k+1}). \quad (8)$$

To achieve the highest sensibility, we choose the initial point of summation to be close to an extremum of the input signal ($\psi \approx \pi/2$ or $T_o = T_{po}/4$, where T_{po} is the nominal value of T_p), thus causing the final point $u(t_{k+1})$ to be close to the extremal value which is usually chosen to be close to 2^{-n-1} (in terms of LSB). Therefore, the relative error is

$$\delta f_p \leq (1/2^n) \cdot (f_p / f_s). \quad (9)$$

This means that for $n = 12$ (the 12-bit AD-converter) and relatively low sampling rate ($f_p/f_s = 1/64$), even the peak-to-peak frequency error estimate ($2 \delta f_p$) obtained by IZC does not exceed 0.001% or 10 ppm.

2.2 Quasisynchronous Sample Interpolation (QSI)

There are two approaches to the second problem, that is, calculating the magnitudes of consecutive harmonics of the actual power-line voltage:

1. Global interpolation (over all of the samples within the actual period) and
2. Local interpolation (using only the last two samples). Let the input signal be of the form:

$$u(t) = \sum_{k=1}^M u_k \cdot \sin(k 2\pi f t + \psi_k), \quad (10)$$

where u_k are the magnitudes of harmonics and f is the fundamental frequency obtained by the IZC.

The initial samples $u(t_i)$ were taken at the moments $t_i = i/f_o N$, where f_o is the nominal value of f . To obtain the actual value of f with no "leakage effect" the samples $u(t'_i)$ are needed, where $t'_i = i/fN$, $i = 0, 1, N$; thus, u_k can be approximately calculated via DFT:

$$u_k \approx (2/N) \sum_{i=1}^M u(t'_i) \exp(-jk\pi f t'_i). \quad (11)$$

By global linear interpolation the (t'_i) values in (11) can be calculated as follows:

$$u(t'_i) = u(t_i) + [u(t_{i+1}) - u(t_i)] \cdot (t'_i - t_i) / T_s, \quad (12)$$

where $T_s = 1/f_s = 1/f_o N$, $t_i < t'_i < t_{i+1}$.

By local linear interpolation, the Fourier coefficients can be calculated via the approximate integration formula through taking into account all the initial samples $u(t_i)$ within the actual period $T = 1/f$ of the input voltage $u(t)$ except the last one which is calculated by linear interpolation over the preceding and following initial samples:

$$u_k \approx (2T_s / T) \left\{ \sum_{i=1}^{N_1} [u(t_i) \exp(-jk2\pi f t_i)] + d' u'(T) \exp(-jk2\pi f t_i) \right\}, \quad (13)$$

where

$$\begin{aligned} t_{N_1} < T < t_{N_2}; t_{N_1} = N_1 \cdot T_s; t_{N_2} = (N_1 + 1) T_s; \\ u'(T) &= [u(t_{N_1})(t_{N_2} - T) + u(t_{N_2})(T - t_{N_1})] / T_s; \\ d' &= (T - t_{N_1}) / T_s. \end{aligned} \quad (14)$$

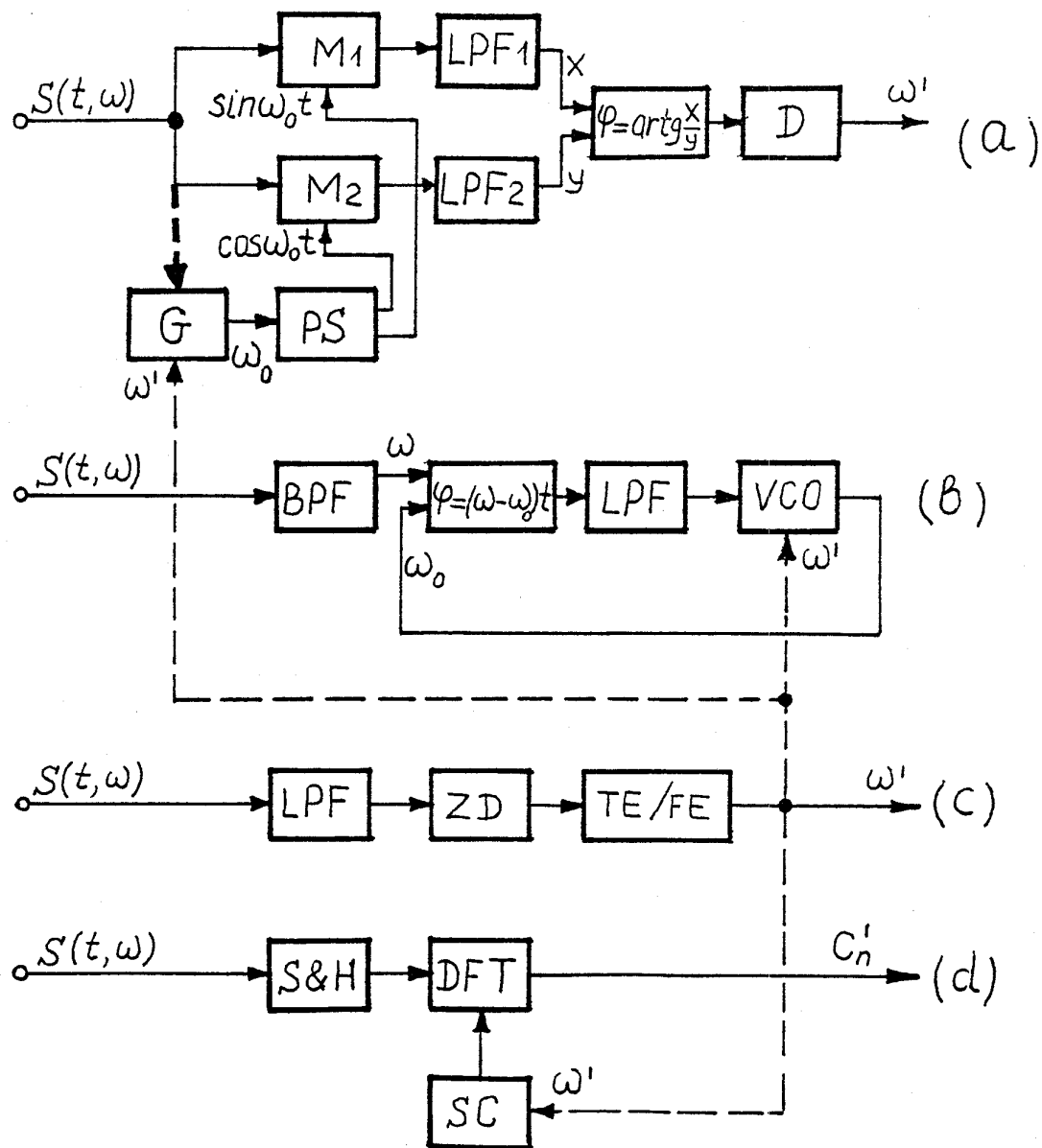
3. MATLAB IMPLEMENTATION OF IZC, QSI

Matlab programs for IZC, QSI implementation are: Basic Matlab version 5.1; Simulink; Signal Processing Toolbox, DSP Blockset Toolbox; for further realization Real-Time Workshop can be used. In the scope of IZC simulation, IZC algorithm is compared with Conventional Zero-Crossing-CZC (including level-crossing with local averaging around zero crossing area). Numerical integration with rectangle and trapezoidal approximation is considered. Round-off and other errors of A/D-conversion and S&H parameters influence are investigated. Affects of high harmonic distortions of the analyzed signal are also taken into account. Concerning dynamic changes of the actual power-line frequency some predictive estimations, for example - based on Least Mean Square prediction, are investigated.

In general, precision of tracking, estimation and short-term prediction of the actual power-line frequency obtained by IZC method is about 0.002 - 0.005%. More precisely, IZC simulation by Matlab shows that this accuracy (0.002-0.005%) is achieved by signal distortion up to 10% for second and third harmonics and up to 0.1% of noise level and also by prediction through (10-15) periods and approximation of frequency transient with 5-order polynomial. Tables and diagrams, which illustrate this estimation, will be given in oral presentation of our lecture.

In the scope of QSI simulation two basic methods of sample interpolation (global - over all of the samples within the actual period, local - using only the last two samples) are compared, as well as different interpolation forms (linear, quadratic). It is shown that for practically used numbers of samples (more than 128) and numbers of harmonics (up to 10) local linear interpolation gives an acceptable result - errors of magnitude and effective value estimation about (0.03 - 0.05)%.

Some considerations about IZC, QSI realisation are discussed, as well as their interconnections with other DSP-methods (see in the Figure).



Interconnections of main DSP-methods:

a - Synchronous Filtering; b - Phase Locked Loop (PLL);
 c - Integrating Zero Crossing (IZC); d - Quasisynchronous
 Sample Interpolation (QSI).

4. CONCLUSION

Simulation results of IZC, QSI methods by Matlab confirm that they give an accurate (about 0.002 - 0.005% for frequency and about 0.03 - 0.05% for magnitude) tracking and estimation under static and dynamic conditions.

Combined use of IZC, QSI allows to eliminate or essentially decrease aliasing and leakage influence. This result is achieved by other methods (for example, above mentioned orthogonal components technique and CFT) only by using additional synchronization (for example, by PLL) with more complicated realization and greater dynamic errors. Therefore IZC, QSI real - time implementation (by Matlab) seems to be very promising and deserving further investigation.

5. REFERENCES

- [1] B.Boashash, Estimating and interpreting the instantaneous frequency of a signal, *Proc. of the IEEE* 80 (4) (1992), 520 - 538.
- [2] A.G. Phadke, J.S. Thorp and M.G. Adamiak, A new measurement technique for tracking voltage phasors, local system frequency and rate of change of frequency, *IEEE Trans. Power Appar. Syst.* 102 (1983), 1025 - 1039.
- [3] A.A. Girgis and T L. Hwang, Optimal estimation of voltage phasors and frequency deviation using linear and non - linear Kalman filtering : theory and limitations, *IEEE Trans. Power Appar. Syst.* 103 (1984), 2943 - 2949.
- [4] M.S. Sachdev and M.M. Giray, A least error squares technique for determining power system frequency, *IEEE Trans. Power Appar. Syst.* 104 (1985), 437 - 443.
- [5] M. Kezunovic, P. Spasojevic and B. Peruncic, New digital signal processing algorithms for frequency deviation measurement, *IEEE Trans. Power Delivery* 7 (3) (1992), 1563 - 1573.
- [6] V.V.Terzija, M.B. Djuric and B.D. Kovacevic, Voltage phasor and local system frequency estimation using Newton type algorithm, *IEEE Trans. Power Delivery* 9 (3) (1994), 1368 - 1374.
- [7] P.J. Moore, R.D. Carranza and A.T. Johns, A new numeric technique for high - speed evaluation of power system frequency, *IEE Proc.- Gener., Transm., Distrib.* 141 (5) (1994), 528 - 536.
- [8] V.Eckhardt, P. Hippe and G. Hasemann, Dynamic measuring of frequency and frequency oscillations in multiphase power systems, *IEEE Trans. Power Delivery* 4 (13) (1989), 95 - 102.
- [9] V.Backmutsky, A. Shenkman, B. Shklyar, V. Zmudikov, D.Kottick and M.Blau, A new DSP - method for frequency average estimation in power systems, *Electric Machines and Power Systems* 23 (1996), 785 - 800.
- [10] M.M. Begovic, P.M. Djuric, S. Dunlap and A.G. Phadke, Frequency tracking in power networks in the presence of harmonics, *IEEE.Trans. Power Delivery* 8 (2) (1993), 481 - 486.
- [11] V.Backmutsky, D. Kottick, B. Shklyar, M.Blau and V.Zmudikov, Comparison of methods and algorithms for accurate frequency trend estimation following emergency transients in power systems, *Proc. of Intern. Power Conf. APT'93, Athens*, 1 (1993), 271 - 274.
- [12] V.Backmutsky and V.Zmudikov, Accurate frequency estimation in power systems by DSP, *Proc. of the 18th IEEE Conf. In Israel, Tel-Aviv*, 5.2.4. (1995), 1 - 5.
- [13] V.Backmutsky and V.Zmudikov, A DSP and Data Acquisition method for application in power systems with variable frequency, *Proc. of the 6th Intern. Conf. on Signal Processing Application and Technology ICSPAT'95 and 1st Ann. Data Acquisition Conf., Boston, USA* (1995), 415 - 419.
- [14] G.Andria, M.Savino and A.Trotta, Windows and interpolation algorithms to improve electrical measurement accuracy, *IEEE Trans.Instrum. Measur.* 38 (4) (1989), 856 - 863.
- [15] A.Ferrero and R.Ottoboni, High accuracy Fourier analysis based on synchronous sampling techniques, *IEEE Trans. Instrum. Measur.* 41 (6) (1992), 780 - 785.
- [16] V.Backmutsky and V.Zmudikov, Simple weight - integrating A/D converters with high noise rejection, *IEEE Trans. Instrum. Measur.* 43 (6) (1994), 912 - 917.
- [17] V.Backmutsky and V.Zmudikov, A simple A/D - converter of weight integration type and some ratio transducers for cooperative application in Digital Multimeters and Data Acquisition Systems, *Proc. of the Intern. Conf. on Industr. Electronics, Control, Instrumentation and Automation, Nov. 1992, San Diego, USA*, 3 (1992), 1577 - 1581.